Integration of GeoGebra-Designed Models in Constructivist Teaching and Learning Geometry

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Kharika Devi Parajuli[1], Bidya Nath Koirala[2]
[1]Ph.D in Mathematics Education, Kathmandu
[2] Professor Ph.D. Central Department of Education, Kathmandu
kharikaparajuli@gmail.com, koiralabidyanath@gmail.com

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Abstract

The main purpose of this research was to improve geometry through constructivist teaching and learning and to explore the students' experiences in learning using GeoGebra-designed modules. The design of this study was a teaching experiment using a qualitative research method. Data was collected through interviews, written text, participant observation, field notes, reflection notes, and field photography. Ninth-grade students at a private school in Pokhara were the research participants. The data was coded into different meaningful themes. The result shows that use of GeoGebra models on triangle and parallel lines properties would be helpful not only in the development of teachers 'efficiency in the process of teaching but also help students' learning cultures through visualization, demonstration, construction, and verification fostering a deeper sense of observation with proper reasoning. Overall, students showed positive and better learning experiences towards learning geometry lines, angles and triangles properties verification with conceptualization through GeoGebra design models. The findings of this study on incorporating GeoGebra developed models into geometry instruction will be beneficial to students, readers, trainers, researchers, and educators to learn and teach geometry more successfully with enthusiasm.

Keywords: GeoGebra, Geometry Triangles, Visualization, Teaching and Learning

Introduction

Mathematics knowledge in general and geometrical knowledge in particular are considered core skills in human life (Baah-Duodu et al., 2020). Geometry is one of the important branches of mathematics that emphasizes children's geometric intuition, knowledge, and skills to explore, think, use logic, and problem-solve. Its goal is to develop students' special geometrical thinking and reasoning skills through dynamic visualization, observation, and measurement with logic (Bergström, 2023; NCTM 1991, 2006) and make students able to cope with the new situation (Herawati, 2010; Mukamba & Makamure, 2020). But the irony is that teachers teach bookish facts, concepts, and theorems through meaningless rote.

School geometry is Euclid geometry. It studies the measurement of shapes, sizes, positions, angles, and dimensions. It deals with properties, relations, points, lines, surfaces, solids, and higher analogs (Oladosu, 2014). This geometry has helped verify relationships, logic, and reasons in our daily activities (Uwurukundo et al., 2022). But studies like TIMSS 2015 (Swedish National Agency for Education, 2016), students' results in nationwide tests (Nordberg, 2019), and Nepal's NEB result of SEE 2019–2022) showed that very poor, which has been considered a difficult subject. Idris's (2006) and Küçük & Gün, (2023) observation that geometry is a visualization subject gives a clue to teaching geometry visually. In other words, we have to change what Belbase et al. (2020) said about the spoon-feeding approach to teaching.

Today's students in the twenty-first century are technology friendly. They prefer to learn visually. The use of GeoGebra in teaching and learning can be one of the ways to visualize geometry (Tambunan & Syahputra, 2023) It can be an effective tool to teach geometry visually (Machaba & Bedada, 2020; Tambunan & Syahputra, 2023). Without the use of GeoGebra, it may not be possible to bring about changes in the mindset that math is hard and abstract (Chai et al., 2011; Wachira et al., 2008).

Mathematics educators have developed lots of free digital mathematics software for visualized teaching and learning. They are GeoGebra, MATLAB, Geometer's Sketchpad, Mathematica, etc. (Examples: Bekene, 2020; Dahal et al., 2019; Mukamba & Makamure, 2020). This software has made a great contribution to effective teaching and self-learning (Tambunan & Syahputra, 2023).

Statement of The Problem

Nepali teachers are very less used to visualize the geometry class, partly because they did not get opportunities for it and partly because they are not trained to do so (Guven & Kosa, 2008). It is one of the reasons why learning geometry is not regarded as easy in the absence of visualization (Antohe, 2010; Zengin et al., 2012). Here, the burning issue is finding ways to help teachers and students visualize geometry (Dahal& Dahal, 2015). Our experience showed that only those we wished to use could produce the desired success (Antohe, 2010). The next problem is to find out who is desirous of using GeoGebra software for teaching (Bekene & Machaba, 2022).

2. Review Literature

Role of Mathematics Software in Geometry Teaching and Learning

There are several studies we reviewed below on the use of GeoGebra in teaching and learning mathematics; however, the results may not be consistent. Some literature claims that the GeoGebra provides a visual and effective learning environment. If students have opportunities to develop knowledge critically, they can gravitate toward a variety of tasks, such as research projects, along with mathematical skills and competencies, in order to obtain knowledge regarding their practical experiences. According to Ottevanger et al. (2007), preparing students to encounter challenges in their future endeavors, technology-based mathematics teaching and learning is also needed by Nepalese schoolchildren. As they suggested, no doubt based on our long years' experiences as'school-level mathematics teachers, educators, trainers, and researchers, those teachers who have been using such software tools in mathematics instruction find that it helps to improve the efficiency of teachers' teaching and students' learning. Software-based models allow both teachers and students to accurately and interactively visualize the work. It also affects students' learning with solutions using mathematical schemes and vital information based not on spoon-fed information claimed by Belbase (2016) and Foley & Ojeda (2007). However, Ziatdinov & Valles, (2022) research result showed that the use of GeoGebra has the effectiveness of its three features like modeling, visualization, and programming in science, engineering, and mathematics.

From the above review, it was figured out that the existing research on the use of GeoGebra so far studied effectiveness in terms of learning outcomes and implementation through simulations during the teaching process but could not find a study that has reported secondary level students 'experiences on learning geometry triangles and line and angle properties verification through GeoGebra visualization modules. Also, most of the students are not benefiting from the opportunities for learning these properties from the root level through GeoGebra-designed models yet in the Nepalese classroom. Therefore, we did this work blending instruction and dynamic software GeoGebra models.

Why GeoGebra in Teaching and Learning in School Mathematics

GeoGebra is a dynamic mathematics software package. It is an interactive geometry, algebra, statistics, and calculus application designed It is available both online and offline in over 100 languages. This tool is extended to use at the school level in mathematics teaching and learning and at the university level in different disciplines such as geometry, algebra, and calculus. The goal of its use is to make mathematical concepts clearer and easier for students to grasp. It provides dynamic associations in mathematics (Ziatdinov & Valles, 2022). It was developed as part of the authors' (2007) master's thesis in computer science and mathematics education at the University of Salzburg in Austria. Markus developed it in 2001/2002. According to Hohenwarter et al. (2007), GeoGebra is designed to enable proactive teaching. Dynamic Geometry Software (DGS) is a tool that has been used to create geometric shapes of 2D and 3D such as lines, points, experimental verification, cylinders, cones, pyramids and other computer-aided designs. It can also be used to create symbolic and graphical elements on two and three-dimensional geometricalfigures. Due to such features, it is extremely useful at the school level to improve both teaching and learning mathematics by allowing students to visualize concepts. According to Ziatdinov and Valles (2022), it additionally provides dynamic interconnections in mathematics. In the same spirit, Hohenwarter & Jones (2007) and other authors (Mukamba & Makamure, 2020; Zengin et al., 2012) suggested that it can be employed by educators, teachers, and experts for successful teaching lessons in a broad range of subjects' areas, designing and demonstrating basic to more complicated ideas to encourage students for mathematical experiments and discoveries (Dahal et al., 2019).

They claimed that GeoGebra's first and foremost goal is to make mathematical concepts clearer and easier for students to grasp through experiments and practical through teachers' support. Likewise, Ziyatdinov et al. (2023) and other scholars (Mukamba & Makamure, 2020; Mwingirwa and Miheso, 2016; Zengin et al., 2012) asserted that teachers, teachers, trainers, and experts could utilize it to facilitate successful educational sessions in diverse fields, building and depicting basic to sophisticated ideas to encourage mathematical explorations and revelations (Dahal et al., 2019).Because of its key features, modeling, visualization, and programming in mathematics, its users also feel that its integration is quite essential in the school mathematics curricula to improve the efficiency of teachers' teaching and students' learning processes, as suggested by Ziatdinov and Valles (2022). As the past literature claimed, our experiences as long-term GeoGebra users as teachers and trainers also feel quite essential to the schoolchildren's ability to improve mathematics in their' learning processes and to improve teachers the efficiency in teaching. Being such a feature, it is useful to improve and enrich both teaching and learning mathematics by allowing students to visualize concept building. Supporting its features, Ziatdinov and Valles (2022) claimed that GeoGebra's first and foremost goal is to make mathematical concepts clearer and easier for students to grasp through experiments and practicals with teachers' support. Similarly, research (Mukamba & Makamure, 2020; Zengin et al., 2012) along with additional literature argue that it can be applied by educators to create and illustrate basic to complex ideas to encourage mathematical experiments and discoveries (Dahal et al., 2019; Tomic, 2013).

GeoGebra is a Visual Tool for Foster Students Learning

Ziatdinov and Valles (2022) on students' perceptions regarding GeoGebra integrated Geometry teaching with models suggested that its use in the mathematics classroom was extremely essential. It allowed students to learn and build abstract concepts through visualization. In order to promote geometrical experiments and discoveries, authors like Hohenwarter & Jones (2007), Mukamba & Makamure (2020), and Zengin et al. (2012) contended that teachers can use it to create animated models that construct and illustrate simple to abstract concepts (Dahal et al., 2019). Arango et al. (2015) and Mayer (2009) argue that the traditional way of instructing and verifying the properties of geometries drawn inappropriately with fixed shapes may be dry and less effective for the students. Nobre et al. (2016) claimed that the use of GeoGebra in teaching geometry students through motivation by relating subject matters and concepts through visualization and experimental verification helps them to understand sequential concepts that help minimize students' meaningless rote. Tatar (2018) also claimed that if teachers and students are engaged in it for teaching and learning properly in the classroom, it provides significant opportunities for both students and teachers for meaningful teaching and learning and concept formation in mathematics in general and geometry triangles in particular at various levels. Seloraji & Eu (2017) found a statistically significant difference in scores among the three student ability groups. In conclusion, the study implies that using GeoGebra enhances students' performance in geometrical studies. According to Muslim et al. (2022), visual learning is a powerful tool to learn conceptual understanding and to develop skills such as observation, recognition, interpretation, perception, self-expression, and belief. These skills provide the learner with the opportunity to see the diagrams, examine them quickly, and then visually recall and interpret information, leading to comprehension and understanding of mathematical concepts in a short time (Muslim et al., 2022). Similarly, Maharjan et al. (2022) reiterated that with visual tools, learners are able to analyze concepts, make conjectures, and convey ideas to others (externalization of ideas). These attributes of visualization point to the fact that the attention of the learners is easily grabbed, hence engaging them in their learning.

However, Mukamba and Makamures (2020) suggested that for effective and quality learning in geometry through visualization, teachers need to enrich their knowledge about how to develop effective models to teach and give instruction. If only it were likely to enhance the mastery and retention of concepts. All the ideas about visualization are confirmed by Edgar Dale's (1969) theory, which theorized that people understand 20% of what they hear, 50% of what they see and hear, and 90% of what they do as a task. The use of visual technology in classrooms, therefore, seems to provide learners with clear explanations and illustrations that could help them explicitly understand mathematical concepts. However, the use of visuals is critical since most educators teach students to memorize and recall information instead of understanding concepts efficiently (Mumcu & Aktürk, 2020; Muslim et al., 2022).

Bedada and Machaba (2022) investigated the effect of GeoGebra application on students' learning of differential calculus and their perceptions regarding learning calculus with GeoGebra and the cycle model in South Africa. They found positive perceptions by the students towards the use of GeoGebra for learning differential calculus. Ziatdinov et al. (2023) have shown positive effects of GeoGebra on the efficiency and effectiveness of learning and teaching topics related to mathematics. The National Research Council (2001) argues that conceptual and procedural knowledge of mathematics are interrelated mechanisms. Software-based activities enable students to explore their ideas and discover new mathematics concepts by themselves (Guven & Kosa, 2008). This idea is consistent with Vygotsky's (1978) classical cognitive constructivist theory, which calls for a gradual transformation in perceptions of the teacher's role from an authoritative source of information (behavioral) to a guide or facilitator of the learner's self-propelled exploration (constructivist). Guven & Kosa (2008) and Mwei et al. (2012) suggest GeoGebra's capacity is for visualization and exploration. It helps reduce cognitive burden during the learning process. Ocal (2017) and Tatar (2018) investigated the effect of GeoGebra students' conceptual understanding and found improvements in learning, but they also found there was no significant difference between the procedural knowledge of the experimental group and the control group.

Purpose and Research Questions

The purpose of this research study was to use the dynamic geometry software GeoGebra models to improve the teaching and learning of geometry lines, angles, and triangle properties and to explore the students' experiences with learning through this approach. To study more systematically and completely, the following research questions work as the guideline for this study:

1. How do GeoGebra-designed models work to improve teaching and learning in the classroom?
2. How do students explore while learning concepts and verifying the geometry lines, angles, and triangle properties using these modules*?*

Theoretical Framework

This study used three theories simultaneously: cognitive theory, radical theory, and Vygotsky's (1978) social learning. These theories help us build geometry models and provide effective delivery to the students in the classroom by linking their prior knowledge of geometry concepts. Cognitive theory has been used to understand students' geometrical thought, reasoning power, logical thinking, and learning processes using GeoGebra models. We generated students' environmental knowledge and processes of knowing for the assimilation of school math. We also used this theory while the students were demonstrating, watching, observing, visualizing, touching, studying, and recalling the information (Belbase & Sanzenbacher, 2016; Zajda, 2023). Radical theory was used when the students were thinking beyond the box of their textbooks. And social learning theory was used when students were producing new knowledge on the basis of the context that they lived in. By following the concept of the above three theories, we developed three models and shared them with students through the projector and laptop in an interactive way (Ellerton & Clements, 1992). We noted that students verified the properties of lines, angles, and triangles through visualization, sharing, asking, testing, and discussing what they were previously familiar with and/or exposed to (Brandt & Chernoff, 2015). Vygotsky's (1978) learning theory concerns the fact that knowledge is socially constructed through the meaning-making process. In such a process, teachers and students are both encouraged to share the animated models in geometry, their new ideas and experiences, expand their knowledge by thinking critically, reflect upon changing ideas, and interact constructively with peers and teachers in teaching and learning. Our focus was on GeoGebra-designed models with instructed pedagogy to create a meaningful learning environment, keeping students at the center (Huang, 2002). In these pedagogical practices, learners construct, reconstruct, and deconstruct their ideas based on their own understanding, and teachers provide support and motivation by experiencing things and reflecting on those experiences from more than one perspective. Following this theory, we asked questions and checked the solutions and their learning methods. . Students visualized and dragged the animated models in seconds to form strong mental images about the properties and relations. The models helped both teachers and students have meaningful discussions in the classroom, where all the students collaborated with each other to construct knowledge in geometry line and angle relations and verified the triangle properties and concepts. We, as transformative teachers, generated students' knowledge and shared ours to advance their didactic, mathematical, and digital competencies (TPACK) (Almulla, 2023; Garcia-Lázaro and Martín-Nieto, 2023). Finally, the teaching experiment approach allowed us to develop models effectively with pre-lesson plans and classroom implementation.

Research Methods

In this study, we adopted the teaching experiment method in a qualitative way (Cobb et al., 1995; Cobb & Steffe, 2011; Steffe & Thompson, 2000). The teaching experiment method consists of a series of teaching episodes, individual interviews, observation, and reflection notes that cover one week's time. The primary purpose of using this method is to investigate "how" and "why" questions (Kang & Zhang, 2023; Mackenzie & Knipe, 2006). This teaching experiment method of research consists of teaching, learning, and research together (Komorek & Duit, 2004). The purpose of choosing this method was to know students' experiences and ways of learning (Aliyu et al., 2015). In this experiment, we first designed the three models on GeoGebra and one PowerPoint presentation, an MCQ, and simple subjective questions related to properties by taking an extra five hours' additional time. The first single model introduced the concepts of parallel lines and relationships between angles. The second model gave parallel lines and alternate angles, straight angles, and additional b) i-iv properties of the triangle. In the third one (c), we prepared to teach that the exterior angle of a triangle is equal to the sum of the opposite interior angles through animation with two sliders defined. We also shared with students the models to learn and teach all together properties. As suggested by Cobb et al. (1995), we prepared and taught a series of three teaching episodes for one week. We also formulated the teaching models and PPT for adequate illustrations in the related problems, pictures, and animations of objects to make abstract concepts easy and simple.

Study Area and Participants

The participants in this study were 38 students who are studying ninth grade with age group as 14-15 years at a private school in Pokhara, Kaski District, in the academic year 2023-2024. Among them, we selected eight students, four girls and four boys of the same class for the interview and their reflective notes who gave rich and thick information.

Process and Data Collection Tools

A teaching experiment involves a sequence of teaching episodes (Cobb & Gravemeijer, 2014; Steffe & Thompson, 2014; Steffe & Ulrich, 2020). Therefore, we divided the whole study period into three teaching episodes. We recorded the anecdotal behavior of students in general and key research participants in particular. In the middle of the presentation and demonstration of the modules, and at the end, we had made arrangements for active observation of the measurements, relationships of sides and angles, and lines. Data were collected using various tools and techniques, like responsive intuitive interactions with the whole class of students, individual interviews, observation of students' behaviors and learning activities, facial expressions, and expressing their experiences in written text. A teaching experiment was carried out to evaluate the student's learning process based on the learning objectives, particularly to understand the concepts of triangle properties experimentally and theoretically to establish students' inductive reasoning, which involved the activities of visualization, exploration, engagement, and observation. The teaching experiment involved 38 students of different ability levels in the regular classroom.

Retrospective analysis is a critical part of teaching experimentation methodology. It is even more laborious and intensive (Steffe & Thompson, 2000). As a researcher and teacher, we analyzed the collected information retrospectively. We reviewed the teaching experiment method critically by analyzing the emotions, activities, and behaviors shown by research participants during the research study. Then, we analyzed the activities of the classes by watching the video recordings as well. We took help from the teacher, who has a degree from the same school, in the experiment classes. The actions included participants' body gestures, facial expressions and their behaviors, classwork, and figure-drawing activities during classes. In addition to that, we also wrote down the information and the students' handwritten text. Similarly, we transcribed interviews with the students. Then, we developed a meaningful theme from the research findings. After analyzing and interpreting the collected data, we finally discussed the findings of the study.

The Process of Developing GeoGebra Model on Geometry Triangles and lines and Angles

We prepared models focused on teaching and learning the following properties: a) Relation between lines, alternate angles, corresponding angles, and the sum of co-interior angles when the lines are cut by transversal; b) Concepts and verified multi-properties of triangles, like i) sum of angles of a triangle is 180; ii) if the triangle is isosceles if and only if base angles are equal; iii) each base angle of an isosceles right angled is 45^0; iv) straight angle be 1800; v) side opposite to the greater angle is longest and side opposite to the similar angle is shortest and vice versa; vi) the sum of any two sides of a triangle is greater than the third side; and c) the exterior angle of a triangle is equal to the sum of interior opposite angles.

We constructed three colorful modules on our laptop and shared them in the classroom (see Figures 1a, 1b, and 1c below). These collaboratively prepared GeoGebra software animation models were used to teach and learn multi-contents and various properties of geometry lines, angles, and triangles (see figures 1, 2, and 3). These widely shared models were developed by students because of their heavy interactions (Cobb & Steffe, 2011). In this process, every student was able to visualize, observe, and test the properties of lines, angles, and triangles (Antohe, 2010; Zengin et al., 2012). Out of these exercises, we, the researchers, became able to ensure a dialectical interaction between the theoretical and empirical aspects of our work. (Cobb & Steffe, 2011; Norton &McCloskey, 2008).

Retrospective and Prospective Analysis Through Interpretation and Meaning-Making

The data were analyzed in retrospective and prospective ways, taking the ideas of Norton & McCloskey (2008) and Steffe and Thompson (2000). We reviewed the teaching experiment methods critically and analyzed the emotions, activities, and behaviors of students during the study. We also analyzed activities from the students' point of view (Thompson, 1979). We went through the recording images, the students' reflective notes and written text as well as our reflective notes. Finally, we derived the findings of our study and reflected them against literature and theories.

Finding and Discussion

As a result, we have GeoGebra-designed animated models to help students' learning processes. With these models, they can figure out angles, measurements, triangle properties, and their relationships by dragging each model figure within a second (see figures 1, 2, and 3). These animated dynamic design models and animated slides of short multiple-choice questions (MCQ) responses contain clear visualizations of proven properties of lines, angles, and their relationships (Figures 1, 2, and 3 are examples of developed models used by Dynamic Geometry Software).

Figure 1

Verifications of Properties of Lines and Angles Relationship Demonstrating Through GeoGebra Model

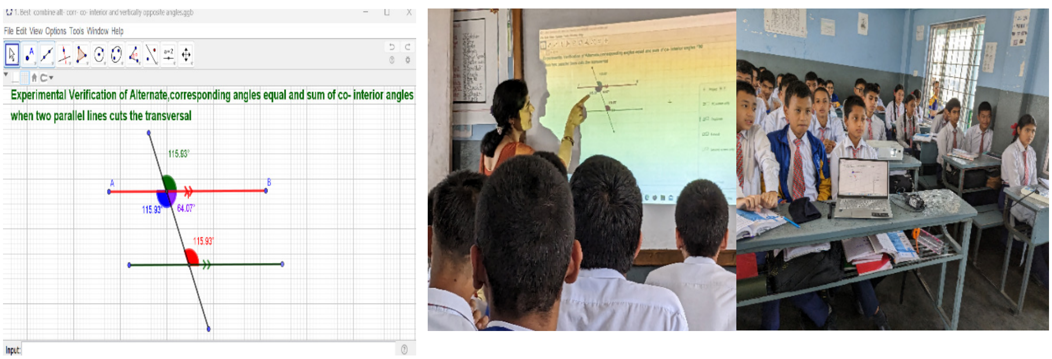


Figure 2

GeoGebra Single Model on Triangle Stringer MultiProperties

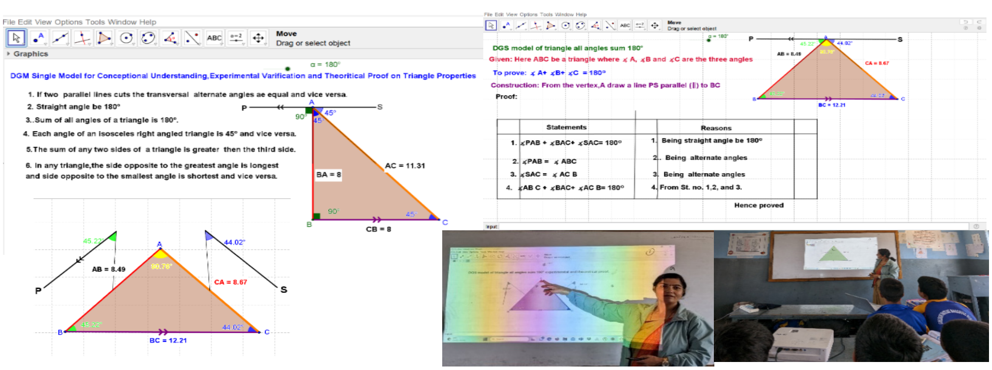
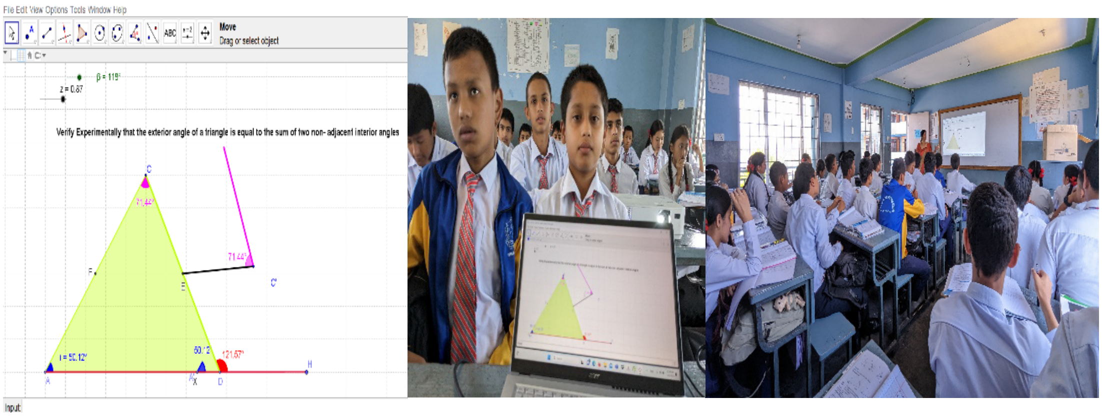


Figure 3

.GeoGebra Animated Model on Triangle Verifying Experimental and Theoretical Proof



Result, 1: GeoGebra Models Enhance Connectional Understanding

We had the experience that many students did not have the basic concepts of the properties of lines, angles, and triangles that they had read in previous classes and solved simple problems. We gave the above models to the same students and asked a number of questions. These questions and the models encouraged them to make movements, mind activity, attenuation, and light facial expressions, and they began to write their observations. Some of them started to share their understanding on the white board as well. They also mentioned an awesome teaching model with the right strategies. In their words, "We are sure that if we learned geometry in such a visual environment, obviously models would give a good impression of learning properties, and we asserted that we would learn a lot of triangle properties." They also acknowledge us by giving thanks for using it, as we did not know about it before. They further said,

> *The eye-catching animation models help our minds be active and help us develop more confidence, meaning less rote and instruction. Therefore, we can grasp property verifications very easily and will have talent too because they are clear and easy to visualize. Therefore, it helps us store and retain the properties for a longer period of time".*

A similar observation was expressed by Suryani and Rofiki (2020): digital GeoGebra integrated teaching modules on triangles can make the learning process more meaningful, effective, and interactive and help to increase the understanding of the concepts and properties, which helps learners gradually increase interest and motivation in the learning process and helps students slowly learn independently. The results of the study also showed that the teaching triangles using GeoGebra models by the teacher helped to develop students' understanding of the concepts and properties.

The students in this study stated that the visual representation of geometric triangles helped them better understand. They acquired a useful comprehension of the links between geometric lines and angles. Along with the theoretical and experimental verification of qualities, they also comprehend the ideas of triangles, alternate angles, and parallel lines. We posed and addressed numerous proving questions, along with their answers, oral testing proficiency, observations of their completed assignments, and prompt answers to inquiries in class. Every time they participated actively in GeoGebra integrated classroom activities, we observed their motivation. As per the research conducted by Arbain & Shukor (2015), Mwei et al. (2012), Saptruto et al. (2018), and Wijaya et al. (2021), it has been shown that GeoGebra integrated models facilitate teacher-student collaboration through exploration and visualization, thereby improving students' conceptual understanding and supporting constructivist learning approaches.

Result, 2: GeoGebra Single Model Strainer Many Properties Lines, Angles and Triangles

We moved the figures to show different properties, and students were surprised and motivated to learn. When we shared these developed models in the classroom by doing the dragging process on the laptop of the first author with a projector and making different shapes in seconds, they observed and verified various triangle and parallel lines and angles, and interestingly, they automatically changed the measurements of angles and sides with clear visualization. They said, "We knew the relationship between the parallel lines and angles and the six triangle properties by shifting a different size of triangle, as the demands of properties and oral presentation attracted us to learn effectively through this visualization process."

They were also of the opinion that they were, in a short period of time, able to learn theorems theoretically and experimentally. They agreed that they could learn very fast because of the eye-catching model that directly focused on the subject matter without causing any disturbance. It was not time-consuming; teachers taught many concepts and theorems in a short time. Because it was attractive, it was very easy and worthy of teaching by standing, teaching with chalk, and talking. They said they could remember very easily for a longer period of time due to visualization.

This finding supports Piaget's (1970) cognitive theory, highlighted in Resnick (2017), and constructivist theory (Steffe & Ulrich, 2014; Steffe &Ulrich, 2020), which holds that learners develop concepts initially in concrete forms before moving on to abstractions. Students encourage all educators, starting at the local level, to use technology-based resources like software. During the interview, one girl and one boy students' participants said that both can easily understand the properties and can do experimentally, theoretically, and related short exercise problems on triangles. This finding also responded to NCTM (2000), which has suggested that technology becomes one of the key tools to enhance quality teaching and learning. Similarly, it complied with Mnguni's (2014) recommendation that "geometric" be visual in nature. Finally, the result resembled what Idris (2006) concluded: that the causes of students' difficulties in geometry learning are visualization abilities and ineffective instruction with the structured nature of drawing figures (Machaba & Bedada, 2022), whose research became true in the case of this study as well.

Result, 3: GeoGebra Promotes Students for Active Learning

This study showed that students were very actively engaged in knowledge construction rather than knowledge transmutation (Bruner, 1966). They constructed objects, found solutions to their problems with correct and proper reasoning, proved the theorems, measured the angles by protractors, tabulated them, and thereby wrote their observations in their exercise book. They tested the angles and side relationships and found them to be 100 percent accurate. Because of the accurate result with accurate shapes changing in seconds, they observed and visualized directly in the student-centered learning environment. This study also paved the way that effective GeoGebra design models can be used for effective strategies for better teaching and learning to improve students level of understanding (Rholey & Picaza, 2023). According to Bruner (1985), learners would better understand abstract concepts if a differentiated learning strategy were planned and implemented according to the learner's strengths. Bruner's (1966) also stated three stages of each mode of thought: active, iconic, and symbolic. According to him, the active stage focused on physical actions: learning happens through movement or actions. This action was seen in the animated GeoGebra models. Here, we noticed that students' learning happened through images and icons. Investigating and verifying the properties is an example of an iconic stage. This stage is interpreted as observing a teacher or peer demonstration on graphs or tables in GeoGebra. Information was stored in codes or abstract symbols. Learning occurred here as well.

Result, 4: GeoGebra Models Empowers Students to Develop Reasoning Abilities and Drawing Skills

This study yielded the information that students developed reasoning. Their reasoning was: a) Why did teachers not teach us this way? b) who is to blame for making mathematics a difficult subject, as we learned it easily; c) who will be the book writer to use such a book so that students get mathematical knowledge easily. Their reasoning encouraged us to develop materials to teach geometry triangle properties by making different episodes, by giving accurate shapes, sizes, measurements, and parallel lines, by making alternate angles equal, and by verifying the properties experimentally and theoretically. It also made students enthusiastic to collect geometrical objects from their culture and learn geometry easily at home. Picaza (2023) supports this approach to students' learning through cognitive processes. This environment became a problem solver for real-life problems, creating good learning environments, and making mediators to facilitate interactive learning with students (Opit, 2012). In this activity, our role was changed from providing information to that of a facilitator, where students are encouraged to explore their ideas. We saw students' participation in learning through software models and Microsoft PowerPoint presentations like graphics, animation, sound, and text, which helped them attract their attention for meaningful learning.

Conclusions

This study was focused on improving the teaching and learning approach from a constructive perspective and exploring students' experiences with learning geometry triangle properties and lines and angles relationships through visualization of GeoGebra-design models. In this study, we used qualitative teaching experiment methodology to understand and observe students' mathematical learning realities. Similarly, we put effort into sequentially innovative, meaningful GeoGebra design models to understand students' geometrical understanding through the meaning-making process. We concluded from the students' experience, discussion, observation, and reasoning while they proved the theorems theoretically and experimentally.

We observed during the study that students were using the visualization of GeoGebra-developed models to understand numerous concepts on lines, angles, and triangle properties, along with their theoretical and experimental confirmations. This involves quickly changing the shapes and sizes, lines, angles, sides, and measurements within seconds using a dynamic GeoGebra model, which makes the mind active and facilitates meaningful interpretation during the learning process. They also seemed to have positive experiences with the GeoGebra models in classroom activities. They also claimed GeoGebra offered lots of learning and made people happy, interested, and motivated to learn geometry by constructing figures with animating and verified properties and answering MCQ questions rapidly and in a short time. Such teaching and learning activities help to develop skills such as drawing skills, thinking, analysis of figures, situations of variations of models, and evaluating properties that hold the right justifications and representations of correct lines and angles with level vertices.

The aforesaid conclusion reminded Luitel (2017), who mentioned that to motivate the students, teachers need to shift their pedagogies from traditional lecture methods to modern pedagogies such as technology-based active participation, collaboration and cooperation, etc. As reflective thinkers with transformational pedagogies instructed our activities, classroom practices, and design model, we came to believe that the GeoGebra-designed model enhanced 21st-century teaching and enabled teachers to increase work efficiency and inspire geometry learning in their students. In addition, GeoGebra can help students and teachers make mathematics classrooms interesting and authentic through a mutual teacher-student relationship (Dahal, 2013). Also, students of any level of geometrical knowledge can be encouraged to study geometry by using this application. One of the conclusions is that learning and teaching mathematics should not be purely theoretical and instructional but also involve a variety of teaching and learning strategies that involve the use of teaching aids proven to help stimulate students' interest in mathematics.

Implications

This study implies that we can offer some insights to readers, novice teachers, novice teacher trainers, and educational researchers about integrating GeoGebra in mathematics teaching and learning. No doubt, mathematics teacher(s) would highly benefit from our research process to strengthen their knowledge of integrating GeoGebra in order to focus on and improve student learning in mathematics in general and geometry in particular while teaching geometry. For those mathematics teachers and trainers who are unaware of GeoGebra, this research work will be eye-opening for them, and even for those who started their teaching career recently. Our research also indicates that teachers have a rich mastery of mathematical concepts and the interconnectedness between different representations and topics to be able to integrate GeoGebra and promote thinking. Similarly, our research will support policymakers and curriculum designers in making some provisions for ICT-integrated pedagogy for 21st-century learners.

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