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# Difficulties of Grade X students in learning Trigonometry

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#### Abstract

This paper aims to explain the difficulties of grade X students in learning trigonometry that were explored from an empirical study in the year 2017 AD. The researcher used explanatory mixed research design where 155 students of grade X were selected randomly from the four schools of Kirtipur municipality to conduct initial survey to find low achievers and most difficult content areas of trigonometry by using achievement test. Then five low achievers and respective three mathematics teachers were selected and interviewed with the help of semi-structured interview guidelines to explore what difficulties had they experienced and encountered in learning trigonometry. The results revealed that mean achievement score of students in institutional schools was greater than that of students of community schools. However, there are some common areas of difficulties of students in both types of schools for learning trigonometry including the difficulties related to memorization of facts; comprehension of new concepts; transformation of verbal problems; derivation and proofs of trigonometric relations and unfamiliarity with the solving process of problems in application level. These difficulties are due to several reasons including abstract nature of contents, poor prerequisites among students, demonstration of teacher centered instruction, and focusing on rote memorization rather than conceptual understanding. Hence, it is recommended that both students and teachers need to focus on conceptual learning rather than rote memorization for meaningful understanding in trigonometry.

*Keywords*: Learning trigonometry, remembering facts, comprehension of concepts, deriving results, applying the formulae

#### Introduction

School mathematics is becoming a fundamental course for the people living in this contemporary world. Teaching such fundamental course and finding difficulties to learn it, is an ongoing phenomenon for the educational research. The curriculum of school mathematics of Nepal, particularly at grade X, consists of several mathematical concepts including Arithmetic, Geometry, Algebra, Trigonometry, and Statistics. Among these core areas of contents, trigonometry covers 20 percentages of all contents. Trigonometry is a branch of mathematics which deals the relationships between lengths, angles, triangles and so forth. Basically, the contents in the trigonometry are based on the concepts of right angle triangle. So, learning trigonometry means to understand symbols, language, relationships, identities, equations and other verbal problems. Also, trigonometry is effective and useful for measuring the height and distance, length of shadow and height of any object from the horizontal line (Ginsburg, 2016). This implies that if school students cannot understand the language, relationships and other useful property of trigonometry ultimately a rises difficulties to learn it.

The evident of the courses of school mathematics of Nepal shows that trigonometry is one of the major content areas of mathematics at secondary level. The study of trigonometry courses is just starting from grade IX and so on for the community school of Nepal. The contents of trigonometry are taught in compulsory mathematics course to some extent and in detail in the optional courses of mathematics at grade IX of Nepal. Teaching such new contents

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at grade IX is a challenging task for the mathematics teachers. The teachers' preparations, knowledge, ideas and methods to teach trigonometry are important variables to make learning meaningful, effective and to reduce students' difficulties to learn it. Teacher's readiness and interest in teaching are also important for teaching learning activities as well as developing the positive attitude in the students towards mathematics learning (Kamber&Takaci, 2018). That means teacher need to enter the classroom with full preparation of the contents and methods to teach mathematics effectively which will develop positive attitude among students in learning mathematics. Teachers need to have capacity to show the application of the mathematics contents, particularly the application of trigonometry. For example, it is used in oceanography for calculating the heights of oceans. The sine and cosine functions are fundamental to the theory of periodic functions which describe the sound and light waves. It has also its applications in satellite systems.

On the other hand, it is expected that learning trigonometry develops the critical thinking power of students in mathematics. Acharya (2015) and Courtney (2016) stated that the study of contents in trigonometry helps students to develop the capacity of visualizing any mathematical contents differently. That means the strong knowledge of trigonometry has wider applications in learning any contents of mathematics from school to higher level. In this context how school students learn these widely useful contents (trigonometry) and what difficulties are arising for them to learn it effectively was the concern of this study. Equally, how the teachers have experienced students' difficulties in learning trigonometry was also considered to find the students difficulties in learning it.

#### **Research Objectives**

The objectives of this study were to find out most difficult content areas of trigonometry and to explore some difficulties of grade X students in learning trigonometry in the course of elective (optional) mathematics.

## Literature Review

There are several literatures which explain the student's difficulties in learning school mathematics, particularly difficulties in learning trigonometry. The study of Ghimire (2013) pointed that cultural background and participation plays the vital role in their mathematics learning. This study also revealed that low participation of Bote students has made them difficult to learn mathematics. Similarly, Bhat (2017) explained that difficulties in learning trigonometry are due to lack of group discussion, improper use of exercise books and lack of using electronic instructional materials in mathematics. Also, the lack of selecting appropriate teaching methods, fewer practices of related examples and lack of practices of project works in school geometry are the major obstacles for meaningful learning (Yadav, 2017). That means the choice of methods of teaching, assigning project works and learning materials applied in the classroom are also creating difficulties in learning mathematics.

The research of Bhatta (2017) revealed that students have faced difficulties in learning mathematics because the text books are not well planned and contents are not arranged sequentially. There is also scarcity of reference books, teachers guide and math lab in rural areas, and the curriculum is more urban oriented which affects the mathematics learning of rural students. The culture of teacher and students also plays an important role in learning mathematics. Gautam (2017) stated that there is not proper interaction between teacher and Tamang students because of their language and culture. Thus the lack of local teachers in

Tamang community and poor interpersonal relation developed among Tamang students are creating difficulties in learning school mathematics, particularly in learning trigonometry. Moreover, mathematics anxiety, negative feeling of mathematics, economic condition, their educational background, school management system, infrastructure of school are the main causes of difficulties in learning mathematics (Acharya, 2017).

On the other hand, symbols and symbolic structure had strong influence on students' choices in problem solving (Kenney, 2008). That means if students cannot understand the language, symbols and its structures then it is difficult to learn mathematics. For example, graphing calculators used at a way to abandon symbolic manipulation, although few connections were made between symbolic and graphic numerical form. Likewise, Ginsburg (1994) pointed that the development of children's construction of knowledge in the context of schooling are the product of classroom instruction, the availability in children of informal knowledge, the role of motivation, and the effects of specific interventions.

Mulwa (2015) had reviewed the literatures pertinent to difficulties encountered by students in learning and usage of mathematical terminology and concluded that students have difficulties in using mathematical terms and their related concepts when learning mathematics. Similarly, the students have faced difficulties in learning higher mathematical concepts like group theory are due to misconception, incapability of making linkage with new concepts, unable to remember mathematical terminology for long time, and lack of suitable learning resources (Bhandari, 2017). The students are becoming lazy and passive in learning mathematics in classroom because of using only lecture methods and not giving individual attention for students by teachers (Thapa, 2016), stimulating difficulties in learning mathematics at school.

The literatures reviewed above display that there are several factors which influence mathematics learning, particularly learning concepts in trigonometry including classroom environment, economic condition, peer groups discussion, pre-knowledge of subject matter, teaching methods and materials, language, culture, classroom participation and regularity. Also, the lack of sufficient learning resources, motivation and regular evaluation of students' progress are also responsible to create difficulties in learning mathematics at school. The reviews also indicate that the selection of appropriate methods in teaching and learning mathematics including the concepts of trigonometry at school level plays a vital role. The use of student centered methods for teaching and learning mathematics like problem solving, project methods, modern techniques (that is blended with ICT), discovery method, collaborative methods, and so forth and the demonstration of concrete materials related to mathematics concepts can make learning more meaningful and conceptual. That is why, this study intended to find the major difficulties in the content areas of trigonometry as experienced by grade X students and respective mathematics teachers. These difficulties might be related to nature of contents, teachers and students' activities, classroom environments, students' backgrounds and others as reflected from the literature reviewed as above.

#### **Research Methods**

This study adopted the mixed research design, particularly the explanatory design for which the researcher surveyed 155 students of grade X at first to find the low achievers in trigonometry. These students were selected randomly from four schools (55 students from two community schools and that of 100 from two institutional schools) where schools were

selected from same location of Kirtipur Municipality in the year 2017 AD. The achievement test was administrated to find out the most difficult contents or problems area of trigonometry as experienced by selected students and from which five low achievers (three from institutional and two from community schools) and three respective teachers who were teaching optional mathematics at grade X were chosen to explore difficulties that were experiencing in the learning process of trigonometry with the help of semi-structured interview guidelines. There were 18 questions in the achievement test which were constructed from the content of trigonometry and prepared on the basis of Bloom's taxonomy. Split-half method was used to compute the reliability of achievement test in which the reliability coefficient was found 0.85 that ensured the tool was reliable. The interview guidelines for both students and teachers were prepared to explore difficulties in relation with the memorization of facts, comprehension of contents, derivation of formulae, justification of theorems, application of contents and transition of verbal problems. The validity of achievement test paper was ensured with the help of mathematics experts and research supervisor, however, that of qualitative information were maintained by using triangulation of respondents. The obtained quantitative data were analyzed and interpreted by using mean and standard deviation and that of qualitative information's were analyzed and interpreted in logical manner by using the general inductive approach (Thomas, 2006).

# **Results and Discussions**

The results were presented and discussed as follows under the separate headings coined from objectives of the study.

### **Analysis of Survey Results**

This subsection analyzed the achievement results of surveyed students which helped to select low achievers and to find most difficult contents area of trigonometry as expressed by grade X students. Trigonometry is one of the major contents in the course of school mathematics in which students need to memorize facts, definitions, and apply these facts to solve the related problems included in the courses. The level of understanding of the concepts in trigonometry and difficulties in learning the contents can be judged by taking achievement test. So, this study considered achievement is one of the major indicators to find the most difficult contents or problem areas of trigonometry as experienced by grade X students and to select low achiever students for the detail explanation of difficulties in learning and solving the problems in trigonometry. The following table reveals the mean and standard deviation of achievement test of selected 155 Grade X students in trigonometry where the full mark of the test was 50.

Types of	Number of	Mean score of obtained	Standard Deviation of obtained
School	Students	mark	mark
Community	55	17.55	12.83
Institutional	100	23.2	10.62
Total	155	21.19	11.76

The results in the table show that mean achievement score of students in community schools is less than that of institutional schools by 5.65 in trigonometry. This implies that students in community schools are poor in learning trigonometry at grade X in comparison to

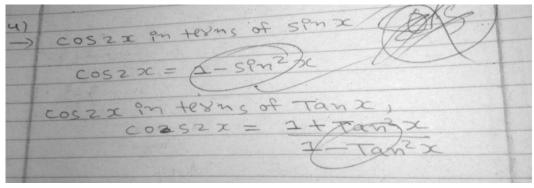
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institutional schools. Likewise, the higher standard deviation in the achievement score of students in community schools indicates the greater variability in achievements from average marks than that of institutional schools. That means there is consistency in achievement results of the students of institutional schools in comparison with community schools. However, there was difficulties found among students in both types of schools in learning trigonometry because they had got low average marks (that is 21.19) of all selected students out of 50 full marks. Also, it was found that no one got the full mark in both type of school. It means that there were difficulties for students in learning and completing achievement test in trigonometry. The standard deviation is very high in both type of school in comparison to mean which justify that there are students having very poor achievement level and greater difficulties in learning trigonometry at grade X.

From the analysis of test papers of sample students, it was found that there were several difficulties related to content areas of trigonometry which are analyzed and interpreted in the following subsections. The test papers revealed that students have difficulties related to remembering the trigonometric formula and definitions, meaningful comprehension of contents, transition the verbal statements into figures, deriving formulae and proving theorems, and application of concepts in trigonometry. These difficulties of content areas were triangulated with the responses of selected five low performer students and respective teachers of grade X to explain the details of those difficulties as they experienced in learning trigonometry.

### **Remembering the Trigonometry Formulae and Definitions**

Memorization of facts in trigonometry such as remembering formulae and definitions are essential mental activities to learn it effectively. According to Bloom's taxonomy, knowledge level is the first level of cognitive domain that includes retrieving, recognizing, and recalling information. The test paper of student showed that they did mistake in remembering formula and they lost the mathematical terminology in definition. The following is one example of student solution in achievement test for the question to express*Cos2X* in terms of *SinX* and *TanX*.



This result shows that student made mistakes to express Cos 2X into Sin X and Tan X. The correct answer is  $Cos 2X = 1 - 2 Sin^2 X$  and  $Cos 2X = (1 - Tan^2 X) / (1 + Tan^2 X)$ . That means students didn't care and remember the correct formula for that concept. They made mistake in using proper sign for the concepts and forget to write coefficient 2 for  $Sin^2 X$ . One

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student expressed his views as: "We have lack of individual attention... so we can't remember the formula for long time". That means students have difficulties in the memorization of the formula and conceptualization of the correct concepts in trigonometry which affects the achievement and overall learning. These findings are also supported by Thapa (2016) which explains the main difficulties in learning trigonometry are due to poor attention in mathematical formula, not caring teachers' voice, problem in using signs and misconception of trigonometry. The long duration between content taught and examination is also responsible to create difficulty among students because this long duration causes to forget in addition to memorize facts, formulae and other concepts in trigonometry.

#### Comprehension of the Content in Trigonometry

Comprehension is a psychological process related to an abstract or physical object, such as a person, situation, or message where by one is able to think about it. So, the difficulties in this sub-section are those in which students are unable to comprehend the concept in trigonometry. Understanding the concepts involves the ranges of different skills. It is the second level of Bloom Taxonomy. One participant student responded as: "It is very hard to understand because most of my friends and teacher say trigonometry is more difficult than other contents. Most of the contents are not useable in daily life. Almost all the contents of trigonometry are abstract and some terminologies are new for me. In government school, optional mathematics started from class nine... so I haven't strong base for optional mathematics".

These views indicate that the trigonometry and its contents are purely new concepts for students in community schools which create difficulties for them to understand. It is just started from grade IX in community school. Also, the friends of the participant and his teacher considered trigonometry as difficult and abstract contents in school mathematics. It is difficult to comprehend trigonometry because the participants have experienced that contents learned in trigonometry have low application in daily life in comparison to geometric and arithmetic concepts. The research of Bhat (2017) pointed that students' difficulties in learning trigonometry are due to lack of group discussion, not proper using exercise book and lack of using electronic instructional materials in mathematics. Thus, the classroom teaching methods and the materials used for teaching are responsible to create the level of difficulties in comprehending the concepts in trigonometry.

### Transition the Verbal Statement in to Figure of Height and Distance

The trigonometry content covers the many sub topics among them 'Height and Distance' is one of the most importance as well as more useable topic. To solve the problems related to height and distance, students need to apply almost all the formulae of trigonometry. One student replied as: "I know the definition of angle of elevation and depression. In the question paper I see both angle . . . I am confusing about the appropriate place of angles". On the same time one selected teacher expressed his views as: "if students understand the question clearly then they can do but now a day's students don't want to learn concept. Student wants rote learning rather than meaningful understanding . . ., when the grading system came in SEE then students became lazy."

These views reveal that students are still confusing to draw the accurate figures of verbal problems in trigonometry even they have good concepts of angle of elevation and depression. The achievement test paper shows not all the students draw the correct figure

related to angle of elevation and depression in the problems of images in water/river, and to find the line of reflection in the triangle. The teachers' views indicate that due to poor language in mathematics, students are unable to understand verbal question in trigonometry which create difficulties in solving and comprehending process of trigonometry. As experienced by teachers, the students are focusing in rote memorization rather than meaningful understanding of the trigonometric concepts. This trend is also motivated by the grading system in SEE examination, which has made students lazier in learning and solving mathematics. As explained in Acharya (2017), anxiety and negative feeling in mathematics help to decrease the mathematics understanding. Likewise, poor basic concepts and language help students to increase difficulties in solving verbal problem related to height and distance in trigonometry. Also, students who are generally more exam oriented think only about that how can they get high grade point in exam, but never think that how can they learn for better understandings (Bhat, 2017; Kenny, 2008). That means students priorities in getting only high marks in exam helps them to increase difficulties in meaningful understanding the concepts in trigonometry.

# Deriving Formulae and Proving Theorem of Trigonometry

Mathematical proof takes the rule of logical deduction. Formula deduction is a logical operation of building the new formula by using definitions, axioms, some logical statement and valid theorems too. This involves well ordering principles, direct proving, indirect proving and giving counter examples. The participant explained that "*I know to prove other numerical problems from the book but these types of formulae like proving trigonometric identities, I didn't do before. I think we use formulae to prove other questions but formula comes obviously*". This implies that students have felt difficulties in proving trigonometric identities in which they need formulae in every step. They have felt that the proving approach is quite different from approaches used in geometry. In trigonometry, very obvious formulae are applied in proving theorems/identities and to derive other formulae. These techniques have made students more confusion and thus difficulties in learning trigonometry as experienced by them.

The teacher replied that "I generally focus on repeatedly asked questions in exam and teach students by looking the trend". This means the school teachers are also focusing to solve frequently asked question rather than focusing in conceptual techniques and creativity to teach trigonometric identities and theorems like formula derivation. Lack of sufficient preknowledge and lack of practice are the main causes of difficulties of students in proving theorem in general (Kenny, 2008; Yadav, 2017) and in particular in trigonometry. Thus it is concluded that students have faced difficulties in proving theorems/identities because of incapability of organizing and applying learned formulae, definitions, concept and knowledge to unknown situations. The other reasons behind such difficulties in proving formulae in trigonometry are due to its abstract nature and verbal (logical) steps in every line of proof rather than numerical calculation. There is rarely explanation by figures as in geometry and less practice of such types of proof create the difficulties in learning proofs in trigonometry. **Application in Trigonometry Problems** 

Application is solving problems in new situations by applying acquired knowledge, facts, techniques and rules. Application problems are the main body of the trigonometry. It

was found that one of the main problems of students in application was to select appropriate formula which can apply to get final answer of different problems. The test paper showed that students solved the questions haphazardly, didn't care the question, and only solved according to their own knowledge and ideas. Students had known some formulae but they didn't know which is appropriate for correct solution. One participant responded that "I know the formula and definition but I can't decide which is applicable in this situation. There are so many formulae in trigonometry so I feel confusion which can I use to get right solution." It means school students are confusing to select suitable concepts to solve trigonometry problems even they have knowledge of these concepts.

Thus, capability of applying required concepts and formula for unknown situation of solving problems in trigonometry is one of the necessary skills for school students. Likewise, the teachers' experiences explained that "some of the students they used their own ideas. They forgot the process which given by teacher. Students solve the questions haphazardly unless they forget to read the total question". This explanation reflects that using own ideas to solve problems in trigonometry emphasize the creativity, however, starting solution process without reading and understanding the whole question can misguide students. So, thinking for correct procedure before solving and do not start haphazardly may help students to apply the right formula for the solution of intended trigonometric problem.

According to Piaget (1952), student makes a plan for learning new knowledge. They make their own mental picture that called by scheme if it is match in real situation it is called assimilation. If it does not match student feel confused and that is the main difficulty of students. It was also found that students who were not familiar to the abstract concept of trigonometry had unsolved questions in application level and they have faced difficulties in solving higher level questions in trigonometry. Bhandari(2017) and Mulwa(2015) pointed that when the grading system came in SEE then student left the hard practice in mathematics. It means the examination system also affects the students' motivation and practices in learning. That is, students have felt comfortable to pass in grading system than previous system of education which has made students lazy in practices. Because of fewer practices in trigonometry problems of application level, students had faced difficulties in learning meaningfully. The students also reported that they can solve only the questions solved by teachers but unable to solve the same type of questions in new situations.

#### Conclusion

Trigonometry is one of the fundamental and major contents included in course of school mathematics, particularly in the course of elective (optional) mathematics. But the achievement result of students on trigonometry was not satisfactory in both types (community and institutional) of schools. Also, the findings revealed that students in community schools have poor achievement level on trigonometry in comparison to institutional schools. There are several areas of difficulties as experienced by grade X students in learning the concepts in trigonometry including the difficulties related to memorization of facts; comprehension of new concepts; transformation of verbal problems of trigonometry into geometric figures; derivation and proofs of trigonometric formulae, identities and theorems; and unfamiliarity with the solving process of problems of application and higher level in trigonometry. The findings also showed that majority of students had difficulties in solving problems of application

level and due to which they were unable to construct appropriate figures to solve the verbal problems as given to them.

The difficulties as experienced by grade X students and their teachers were due to varieties of causes as they reported and expressed with the researchers. The long duration between content taught and examination has made students difficulty in the memorization of facts, definitions, and formulae to solve problems of trigonometry in exam. Similarly, the abstract nature of concepts, low application in daily life, teachers' methods of teaching and materials used in teaching/learning trigonometry were responsible to create difficulties and meaningful comprehension of trigonometric concepts as experienced by students. Also, due to poor English and mathematical language among students have faced difficulties in transforming verbal questions such as problems of height and distance in trigonometry into geometric figures to give correct solution and meaningful comprehension. But teachers blamed that students have emphasized on rote memorization rather than meaningful understanding; poor basic concepts; and language; prioritized only on exam oriented questions; do not care totality of concepts; and given little amount of time in conceptual understanding of trigonometric concepts which ultimately have been created difficulties among students in learning and thus developed negative attitudes towards trigonometry.

The poor skills of students in organizing and utilizing of learned facts, formulae and concepts of trigonometry into new situation such as for solving verbal problems, deriving new formulae and proving theorems/identities had made difficulties to learn trigonometry. The major causes of difficulties in learning trigonometry are due to poor attention in mathematical formulae, not caring teachers' voice, and misconception in trigonometric concepts (Thapa, 2016); give priorities on how to get highest mark on exam rather than conceptual understanding (Bhat, 2017; Kenny, 2008); and lack of practices and pre-knowledge (Yadav, 2008). These empirical research evidences also have been supported to the results of this study. Finally, it is concluded that the aforementioned difficulties and their causes could have developed anxiety and frustration towards mathematics learning for school students. Hence, it is recommended that both students and teachers of trigonometry should have to focus on conceptual learning rather than rote memorization for meaningful understanding.

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References

Acharya, B.R. (2015). *Foundation of Mathematics Education*.Kathmandu:Dikshant Prakashan.

Acharya, B.R. (2017). Factors affecting difficulties in learning mathematics by mathematics Learners. *International Journal of Elementary Education*, 6 (2),8-15,DOI: 10.11648/ j.ijeedu.20170602.11

Bhandari, T.R. (2017). *Difficulties in Learning Group theory*. An Unpublished Master's Degree Thesis, T.U., Kathmandu.

Bhat, D.S. (2017). Problem Faced by Students in Learning Sets. An Unpublished Master's Degree Thesis, T.U. Kathmandu.

Bhatta, S. (2017). Problem Faced by Mathematics Teacher at Basic Level in Teaching Geometry. An Unpublished Master's Degree Thesis, T.U. Kathmandu.

Courtney,S. (2016). Developing meaning in trigonometry. *Journal for research in mathematics education, 28* (3), 258-277.

Gautam, B. (2017). *Tamang Student's Learning Difficulties*. An Unpublished Master's Degree Thesis, T.U., Kathmandu.

Ghimire, S. (2013).*Difficulties of Bote Student's in Learning Mathematics*. An Unpublished Master's Degree Thesis, T.U., Kathmandu.

Ginsburg, H.P. (2016). Mathematics learning disabilities: A view from developmental psychology. *Journal of Learning Disabilities*, *30*, 20-30. Retrieved from: https://journals.sagepub.com/home/ldx

Kamber, D. & Takaci, D. (2018). On problematic aspects in learning Trigonometry. *International Journal of Mathematical Education in Science and Technology*, 49, 161-175, DOI: 10.1080/0020739X.2017.1357846

Kenny,R. (2008).*Influence of Symbols on Pre –calculus Students; Problem Solving Goals and Activities*. Doctoral Dissertation Submitted to Graduate Faculty of North Coralina State University.

Mulwa, E.C. (2015). Difficulties encountered by students in the learning and usage of mathematical terminology. *Journal of Education and Practice*, 6 (13).

Piaget, J. (1952). *The Origins of Intelligence in Children*. New York: International Universities Press.

Thapa, R. (2016). *Learning Difficulties in Trigonometry*. An Unpublished Master's Degree Thesis, T.U. Kathmandu.

Thomas, D. R.(2006). A General Inductive Approach for Analyzing Qualitative Evaluation Data. *American Journal of Evaluation*, 27, 237-246. http://dx.doi.org/ 10.1177/1098214005283748

Yadab, A.K. (2017). *Students' Difficulties in Proving Theorems of Geometry*. An Unpublished Master's Degree Thesis, T.U. Kathmandu.

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