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Mathematical Insights into Indoor Air Quality and Pollutants Behavior

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ABSTRACT

Air pollution is a key threat to human life as we spend about 90% of time in indoor. Mathematical analysis is a tool to analyze and predict air quality and its distribution. This paper investigates the air pollution dynamics in room, focusing on their impact on indoor air quality. We have explored the role of the air pollutants to affect the indoor air quality of the room. In two sets of environment, we observed and compared the correlation among particular matters of size 1, 2.5 and 10 microns; PM_1, $PM_{2.5}$, PM_{10} and AQIUS. It is observed that PM_1 and $PM_{2.5}$ have high impact on AQI with correlation coefficient 0.99 and 1 respectively whereas PM_{10} has less than that of them with correlation coefficients 0.97. This study leads to a recommendation to policy makers and users to focus more in reducing the $PM_{2.5}$ concentrations by identifying sources and using suitable technologies.

Keywords: air pollution, air quality, pollutants, particulate matter, indoor

INTRODUCTION

The quality of air in indoor environment has significant implications for human health, comfort and overall well-being. There are natural and mechanical removal system for the air pollution in an indoor environment. Natural ventilation refers to the process of supplying and removing air in an indoor space without the use of mechanical systems that depends on wind pressure and buoyancy effects, which are affected by temperature differences between indoor and outdoor. Indoor air often contains higher concentrations of pollutants compared to outdoor air. There are wind-driven and buoyancy-driven natural ventilations. First type occurs when wind flows around and through openings such as windows and doors, creating pressure differentials that drive airflow (Santamouris, 2007). Buoyancy-driven ventilation, occurs due to temperature gradients between the interior and exterior, leading to the stack effect, which facilitates vertical airflow through openings like chimneys or high vents (Etheridge, 2012). Natural ventilation can remove airborne pollutants, moisture, and excess heat generated. Its effectiveness depends on architectural features such as the size, location, and orientation of openings (Li and Delsante, 2007). Cross-ventilation significantly improves airflow efficiency and pollutant removal (Mazzarella and Peron, 2009). It is possible to quantify the effect of diverse sources of contaminants, including which will illustrate how mathematical frameworks can aid in predicting pollutant behavior and assessing risks. Particular matters of size less than or equal to 1, 2.5 and 10 microns are denoted by particular matters, PM_1, $PM_{2.5}$ and PM_{10} respectively. Mathematical modeling, numerical simulation and analysis can analyze its concentration trends, correlation with various pollutants, their estimations, ventilation effectiveness and removal strategies. Effective management of particular matters, heat and odors is essential to ensure occupant health, comfort, and safety. Heat and odor control mechanisms, including thermal management, well-designed exhaust systems and activated carbon filters, are essential to prevent odor accumulation and ensure a pleasant indoor atmosphere (Chen and Zhang, 2021). Particulate matters, $PM_{2.5}$ and smaller are small enough to penetrate deep into the respiratory system, posing severe health risks, including respiratory and cardiovascular diseases (Bruce

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et al., 2015). Proper ventilation systems, including range hoods with high-efficiency particulate air (HEPA) filters, can effectively reduce PM levels in kitchen or an indoor space.

High concentrations of CO in indoors can lead to poisoning, causing symptoms ranging from headaches and dizziness to severe neurological damage or even death. While carbon dioxide (CO_2) is less toxic than CO, elevated levels in kitchens can contribute to poor indoor air quality and discomfort. (Santamouris, 2007). Ensuring proper airflow and the strategic placement of ventilation systems are effective strategies for maintaining CO_2 levels within acceptable limits. Mechanical systems such as exhaust fans and range hoods can target specific pollutants (Etheridge, 2012). Kitchens represent a critical area due to the frequent emission of heat, PMs, CO, CO_2, and volatile organic compounds (VOCs) (Bruce et al., 2015).

For the mathematical modeling of fluid flow or mass transport, we take advection-diffusion reaction equation

$$\frac{\partial c}{\partial t} = \nabla \cdot (D\nabla c) - \nabla \cdot (\mathbf{v}c) + S, \tag{1}$$

where c represents the concentration, D the diffusion coefficient, $\mathbf{v}$ the velocity field and S the source term (Buonanno et al., 2009; Zhang et al., 2010). This equation considers the contributions from diffusion, advection, and pollutant sources, which affects the concentration values of the pollutants such as $PM_{2.5}$, CO, and VOCs etc. There are different forms of this equation that depends on the aim of the study, and the boundary conditions. Heat transfer is driven according to the laws of thermodynamics and the principle of conservation of energy as given by the heat equation:

$$\frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T) + Q$$

where, $T(x,t)$ = temperature, k = thermal conductivity, Q = heat source. In the atmosphere, heat transfer is controlled by diffusion and advection.

Thermal comfort is affected by external and internal factors, like outdoor temperature, the heat generated from cooking, and the design and operation of ventilation systems. Mathematical models of thermal comfort often use the heat balance equation, which considers the heat gain from sources such as cooking appliances and the heat loss via ventilation. The equation for the heat balance of a room can be expressed as

$$Q_{\text{gain}} - Q_{\text{loss}} = \Delta T$$

where Q_{gain} is the heat generated by internal sources, and Q_{loss} is the heat lost through ventilation, conduction, and radiation to the environment.

The above or other equations can be solved using computational fluid dynamics (CFD) models or simpler analytical models to predict pollutant concentrations and removal efficiencies. First discretize the domain into computational 2D or 3D grids and then solve the partial differential equation using numerical techniques such as finite difference, finite element, finite volume or other methods. These methods approximate continuous form of derivatives into discrete form. We use the initial and boundary conditions like pollutant concentration or flux at walls, windows, and doors. For unsteady flow, we use explicit or implicit time-stepping methods to compute future concentrations using current values by solving discrete equations. Post-processing tools visualize/plot of pollutant concentrations or time series data analysis or animations to show pollutant dispersion over time. Results are validated by comparing them with experimental measurements or analytical solutions.

LITERATURE REVIEW

Numerous studies have explored the mechanisms of pollutant emissions, airflow dynamics, and the effectiveness of various ventilation strategies in mitigating the adverse effects air pollution. Cooking activities are a major source of indoor air pollutants, including PM, carbon monoxide (CO), and volatile organic compounds (VOCs). When frying and grilling, a significantly higher level of $PM_{2.5}$ (Buonanno et al., 2009) than boiling and steaming. Zhang et al. (2010) revealed, the emission rates of total PM and VOC are related to the species of laden emissions, and unsaturated oils are found to be the most harmful. Gas stoves are one of the contributors to the generation of CO and NO_2, and exposure to these pollutants is directly linked to health effects associated with indoor air if not adequately vented (Logue et al., 2014). The health implications of poor ventilation are well known. Bruce et al. (2015) found that exposure over a long time period to cooking-related pollutants, in particular $PM_{2.5}$ and CO causes respiratory and cardiovascular diseases. The World Health Organization (WHO) guideline for indoor $PM_{2.5}$ $\mu g/m^3$ and CO levels 10 ppm in 8 hours mean (Bruce et al., 2015). Regardless of these dictates, the concentration of these pollutants has been reported to vary from few to several times higher than their limits inside poorly-ventilated rooms, underlining the need for developing more effective ventilation strategies.

Zhang and Chen (2010) studied various ventilation strategies and discovered that the hybrid ventilation that combines natural ventilation with mechanical ventilation would acquire the best compromise between energy saving and air quality improvement. Natural ventilation is a cheap method to provide removing pollutants in which natural forces, such as wind and buoyancy, are used (Allocca et al., 2003). In those room spaces where pollutant generation and heat losses occur, we can go for natural ventilation systems. Buoyancy-driven natural ventilation has special significance in kitchens where cooking-based heat loads can produce large buoyancy forces for pollutant extraction. The wind-induced flow, on the other hand, is that which acts due to wind pressure from the outside to the interior by means of outlets such as windows, and inlets such as slits. It is also very sensitive to the building orientation, wind from outside and the position of the windows. The buoyancy and wind-drive effects of ventilation are combined in hybrid ventilation to make use of the different weather conditions (Heiselberg, 2002).

Wind-induced natural ventilation utilizes negative and positive pressure areas created due to wind impact on the outer walls of a building. Windows, fins or louvers can increase air circulation – which can help pollutants, such as PM and CO, to dissipate. It is known that the wind-driven ventilation is very effective in relatively stable and moderate wind areas (Santamouris and Kolokotsa, 2013. These are most valuable where natural driving forces are insufficient for ventilation, particularly in cases of high cooking time loads (Chenari et al., 2016).

Architectural features play a significant role in the effectiveness of natural ventilation systems. Factors such as window size, chimney design, and cross-ventilation can greatly influence the natural airflow within a room or kitchen (Zhang et al., 2010). The size and positioning of windows affect the amount of natural airflow, with larger windows generally promoting better ventilation. Cross-ventilation, which relies on having openings on opposite sides of a room, allows for more effective air exchange and pollutant removal. Previous studies have laid a strong foundation for understanding ventilation, highlighting the importance of effective airflow management and pollutant removal (Allocca et al., 2003; Heiselberg, 2002). As a general phenomenon, the greater the temperature difference in the building, the more predominant the buoyancy force and, therefore, the mechanical ventilation effect (Buonanno et al., 2009). Effectiveness of the stack effect depends on several factors, including the height of the building, the temperature difference between the indoor and outdoor environments, and the design of the ventilation openings (Logue et al., 2014). For kitchens and rooms, buoyancy-driven ventilation with wind pressure can effectively remove pollutants and improve air quality (Buonanno et al., 2009). Out of different pollutants from different sources, there is very less research are made to evaluate the relation among the dependent variable AQI and independent variables PM_1, $PM_{2.5}$, PM_{10}, particularly in Nepalese context. In this paper we explore the relation among the above variables and find which is the most contributing pollutant on air quality index. The primary objectives of this study are to investigate the correlations of PM1, PM2.5 and PM10 in air quality index and most influential pollutant for the increase of AQI.

METHODS

The residential room environment is taken for the study. Two set of data in low and moderate air pollution are taken with 40 and 598 readings of PM_1, $PM_{2.5}$, PM_{10}, AQIUS. We have assumed the well-mixed condition for the distribution of the air pollution inside the room. To check correlations, we have used a correlation matrix to check linear relationships. We have plotted scatter plots to explore relationships between AQI and each predictor (e.g., PM_1, $PM_{2.5}$, PM_{10}). Scatterplots have visualized how air quality index in US standard (AQIUS) is related to PM_1, $PM_{2.5}$, PM_{10}. Pairwise scatter plots of all variables for a comprehensive view of relationships are plotted using pair plots. Our study is limited to indoor environment not compared with outdoor AQI data to see if the indoor air sources dominate. Also we have not checked cofounding factors like humidity, temperature and room occupancy.

RESULTS AND DISCUSSIONS

The data collected is analyzed for different scenarios. Comparisons are made among the parameters for evaluating associations, concentrations, comparisons of experimental and simulation data. There results of the study are presented in next section followed by some discussions.

Results

In case of first set the average PM_1, $PM_{2.5}$, and PM_{10} concentration are found to be 13.95, 16.75, and 53.80 respectively whereas the AQIUS value is 60.05. These values suggest that PM_{10} and $PM_{2.5}$, PM_{10} are 4.41, 5.11 and 44.17 reflecting greater variation in PM_{10} whereas AQI has 11.73 (Table 1).

Table 1

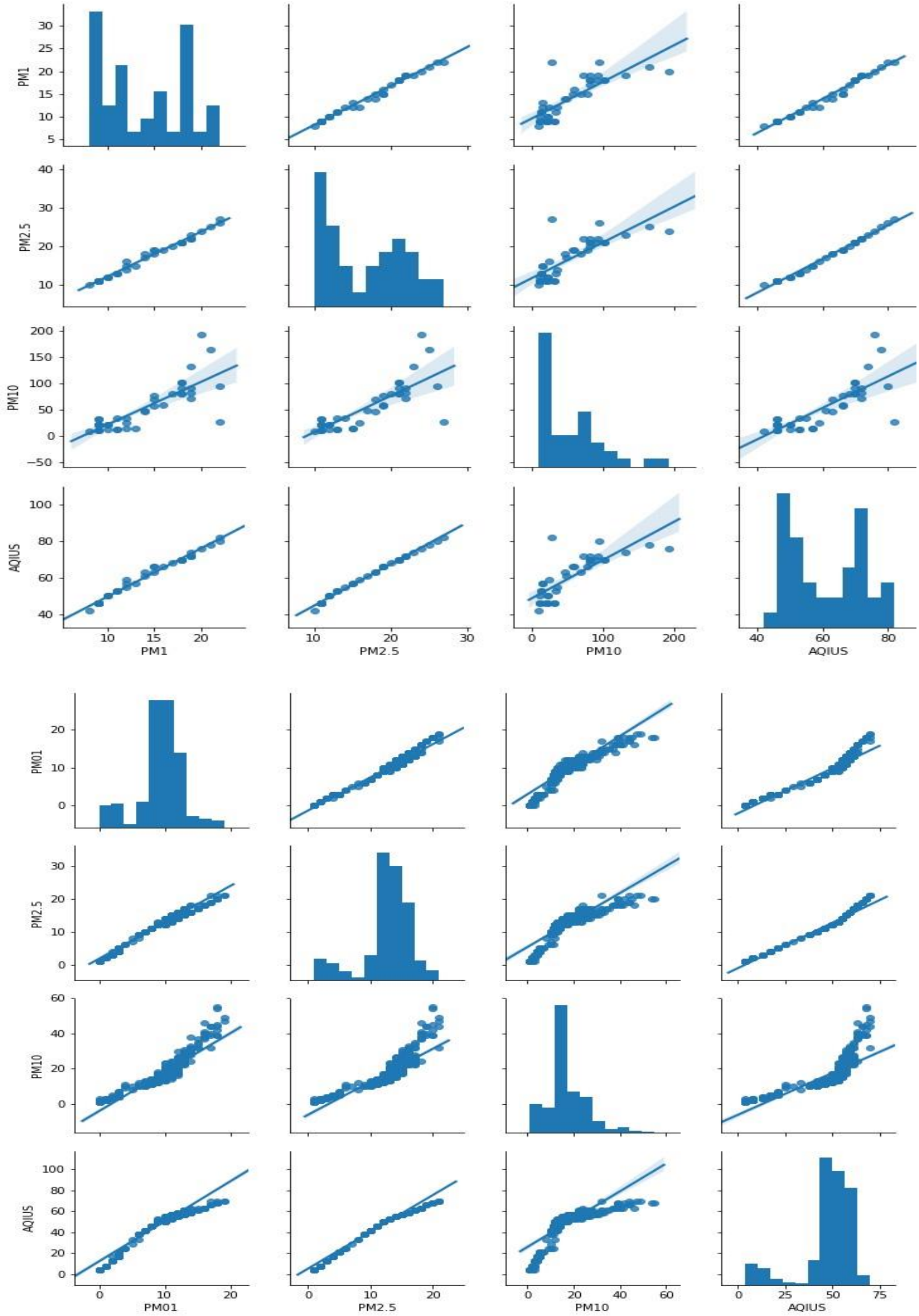
Descriptive Statistics of Air Pollution Variables for Data Set-I and Set-II

	Data Set-I				Data Set-II			
Statistic	PM_1	$PM_{2.5}$	PM_{10}	AQIUS	PM_1	$PM_{2.5}$	PM_{10}	AQIUS
Count	40	40	40	40	598	598	598	598
Mean	13.95	16.75	53.80	60.05	9.37	12.17	16.65	48.17
Std	4.41	5.11	44.17	11.73	3.47	3.90	8.33	13.88
Min	8.00	10.00	10.00	42.00	0.00	1.00	1.00	4.00
25%	9.75	11.75	21.00	49.00	8.00	11.00	12.00	46.00
50%	13.50	16.50	34.50	60.00	10.00	13.00	15.00	53.00
75%	18.00	21.00	81.00	70.00	11.00	15.00	21.00	57.00
Max	22.00	27.00	193.00	82.00	19.00	21.00	55.00	70.0

The sub-figures in pair plots (Figure 1), illustrate the relationships between PM_1, $PM_{2.5}$, PM_{10}, and the Air Quality Index (AQI). The diagonal histograms show the distribution of each variable and the off diagonal scatter plots indicate the correlation of the variables with one another. There are strong linear relationships between PM_1, $PM_{2.5}$, PM_{10}, and AQI, suggesting the AQI tends to rise as particulate matter levels increase. The relationships in the PM_1 and AQIUS complement these distributions by showing pairwise trends, such as linear relationships or clusters.

Figure 1

Pairwise Correlation plots



In Figure 1, scatter plots include a regression line, highlighted by a shaded region indicating the confidence interval. The overall trends highlight that higher concentrations of particulate matter (PM_1, $PM_{2.5}$, PM_{10}) correspond to higher AQI values, where increased pollution levels are associated with poor air quality. $PM_{2.5}$ has a high positive correlation +1 indicating $PM_{2.5}$ strongly increases as AQI. PM_1 has also similar correlation (0.99) whereas PM_{10} has less effect on AQI than that of $PM_{2.5}$ and PM_1.

Table 2

Pairwise Correlation Matrix

	PM_1	**$PM_{2.5}$**	**PM_{10}**	**AQIUS**
PM_1	1.00	0.99	0.81	0.99
$PM_{2.5}$	0.99	1.00	0.80	1.00
PM_{10}	0.81	0.80	1.00	0.79
AQIUS	0.99	1.00	0.79	1.00

The correlation matrix (Table 2) presented shows that there are very high associations between pollutants (PM_1 and $PM_{2.5}$, PM_{10}) and AQIUS indoors with observations as mentioned below:

Table 3

Key Observations from Correlation Matrix

Variable	Correlation (r)	Interpretation
$PM_{2.5}$–AQIUS	1.00	Perfect linear correlation. $PM_{2.5}$ is the primary driver of AQIUS
PM_1–AQIUS	0.99	Nearly identical to $PM_{2.5}$, suggesting PM_1 is also a dominant factor
PM_{10}–AQIUS	0.79	Strong but weaker influence compared to PM_1/$PM_{2.5}$

AQIUS is almost entirely determined by PM_1 and $PM_{2.5}$ ($r \geq 0.99$). It gives a practical meaning that reducing these fine particles will directly lower AQI. PM_{10} matters less (r = 0.79), but still contributes to dust levels. Since PM_1 and $PM_{2.5}$ are almost interchangeable (r = 0.99), to solve the issue of 151ulticollinearity and problem to separate their individual effect, we need to use only $PM_{2.5}$ in models as AQIUS prioritizes it.

Discussions

We have some insights from the results. PM_{10} has the highest mean value in both of the set of data (53.8 and 16.65 respectively). The deviation of PM_{10} and AQI is much higher than that of PM_1 and $PM_{2.5}$. But the correlation matrix indicates more effect of $PM_{2.5}$ than that of PM_{10}. In descriptive statistics gives large decrease of PM_{10} from 53.8 to 16.75 does not have that much effect than decrease of PM_1 and $PM_{2.5}$ from 13.95 and 16.75 to 9.37 and 12.75. The average AQIUS in first and second data set are 60.05 and 48.17 respectively. This issue is addressed by the correlation coefficients among dependent variable AQIUS and independent variables PM_1, $PM_{2.5}$ and PM_{10}. In first set of data the PM_1, $PM_{2.5}$ and PM_{10} have correlation values 0.99, 1 and 0.79 respectively with AQI. In second set of data they are 0.95, 0.98 and 0.79 respectively. In the meantime, $PM_{2.5}$ has high correlation 0.98 with PM_1 than PM_{10} (0.88) in second set of data. In first set, it is 0.99 and 0.80 respectively. This can be observed in the pairwise correlation and regression shown in Figures 1and 2.

This is important for indoor air quality as $PM_{2.5}$ and PM_1 deeply penetrate into the lungs respectively; their management are also urgent. For pollution control we need to focus on $PM_{2.5}$ and PM_1 sources, ban indoor combustion (smoking, candles). Use air purifiers with HEPA filters to control indoors. Prohibit indoor combustion (smoking, candles). Increase ventilation (use more exhaust fans, keep windows open when outdoor air is clean). For PM_{10} control we need attention to dust control (vacuuming, reducing carpeted areas).

CONCLUSIONS

Understanding IAQ and the pollutants that inhabit our living spaces is critical for health of the people and environmental sustainability. $PM_{2.5}$ is the most dangerous pollutant which penetrates deep into the lungs and enter the blood stream in comparison to the PM_{10} which affects upper and central part of the respiratory system. As $PM_{2.5}$ higher contributions in air quality than PM_{10}, our focus mainly should be in controlling the $PM_{2.5}$ through the control over source or using the protective measures such as quality masks and air purifiers. For quality indoor air, future research should focus on developing adaptive ventilation strategies that incorporate real-time monitoring of indoor air quality and environmental factors and climate factors.

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