Difficulties in Studying Abstract Algebra at the Undergraduate Level

Yagya Prasad Gnawali
Mahendra Ratna Campus, Tahachal, Tribhuvan University
Correspondence: gnawali.yagya@gmail.com

Article info:
Received: February 9, 2024
Revised: March 11, 2024
Accepted: April 2, 2024

Keywords:
Abstract algebra, challenging in learning, quantitative research, undergraduate level

Abstract: This study has attempted to explain the major challenges that undergraduate students face when learning abstract algebra. Students at the undergraduate level totaling one hundred twenty (120) were chosen as the sample size. In this study, the survey design is based on the positivist paradigm. Probability sampling was used to select the students of undergraduate level. For the validation of this study, self-administrated Likert type questionnaires were employed, along with extensive discussion with mathematics teachers who have taught and are presently teaching algebra at the undergraduate level. According to survey results, there were four main reasons—Formalism *and Rigor, Lack of Prior Knowledge, Teaching and Learning Technique, Assignment and Assessment,* -that hindered the students' ability to learn abstract algebra. The research outcomes in terms of motivation, self- esteem and challenges of the respondents will be helpful to understand the nuances of teaching and learning abstract algebra at higher education in Nepal.

Introduction

Almost all ancient civilizations, including Babylonian, Egyptian, Chinese, and Hindu, have techniques for resolving polynomial problems, primarily linear and quadratic ones. Particularly, Babylonians, were competent algebraists as well as they were able to solve linear and quadratic equations such as $x + y = a$, $x^2 + y^2 = b$ and $ax^2 + bx + c = 0$. The issues and their solutions were completely verbal in the original context, hence there was no algebraic notation. It is not justified, and the answers were prescriptive. In the Babylonian setting, there were no zeros, negative numbers, or irrational numbers. The "father of algebra" has traditionally been identified as the Greek mathematician Diophantus, when algebra is understood to be the theory of equations. However, there is a lot of debate today regarding whether al-Khwarizmi, who founded the academic discipline of al-jabr, deserves that title instead (Herstein, 2006). Those who prefer Diophantus contend that Al-Jabr's algebra is a little more fundamental than Arithmetica's and that Al-Jabr's approach is fully rhetorical as opposed to Arithmetica's syncopated manner (Howe, 2022).

In the twenty-first century, generalized and abstract algebraic structures are studied using a field of mathematics known as abstract algebra (Hungerford, 1974). It focuses on the study of algebraic systems such as groups, rings, fields, vector spaces, and modules rather than exact numerical computations (Vasishtha, & Vasishtha, 2000). These structures are defined by a group of elements and procedures that follow a set of precepts or characteristics. It is commonly known that abstract algebra is important for math classes (Israel, 2007). A number of mathematicians concurred that the axiomatic approach is the most beneficial and successful strategy for teaching.

Abstract algebra is frequently taught as a course or a sequence of courses in mathematics programs at the undergraduate level. It gives students a strong foundation in the core ideas and methods of abstract algebra and acts as an introduction to algebraic structures (Agustyaningrun, Abodi, Sari, & Mahmudi, 2018). In fact, even while the vocabulary and methodology of abstract algebra are used more widely in disciplines like computer science, physics, chemistry, and data communications, abstract algebra still plays a vital role in advanced mathematics itself (Gallian, 2010). Abstract algebra is an essential part of the undergraduate mathematics and mathematics education curricula.

This course serves as one of the first opportunities for most third-year students of Bachelor of Education to interact with the difficult concepts of formal proof and mathematical abstraction. It implies that students must acquire a thorough understanding of the many concepts of abstract algebra, including groups, characteristics of groups and subgroups, modulo 5, and $Z_5 = \{0, 1, 2, 3, 4\}$ with binary operation (+, X) (Kanna, & Bhambri, 2013). Students must be aware of the group's generators in order to participate in cyclic groups, Z_5, on the other hand, is a cyclic group produced by every nonzero element. Examples of quotient groups are crucial for undergraduate students. Furthermore, the center of permutation group $S_3 = \{(1), (12), (13), (23), (123), (132)\}$ is (1), and it may be solved in $n \leq 5$. Undergraduates are taught the theoretical idea, but they are not able to provide appropriate instances. For example: The group's order and its generator's order are the same. In particular, abstract algebra commits the theorem to memory in order to focus on the examination viewpoint (Lanski, 2010).

Why Does it Become Abstract?

Regardless of the particular elements involved, abstract algebra studies algebraic structures that share certain fundamental qualities, adopting a more general and abstract approach (Vasishtha, & Vasishtha, 2000). Because of this abstraction, mathematicians are able to recognize and examine common patterns and structures seen in various mathematical systems. In this context, algebraic structures explore like rings, fields, and groups is a common first step in the study of abstract algebra (Leron, & Dubinsky, 1995). By focusing on the abstract features of algebraic structures, mathematicians can build a deeper grasp of the structural similarities and differences between different mathematical systems (Manandhar, & Sharma, 2021). This makes it easier to find connections across seemingly unrelated fields of mathematics. (This para is not well tuned up with the heading?)

Though it has abstract ideas but they are frequently applied to other areas of mathematics and beyond (Saracino, 2017). For example, group theory is frequently employed in physics, cryptography, and computer science, whereas ring and field theory is crucial in disciplines such as coding theory and algebraic geometry (Sleeman, 1986). Many fields of mathematics have a strict and formal foundation due to abstract algebra. Mathematicians can establish results and prove theorems that are valid in a variety of situations by studying abstract algebraic structures, so abstract algebra is a starting point for more complex mathematical subjects (Wasserman, 2017). For example, the foundations of abstract algebra are built upon in algebraic geometry and algebraic number theory. Abstract algebra is distinguished by its rigorous proof methodologies. A mathematical argument must be rationally supported at every stage in a formal, rigorous setting. This guarantees that reasonable reasoning and sound principles form the foundation of the findings reached, consequently, more complex mathematical topics frequently use abstract algebra as their base (Xue, 2022). Students can investigate other areas of mathematics that depend on algebraic structures with a firm foundation in abstract algebra, which is established by formal reasoning.

Statement of the Problem

For undergraduate students, abstract algebra is crucial because it offers a basic grasp of the basic algebraic structures and ideas that form the basis of many mathematical fields. Groups, rings, and fields are studied to help students gain a better understanding of the symmetry and structure that are present in mathematical systems. This abstraction makes it possible to investigate general ideas that go beyond the particular numerical examples seen in basic algebra. Furthermore, abstract algebra gives students the tools they need to solve problems, reason logically, and build strong mathematical arguments. These are transferable skills that may be used in a variety of mathematical contexts and serve as a foundation for more complex subjects covered in undergraduate-level coursework.

Abstract algebra is a useful and adaptable component of a variety of fields, including computer science, cryptography, physics, and engineering (Ball, Thames, & Phelps, 2008). Abstract algebra extends from elementary algebra's real numerical; it might be difficult for undergraduate students to understand through which it might be confusing and take a big mental leap to go from concrete concepts to extremely abstract ones like rings, fields, and groups (Zaffar, Qureshi, & Ansari, 2013). Students used to more computational approaches may find it frightening due to the emphasis on formal arguments and rigorous logical reasoning.

Furthermore, students must become proficient in a specialized language because abstract algebra frequently introduces unfamiliar and novel terms. Some students may find abstract algebra less tangible than other applied parts of the curriculum because there aren't as many immediate real-world applications (Goyal, Gupta, Gupta, & Gupta,2015). Overcoming these obstacles is crucial, though, as abstract algebra provides a springboard for more advanced mathematical studies and imparts useful thinking and problem-solving abilities that cut across many academic fields.

Theoretical Framework

Undergraduate students may consider many educational philosophies and psychological characteristics as impediments to their ability to master abstract mathematics. The "Cognitive Load Theory" (CLT), which sheds light on the mental work students do during the learning process, is one such theoretical framework that might be pertinent (Feldnon, 2007). The relationship between cognitive load, affect, and student decision-making varies depending on the decisions made and the methods every individual use when evaluating the possibilities (Mc Carty, Redmond, & Peel, 2021). The cognitive load theory (CLT) offers recommendations for how the material should be presented to support student activities that exploit intellectual achievement. The key principle of CLT is that human cognitive architecture ought to be taken into account extensively when creating content for learning (Kirschner, Kirschner, & Pass 2009). This cognitive architecture comprises of a relatively limitless long-term memory that interacts with a limited working memory (WM) that becomes cognitively overworked when executing a high-complexity task. In other words, particular learning objectives should guide the creation of training exercises and learning tasks (Leppink, 2017). In order to effectively deal with the difficulties of learning abstract algebra at the undergraduate level, concentrate on establishing a solid foundation in fundamental algebraic structures and ideas, including fields, rings, and groups.

It starts with having a firm grasp of and experience with basic theorems and proofs, so it also uses of online resources, lecture notes, and textbooks to enhance your comprehension and solve problems to solidify your learning. Moreover, cognitive load theory can provide guidelines to assist in a manner which encourages students activities for performance. Students seek out extra help to clear up any unclear topics through online forums, study groups, or conversations with academics. For improvement students problem-solving abilities, need to solve a range of challenges because practice makes perfect. Additionally, wherever possible, relate abstract algebraic principles to real-world instances. This can help with comprehension by offering a practical viewpoint. patience, perseverance, and a proactive approach to asking for assistance when needed….?? How would you use this theory and relate with this?

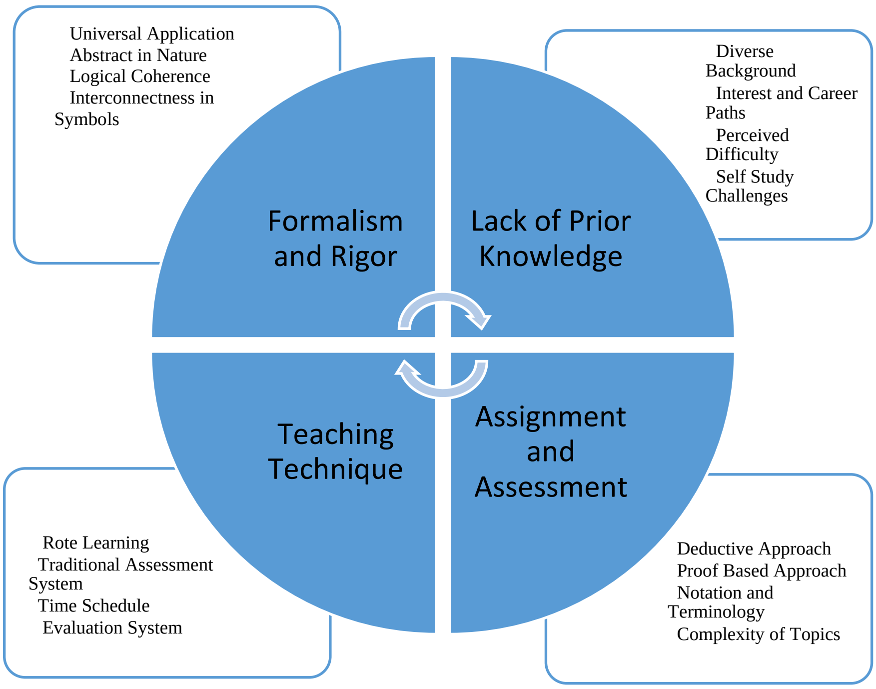
Conceptual Framework

A variety of cognitive, pedagogical, and motivational aspects are frequently included in the conceptual framework pertaining to the obstacles undergraduate students experience when attempting to study abstract algebra (Adom, & Hussein, 2018). Students are introduced to a higher degree of mathematical abstraction in abstract algebra than they have encountered in concrete concepts in the past. It might be difficult to transition from working with numbers to understanding abstract structures and algebraic systems. Additionally, students who are not yet familiar with the complexities of mathematical proof and abstraction may find abstract algebra's rigid formalism challenging.

Pedagogically, understanding is supported or hindered by the way abstract algebraic concepts are presented and the resources used in the classroom. Additionally, students' involvement and performance in abstract algebra can be influenced by their motivation and sense of the subject's significance to their larger academic or professional objectives. In order to overcome these obstacles, a well-rounded strategy that tackles the pedagogical techniques used to teach these difficult topics as well as the cognitive difficulties associated with abstraction is needed.

Undergraduate students typically find it difficult to study abstract algebra because of the difficult shift from concrete mathematical notions to abstract structures and rigorous proofs. In order to effectively communicate the complexities of abstract algebraic notions, it becomes more difficult, as a solid basis in mathematical abstraction is required, along with a change in educational approaches (Ticknor, 2012).

To put it briefly, because abstract algebra is by nature itself abstract, it may be applied to any situation. When instructing in the classroom, teachers must employ interconnected symbols. It is also challenging for students to learn because they typically have little previous experience for independent study. Because rote learning, traditional evaluation methods, and classroom timetables create obstacles to learning abstract algebra, the necessary conceptual framework is provided below.



Methodology

When analyzing quantitative data for Likert-type scales, it is common practice to combine participant responses and compute descriptive statistics for each item on the scale, including means, standard deviations, and frequencies. Furthermore, associations between Likert-scale items can be examined, and hypotheses can be tested, using inferential statistical methods like t-tests. It's critical to confirm the scale's validity and reliability using methods like factor analysis and Cronbach's alpha. Furthermore, quantitative analysis makes objective comparisons possible, which makes it easier to assess hypotheses and extrapolate results to larger populations. Consequently, I apply positivist paradigm and random sampling technique for the collection of samples and taking 120 undergraduate students who were involved in final examination. I used one sample t- test for descriptive statistics for the test of hypothesis about the four dimensions.

Results and Discussion

In this section, I used Cronbach's Alpha to calculate the survey's reliability based on the data. I had employed Likert-scales with five points. I discovered that the students' perception Cronbach Alpha rating was 0.802 out of 50 items. A reliability coefficient of more than 0.6 indicated a high level of reliability. The values of Cronbach's Alpha are provided in Table No. 1 below.

Initially, I calculated the mean and standard deviation of the descriptive statistics as well as the one-sample t-test to determine the significance for each component item. I performed a Null Hypothesis test to see if there were any significant differences between the perspectives of the students' Difficulties in Studying Abstract Algebra at the Undergraduate Levels

Using a table and figure and I was able to independently describe the aspects that affected the students in each segment. In order to compare the neutral value (test value = 3), which was based on the average value of five points on Likert scales, descriptive statistics (mean and standard deviation) were computed for each of the six components. A one-sample t-test was performed to determine whether or not the mean differences were significant at the significance level.

Table 1*: Cronbach's Alpha value*

	Reliability Statistics		
Category	Cronbach's Alpha	No of items	Sample Size
Undergraduate Students	0.802	40	120

After evaluating four distinct elbows with eigenvalues greater than one, I was able to conclude that there may be four alternative combinations of components, nine of which would have out-of-layer items and just 34 loaded items overall. The scree plots are shown in Figure 2 below.

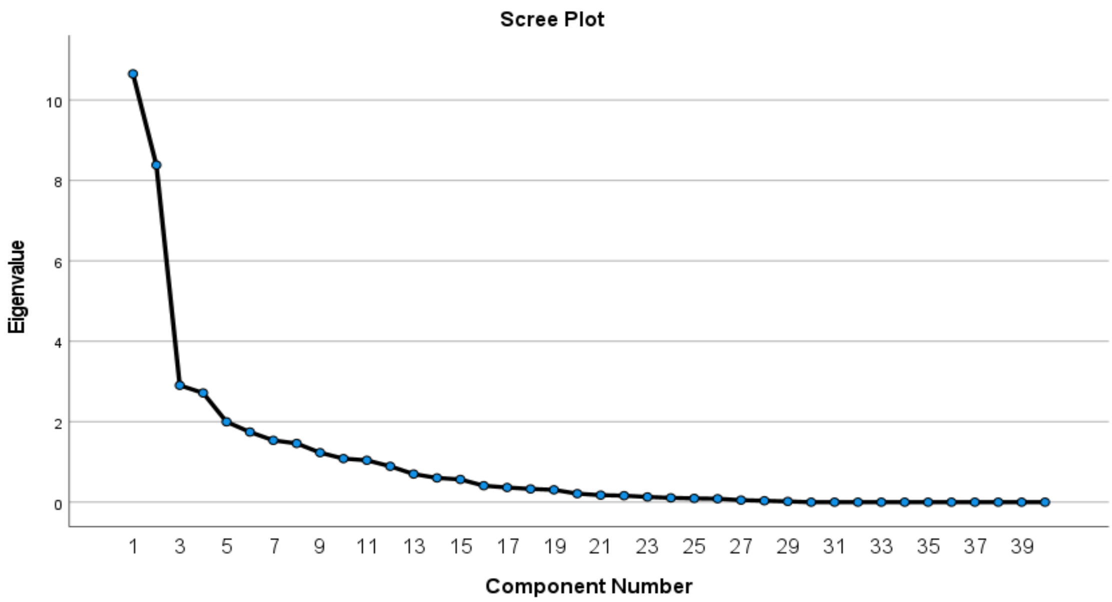


Figure 2: *Scree plot of the component numbers*

I listed all four components in Table 2 together with factor loading and reliabilities values (Cronbach's Alpha) for each component related to the difficulties in studying abstract algebra at the undergraduate levels

Table 2: *Principal component analysis of use of difficulties in studying abstract algebra at the undergraduate level*

Factor Loading from Rotated Components Rotated Component Matrix Items	Factor Loading	Components
1. The complexity of the mathematical notation used in abstract algebra is a barrier to my learning.	0.901	Factor-1 *Formalism and Rigor* (Cronbach's Alpha=0.941)
2. I found that abstract algebraic proofs are challenging due to their level of formalization and rigor.	0.883	
3. I'm not confident in my capacity to comprehend abstract algebra and create sound proofs	0.880	
4. Abstract algebra is hampered by the need for exact mathematical language.	0.838	
5. I find it is challenging to understand the real-world applicability of the principles.	0.833	
6. I feel that formality and rigor of reasoning in abstract algebra improves my learning experience.	0.777	
7. Application of abstract algebraic formal notions are impacted by the speed of learners.	0.750	
8. The formalism needs in abstract algebra seems difficult to me when I go from mathematical courses.	0.747	
9. Abstract algebra course is influenced by how well formal topics are explained and illustrated.	0.743	
10. I have previously studied algebraic structure course help me to understand abstract algebra.	0.740	
11. I find the abstract algebra course challenging to follow because it requires prerequisite knowledge that I do not have.	0.681	
1. I have found the background knowledge needed to start course of abstract algebra.	0.814	Factor 2: Lack of Prior Knowledge Cronbach's Alpha = 0.837
2. I have trouble been exposed to complex mathematical topics before abstract algebra.	0.754	
3. It is difficult for me to relate abstract algebraic ideas to the mathematical calculation.	0.706	
4. I found it is bridging the knowledge gap between my previous and abstract algebraic notions.	0.704	
5. I did not find the degree of mathematical structural knowledge to link abstract algebra.	0.693	
6. Inlearning, I found my inability to link abstractly my progress in abstract algebra ????.	0.628	
7. The speed in abstract algebra assumes prior knowledge in related mathematical fields overcomes me…???	0.593	
8. I'm completely taken aback by abstract algebra's quickness, which implies a background in related math subjects.	0.542	
9. I encounter with abstract algebra in day to classroom activities.	0.510	

1.	The lack of the assignment tasks in the abstract algebra course makes it difficult for me in final examination.		Factor-3 *Assignment and Assessment* (Cronbach's Alpha=0.723)
2.	The lack of clear guidelines for assignments in the abstract algebra course hinders my ability to complete them effectively.	0.724	
3.	The assessments (exams, quizzes, etc.) in the abstract algebra course adequately reflects the concepts covered in the assignments.	0.706	
4.	The grading criteria for assessments in the abstract algebra course are unclear, making it challenging to meet the expectations.	0.664	
5.	The objective questions in the abstract algebra course adds unnecessary stress and pressure.	0.566	
6.	Theoretical teaching styles and final examination system in the abstract algebra makes me difficult to learn.	0.532	
7.	The abstract algebra course does not provide sufficient opportunities for practical application of theoretical concepts.	0.515	
1.	Traditional teaching methods does not support my understanding of abstract algebra concepts.	0.693	Factor-4 *Traditional Teaching Methods* (Cronbach's Alpha=0.62)
2.	Traditional teaching techniques discourage in active participation and discussion among students in the abstract algebra.	0.623	
3.	Traditional teaching methods does provide to different learning styles and preferences in the abstract algebra course.	0.550	
4.	Traditional teaching methods in helping you retain and apply abstract algebra concepts outside of the classroom:	0.526	
5.	Interaction and engagement are not facilitated by traditional teaching methods in the abstract algebra course.	0.482	
6.	Traditional teaching techniques are in helping you achieve your learning goals in the abstract algebra course.	0.474	
7.	Traditional teaching techniques accommodate the diverse backgrounds and experiences of students in the abstract algebra course.	0.430	

Factor-wise Analysis

In this case, I have prepared the four components shown below in accordance with the scree plot. The initial element is called *Formalism and Rigor*, and it has the lowest mean of the four at 2.57. Likewise, 4.48 is the highest average value that the respondent evaluated. One such aspect, which makes up the second factor, is called *Lack of Prior Knowledge*. Once more, Assignment and Assessment are the third factor, its' average value is 3.92, which is higher than the average value of 3. In addition, *Traditional Teaching Methods*, the fourth factor, has an average value of 3.86, which is greater than the test result 3 average. It is determined that, when compared to the test, all four criteria difference significantly. Teachers can use a variety of strategies to lower the barrier to studying abstract algebra at the undergraduate level. This includes utilizing real-world applications to inspire students, giving them concrete examples and visualizations, involving them in interactive learning activities, scaffolding the content to build upon prior knowledge, giving them clear explanations and connections, offering frequent feedback and support, enhancing learning with examples, and promoting creativity and exploration (McGinn, Lange, & Booth, 2015). Through the implementation of these tactics, teachers can enhance undergraduate students' comprehension and appreciation of abstract algebra by making it more approachable, clear, and captivating. (See Table 3)

Table 3: *Descriptive statistics and one- sample t-test in Four Factors*

Test value= 3								95% Confidence Interval of the Difference	
Components	N	Mean	S D	t	df	Sig(two-tailed)	Mean Difference	Lower	Upper
Factor 1	120	2.5750	.90086	-5.168	119	.000	-.42500	-.5878	.2622
Factor 2	120	4.4852	.41994	38.742	119	.000	1.48519	1.4093	1.5611
Factor 3	120	3.9238	.64660	15.651	119	.000	.92381	.8069	1.0407
Factor 4	120	3.8611	.45750	20.619	119	.000	.86111	.7784	.9438

Formalism and Rigor

Among the loaded 11 elements, the reliability value Cronbach's Alpha = 0.941 is significant because it is higher than 0.6. The average rated value of this factor is 2.57, with the mean difference of -0.425 and a standard deviation of 0.90086, and $p<0.05$. Likewise, the highest rated average value of this component is 2.83, whose standard deviation is 1.072 and mean difference is -0.167. Similarly, the lowest rated average value of this component is 2.13, whose standard deviation is 1.209 and mean difference is -0.867. All the items were significantly difference at 0.05 ($p<0.05$) except the item undergraduate students have previously studied algebraic structure course help me to understand the abstract algebra. Respondents rated below average value about the given items the complexity of mathematical notation used in abstract algebra is a barrier for learning, algebraic proofs are challenging due to their level of formalization and rigor, proof is hampered by need for exact mathematical language, previous studies helped me for the explanation about the mathematical course and so on.

In abstract algebra, formality and rigor are frequently stressed in order to guarantee the accuracy, precision, and correctness of reasoning based on mathematics (Dummit, & Foote, 2008). However, an overemphasis on formalism and rigor can also cause learning difficulties for some students, especially those who are unfamiliar with the material. Groups, rings, and fields are a few examples of the very abstract ideas and structures covered by abstract algebra (Faizah, Nusantara, Sudirman, Rahardi, 2020). Formalism and rigor that are introduced too soon may make these ideas appear even more daunting and difficult, which will discourage students from actively participating in the subject matter (See Table 4).

Table 4**:** *Descriptive statistics and one- sample t-test in formalism and rigor*

Test Value =3								95% Confidence Interval of the Difference	
Components	N	Mean	S D	t	df	Sig(two-tailed)	Mean Difference	Lower	Upper
For& Rig 1	120	2.37	1.309	-5.301	119	.000	-.633	-.87	-.40
For& Rig 2	120	2.33	1.252	-5.831	119	.000	-.667	-.89	-.44
For& Rig 3	120	2.13	1.209	-7.854	119	.000	-.867	-1.09	-.65
For& Rig 4	120	2.53	1.152	-4.438	119	.000	-.467	-.67	-.26
For& Rig 5	120	2.37	1.053	-6.591	119	.000	-.633	-.82	-.44
For& Rig 6	120	2.53	1.236	-4.135	119	.000	-.467	-.69	-.24
For& Rig 7	120	2.77	1.121	-2.281	119	.024	-.233	-.44	-.03
For& Rig 8	120	2.57	1.179	-4.026	119	.000	-.433	-.65	-.22
For& Rig 9	120	2.73	1.035	-2.823	119	.006	-.267	-.45	-.08
For& Rig 10	120	2.83	1.072	-1.704	119	.091	-.167	-.36	.03

For& Rig 11	120	2.70	1.248	-2.634	119	.010	-.300	-.53	-.07
Factor 1	120	2.5750	.90086	-5.168	119	.000	-.42500	-.5878	.2622

Interpretation of the Table

Lack of Prior Knowledge

Among the loaded 9 elements, the reliability value Cronbach's Alpha = 0.837 is significant because it is higher than 0.6. The average rated value of this factor is 4.482, with the mean difference of 1.48 and a standard deviation of 0.414994, and $p<0.05$. Likewise, the highest rated average value of these two components is 4.67, whose standard deviation is 1.072 and mean difference is 1.67. Similarly, the lowest rated average value of this component is 4.03, whose standard deviation is 0.703 and mean difference is 1.03. Respondents agreed that the background knowledge needed to start course of abstract algebra, the difficulties of abstract algebra to relate ideas for mathematical calculation, the bridging the knowledge gap between the previous knowledge and abstract algebra, it is found that the inability to link abstract algebra with basic knowledge, and students always encounter with the abstract algebra in day-to-day activities in classroom learning. All the items were significantly difference at 0.05 ($p<0.05$).

Lack of enough prior knowledge is one of the main obstacles to studying abstract algebra. In contrast to more basic areas of mathematics, abstract algebra necessitates a certain degree of mathematical maturity and acquaintance with sophisticated ideas from subjects like reasoning, elementary number theory, and group theory (Fraleigh, 1984). Students could find it difficult to understand the complex linkages and abstract structures found in algebraic notions such as fields, rings, and groups without this fundamental knowledge (Gallian, 2013). In addition, students may find it difficult to move from practical, computational mathematics to algebra's more abstract and theoretical structure, which is made worse by the requirement to hone their proof-writing abilities (Hausberger, 2020). As such, conceptual fluency and past mathematical experience are two things that might seriously hinder one's ability to learn abstract algebra (See Table 5).

Table 5: *Descriptive statistics and one- sample t-test in lack of prior knowledge*

Test Value =3								95% Confidence Interval of the Difference	
Components	N	Mean	S D	t	df	Sig(two-tailed)	Mean Difference	Lower	Upper
LPK 1	120	4.53	.621	27.055	119	.000	1.533	1.42	1.65
LPK 2	120	4.67	.540	33.826	119	.000	1.667	1.57	1.76
LPK 3	120	4.60	.492	35.628	119	.000	1.600	1.51	1.69
LPK 4	120	4.57	.618	27.765	119	.000	1.567	1.45	1.68
LPK 5	120	4.27	.775	17.905	119	.000	1.267	1.13	1.41
LPK6	120	4.63	.484	36.974	119	.000	1.633	1.55	1.72
LPK7	120	4.40	.556	27.578	119	.000	1.400	1.30	1.50
LPK 8	120	4.03	.709	15.959	119	.000	1.033	.91	1.16
LPK 9	120	4.67	.473	38.568	119	.000	1.667	1.58	1.75
Factor 2	120	4.4852	.41994	38.742	119	.000	1.48519	1.4093	1.5611

Assignment and Assessment

Among the loaded 7 elements, the reliability value Cronbach's Alpha = 0.723 is significant because it is higher than 0.6. The highest rated value of this factor Assignment and Assessment is 4.37, with the mean difference of 1.37 and a standard deviation of 0.66, and $p<0.05$. Likewise, the Lowest rated average value of this component is 3.57, whose standard deviation is 1.15 and mean difference is 0.57 Similarly, the average rated factor value of this component is 3.92, whose standard deviation is 0.646 and mean difference is 0.92. It means the respondent rated higher value greater than test value (neutral value =3). Moreover, respondents agreed over the statement which are related with assignment and assessment. Respondents were agreed about the lack of the assignment tasks in the abstract algebra course makes it difficult for me in final examination, lack of clear guidelines for assignments in the abstract algebra course hinders my ability to complete them effectively, the assessments (exams, quizzes, etc.) in the abstract algebra course adequately reflects the concepts covered in the assignments and the grading criteria for assessments in the abstract algebra course are unclear, making it challenging to meet the expectations, objective questions in the abstract algebra course adds unnecessary stress and pressure, theoretical teaching styles and final examination system in the abstract algebra makes me difficult to learn and the abstract algebra course does not provide sufficient opportunities for practical application of theoretical concepts.

By using a variety of pedagogical strategies designed to accommodate students' varied learning needs, barriers to abstract algebra comprehension at the college level can be removed (Hazzan, 1999). This entails giving students a supportive learning environment where they feel comfortable asking questions and seeking clarification, giving clear explanations of abstract concepts, offering plenty of opportunities for active engagement through problem-solving exercises and group discussions, and using concrete examples and visual aids to illustrate complex ideas (Gaonkar, 2017).
(See Table 6).

Table 6: *Descriptive statistics and one- sample t-test in assignment and assessment*

Test Value =3								95% Confidence Interval of the Difference	
Components	N	Mean	S D	t	df	Sig(two-tailed)	Mean Difference	Lower	Upper
A &A1	120	3.83	.901	10.128	119	.000	.833	.67	1.00
A&A2	120	3.57	1.150	5.396	119	.000	.567	.36	.77
A&A3	120	3.87	.925	10.261	119	.000	.867	.70	1.03
A&A 4	120	4.03	.755	14.989	119	.000	1.033	.90	1.17
A&A 5	120	3.93	1.002	10.204	119	.000	.933	.75	1.11
A&A 6	120	3.87	.888	10.690	119	.000	.867	.71	1.03
A&A 7	120	4.37	.660	22.677	119	.000	1.367	1.25	1.49
Factor 3	120	3.9238	.64660	15.651	119	.000	.92381	.8069	1.0407

Traditional Teaching Methods

Among the loaded 7 elements, the reliability value Cronbach's Alpha = 0.62 is significant because it is higher than 0.6. The average rated value of this factor is 3.86, with the mean difference of 0.86 and a standard deviation of 0.45, and $p<0.05$. Likewise, the highest rated average value of this component is 4.57, whose standard deviation is 0.561 and mean difference is 1.57. Similarly, the lowest rated average value of this component is 2.70, whose standard deviation is 1.16 and mean difference is -0.30. The respondents were agreed that the items traditional teaching methods does not support students' understanding of abstract algebra concepts, traditional teaching techniques discourage active participation and discussion among students in the abstract algebra, traditional teaching methods does not provide to different learning styles and preferences in the abstract algebra course, interaction and engagement are not facilitated by traditional teaching methods in the abstract algebra course. Likewise, respondents disagreed that in traditional teaching techniques are helping the students to achieve their learning goals in the abstract algebra course and traditional teaching techniques accommodate the diverse backgrounds and experiences of students in the abstract algebra course. In this regards all items were significantly difference at 0.05 ($p < 0.05$) (See Table 7). Students can build a strong foundation in advanced mathematics by breaking down the complexity of abstract algebra, especially in the areas of group, ring, and field structure comprehension (Hausberger, 2020). This will empower them to confidently and proficiently pursue higher-level studies and applications. Removing obstacles to understanding these intangible ideas fosters inclusion and more involvement in mathematical discourse, which in turn improves our collective comprehension and application of mathematical concepts in a variety of contexts.

Table 7: *Descriptive statistics and one- sample t-test in traditional teaching methods*

Test Value =3								95% Confidence Interval of the Difference	
Components	N	Mean	S D	t	df	Sig(two-tailed)	Mean Difference	Lower	Upper
T &T1	120	4.37	.660	22.677	119	.000	1.367	1.25	1.49
T&T2	120	4.10	.749	16.083	119	.000	1.100	.96	1.24
T&T3	120	4.57	.561	30.586	119	.000	1.567	1.47	1.67
T&T 4	120	3.20	1.017	2.153	119	.033	.200	.02	.38
T&T 5	120	4.10	.834	14.446	119	.000	1.100	.95	1.25
T&T 6	120	2.83	1.162	-1.571	119	.119	-.167	-.38	.04
T&T 7	120	2.70	1.164	-2.824	119	.006	-.300	-.51	-.09
Factor 4	120	3.8611	.45750	20.619	119	.000	.86111	.7784	.9438

Conclusion

Abstract algebra can be difficult to understand at the undergraduate level since it differs from practical mathematical topics that are taught earlier in the curriculum. Students studying abstract algebra are exposed to algebraic theories and structures that prioritize abstraction and generalization, which might be difficult at first to comprehend. Students studying abstract algebra must think in terms of groups, ring, field of their operations, and axioms, which may appear unrelated to their previous mathematical experiences, in contrast to mathematics or basic algebra. Furthermore, a greater degree of mathematical maturity and logical rigor are required due to the emphasis on proofs and theoretical reasoning. In order to bridge the gap between concrete and abstract mathematical thinking and ultimately get a deeper knowledge of algebraic concepts, overcoming this barrier will require time, patience, practice, and exposure to a variety of examples and applications.

Several strategies must be used to assist students in properly understanding the abstract concepts in order to overcome the obstacle of learning abstract algebra at the undergraduate level. First of all, giving students a strong foundation in preparatory mathematics subjects like basic concept of group, action of set on a group, ring, field and extension field might aid in their understanding of the basic ideas behind abstract algebra. Students can have a deeper comprehension of the content by finding abstract concepts more relatable and approachable when they are taught through the use of concrete examples and applications.

Problem-solving sessions, and interactive conversations are examples of active learning strategies that can captivate students and motivate them to actively investigate and apply abstract algebraic concepts. It will take time, effort, and exposure to a variety of examples and applications to get obstacle and close the gap between concrete and abstract mathematical thinking. Students can improve their understanding of abstract algebra and detect and correct misunderstandings by receiving frequent feedback and practice in assignments and self-evaluation. Encouraging a welcoming and inclusive classroom where students are at comfort asking questions and looking for explanation will help students become more confident and motivated to take on the challenges of abstract algebra.

When students approach abstract algebra with an open mind, free from the burden of prejudices from past mathematical experiences, their existing knowledge can sometimes reduce the barrier to studying the topic like group, ring and field. Regular assignment and assessment programs facilitate the learning of abstract algebra by giving students continual opportunity for practice, reinforcement, and feedback. This helps them recognize and correct misunderstandings and strengthen their grasp of abstract algebra concepts.

References

Adom, D., & Hussein, E. K. (2018). Theoretical and conceptual framework: Mandatory ingredients of a quality research. *International Journal of Scientific Research, 7(*1), 437-441

Agustyaningrun, N., Abodi, A. M., Sari, R. N & Mahmudi, A. (2018). An analysis of students' error in solving abstract algebra tasks. Journal of Physics: Conf. Series https://doi.org/10.1088/1742.65961109711/012118

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching. *Journal of Teacher Education, 59*(5), 389-407, https://doi.org/10.1177/0022487108324554

Dummit, D. S., & Foote, R. M. (2008). *Abstract algebra*. India: Wiley Wast House.

Faizah, S., Nusantara, T., Sudirman, S., & Rahardi, R. (2020). Exploring students' thinking process in mathematical proof of abstract algebra based on Mason's framework. *Journal for the Education Gifted, 8*(2), 871-884, https://doi.org/10.17478/jegys.689809

Feldnon, D. F. (2007). Cognitive load and classroom teaching: The double- edged sword of automatically. *Educational Psychology, 42* (3), 123- 137 https://doi.org/10.1080/00461520701416173

Fraleigh, J. B. (1984). *A first course in abstract algebra*. New Delhi: Narosa Publishing house.

Gallian, J. A. (2013). *Contemporary abstract algebra*. Delhi: Cengage Learning India Pvt.

Gaonkar, V. S. (2017). A study on algebra of groups and rings structures in mathematics. *International Journal of Scientific and Innovative Mathematical Research, 5*(1), 24-29 https://doi.org/10.20431/2347-3142.0501006

Goyal, J. K., Gupta, K. P., Gupta, G. S., & Gupta, S. S. (2015). *Advanced course in modern algebra.* Meerut: Pragati Prakashan.

Hausberger, T. (2020). Abstract algebra teaching and learning. In: Lerman, S. (eds) *Encyclopedia of Mathematics Education.* Springer, Cham https://doi.org/10.1007/978-3-030-15789-0_100022

Hazzan, O. (1999). Reducing abstraction level when learning abstract algebra concepts. *Educational Studies in Mathematics, 40*(1), 71-90, https://doi.org/10.1023/1003780613628

Herstein, I. N. (2006). *Topic in algebra.* India: John Wiley and Sons.

Howe, R. M. (2022). *The group algebra.* In an invitation to representation theory. Springer undergraduate mathematics series. Springer, Cham. https://doi.org/10.1007/978-3-030-98025-2_6

Hungerford, T. W. (1974). *Algebra.* New York: New York Inc. Springer Verlog.

Israel, K. (2007). A history of abstract algebra. https://doi.org/10.1007/978-0-8176-4685-1

Johnson, E., Keller, R., Fukawa, - Connelly, T. (2017). Results from a survey of abstract algebra instructors across the United States: Understanding the choice to (Not) Lecture. *Int. J. Res. Undergraduate Math Education,* 1-32 https://doi.org/10.1007/s40753-017-0058-1

Kanna, V. K., & Bhambri, S. K. (2013). *A course in abstract algebra.* India: Vikas Publishing House Pvt Ltd.

Kirschner, P. A., Kirschner, F., & Pass, F. (2009). Cognitive load theory. In E. M. Anderman, & L. H. Anderman (Eds). Psychology of classroom learning: *An encyclopedia (*Vol1 (a-j), pp205-209

Lang, S. (1994). *Algebra*. New York: New York Inc, Springer Verlog.

Lanski, C. (2010). *Concepts in abstract algebra*. American Mathematical Society.

Leppink, J. (2017). Cognitive load theory: Practical implications and an important challenge. *Journal of Taibah University Medical Sciences, 12*(5), 385-391

Leron, U., & Dubinsky, E. (1995). An Abstract Algebra Story, The American Mathematical Monthly, 102:3, 227-242, https://doi.org/10.1080/00029890.1995.11990563

Manandhar, R., & Sharma, L. (2021). Strategies of learning abstract algebra. *International Journal of Research, GRANTHAALAYAH*, *9*(1), 1-6 https://doi.org/10.29121/granthaalayah.v9.i1.2021.2697

Mc Carty, C., Redmond, P. & Peel, K. (2021). Teacher decision making in the classroom: the influence of cognitive load and teacher affect. *Journal of Education for Teaching*, 1-15 https://doi.org/10.1080/02607476.2021.1902748

McGinn, K. M., Lange, K. E., & Booth, J. (2015). A worked example for creating worked examples. *Mathematics Teaching in the Middle School, 21*(1), 27-33 https://doi.org/10.5951/mathteacmiddscho.21.10026

Saracino, D. (2017). *Abstract algebra*. New Delhi: Waveland Press, Inc.

Setianingrum, R. S., Syamsuri, S., & Setiani, Y. (2020). Analyzing the students' learning difficulties in algebra. *Jurusan Pendidican Matematika, 8*(1), 19-34

Sleeman, D. (1986). Introductory algebra: A case study of student misconceptions. *Journal of Mathematical Behavior*, *5*(1), 25-32

Ticknor, C. S. (2012). Situated learning in an abstract algebra classroom. *Educ. Stud Math, 81*, 307-323. https://doi.org/10.1007/s10649-012-9405-y

Vasishtha, A. R., & Vasishtha, A. K. (2000). *Modern algebra*. India: Krishna Prakashan Media (P) Ltd, Meerut.

Wasserman, N. H. (2017). Making sense of abstract algebra: Exploring secondary teachers understanding of inverse functions in relation to its group structure. *Mathematical Thinking and Learning, 19*(3), 18201, https://dx.doi.org/10.1080/10986065.2017.1328635

Xue, Z. (2022). Group theory and ring theory. *Journal of Physics Conference Series, 2386*(1), 012024 https://doi.org/10.1088/1742-6596/2386/1/012024

Zaffar, A., Qureshi, S. M., & Ansari, R. K. (2013). Teaching of abstract algebra at undergraduate level. *The S. U. Journal of Edu*, 39, 1-12 https://shorturl.at/cekqs