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Mathematical Models on Mechanics of Biofluids

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Abstract

In this work, we study the mathematical models of flows for some biofluids. In biomechanics, peristaltic flow plays an important role in which the motion generated in the fluid contained in a distensible tube when a progressive wave of area contraction and expansion travels along the wall of the tube. We consider theeffectof elasticity of the tube wall in the flow through the progressive wave travelling along its length without its direct calculation. Since the no – slip condition has used on a moving undulating wall surface, it determines the sinusoidalboundary conditions on the upper andlower wall of the tube.The wide occurrence of peristaltic motion gives its result physiologically from neuro-muscular properties of any tubular smooth muscle.

Key Words: Biofluids, Progressive wave, Undulating wall surface, Peristaltic motion.

Introduction

Peristaltic flow is the motion generated in the fluid contains in a distensible tube when a progressive wave of area contraction and expansion travels along the wall of the tube [1]. The elasticity of the tube wall does not directly enter into our calculations, but it affects the flow through the progressive wave travelling along its length. This wave determines the boundary conditions since the no-slip condition has to be used now on a moving undulating wall surface. Peristaltic motion is involved in expansion and contractions (or vasomotion) of small blood vessels, Celia transport through the ducts efferentes of the male reproductive organs, transport of spermatozoa in cervical canal, transport of chime in small intestines, function of ureter, and transport of bile etc. [1] [2]. The wide occurrence of peristaltic motion gives the result physiologically from neuro-muscular properties of any tubular smooth muscle. Physiological fluids in humans or animals are, in general, propelled by the continuous periodic muscularcontraction or expansion (or both) of the ducts through which the fluid passes. In particular, peristaltic mechanismsmay be involved in the swallowing of food through the esophagus, vasomotion of small blood vessels,spermatic flows in the ductus efferentes of the male reproductive tract, embryo transport in the uterus, andtransport of urine through the ureter, among others [3][4][5]. Although physiological fluid flows are similar with respect to peristalsis, their main differences lie in thefluid being transported, the geometry of the vessel or cavity, and the wave form [6]. Newtonian and non-Newtonian fluids have been considered in ureteral, esophageal, and vasomotion peristalsis, e.g., Newtonianfor urine, a power-law for the food bolus, and a Casson fluid for blood. Mostly, two-dimensional and axisymmetricgeometries are studied, and a sinusoidal wave form is generally employed [7] [8] [9]. The main motivationfor any mathematical analysis of physiological fluid flows is to ultimately have a better understanding of theparticular flow being modeled. There are some differences between peristalsis in different physiological systems. Thus we have chosen to concentrate on the mathematical models that describe the peristaltic motion in tube and channel with long-wavelength. The model is solved by expanding the stream function, which determines the flows. In the flow phenomena, there is a prescribed pressure gradient along the tube or channel and a progressive wave passes through the walls [10].

In this paper, we consider peristaltic motion in channels or tubes. The fluid involved may be non-Newtonian e.g. power-law, viscoelastic, or micropolar fluid or Newtonian, and the flows may take place in two layers a core layer and a peripheral layer. The equations of motion in their complete generality do not admit of simple solutions and we have to look for reasonable approximations.

1. Peristaltic Motion in a Channel.

Let u(x, y, t), v(x, y, t), and p(x, y, t) denote respectively the two velocity components and pressure at the point (x, y) at time t in a fluid with constant density ρ and viscosity coefficient μ. Then the equation of continuity, which expresses the fact that the amount of fluid entering a unit volume per unit time is the same as the amount of the fluid leaving it per unit time, is given by[1]

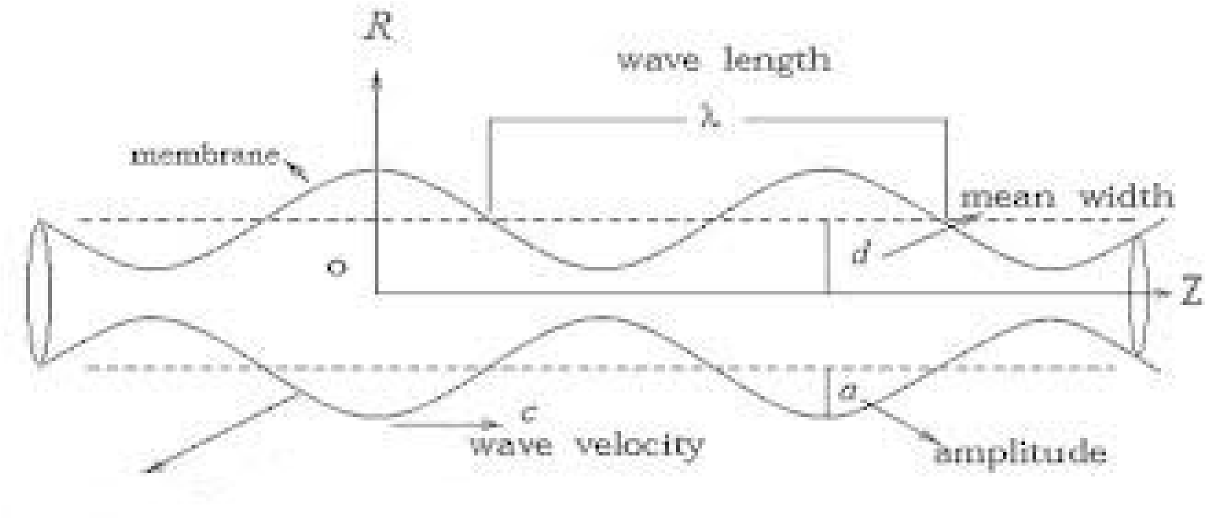
$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \qquad (2)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right). \qquad (3)$$

We consider the flow of a homogeneous Newtonian Fluid through a channel of width 2a. Travelling sinusoidal waves are superposed on the elastic walls of the channel. Taking the x -axis along the center line of the channel and the y-axis normal to it. The equations of the walls are given by [1] [2]

$$Y=\eta(X,T)=\pm a\left[1+\boldsymbol{\epsilon\cos}\left\{\left(\frac{2\pi}{\lambda}\right)(\boldsymbol{X}-\boldsymbol{cT})\right\}\right] \qquad (\mathbf{4})$$



Where $\in$ is the amplitude ratio, λ the wave length, and c the phase velocity of the waves.Now equation (1) can be satisfied by introducing the stream function $\varphi(x, y)$ which is such that

$$\nu\nabla^4\Psi = \nabla^2\Psi_T + \Psi_Y\nabla^2\Psi_X - \Psi_X\nabla^2\Psi_Y$$

Or

$$\nu\nabla^2\Psi = \frac{\partial}{\partial T}\nabla^2\Psi + \frac{\partial\Psi}{\partial Y}.\frac{\partial}{\partial X}\nabla^2\Psi - \frac{\partial\Psi}{\partial X}.\frac{\partial}{\partial Y}\nabla^2\Psi \quad (5)$$

Where the velocity components are given by

$$u = \Psi_Y = \frac{\partial\Psi}{\partial Y}, v = -\Psi_X = -\frac{\partial\Psi}{\partial X} \quad (6)$$

Assuming that the walls have transverse displacements at all times, we get the boundary conditions as

$$u = 0, v = \pm\frac{2\pi ac\in}{\lambda}\sin\{\frac{2\pi}{\lambda}(X - cT)\} at Y = \pm\eta(X, T) \quad (7)$$

Where

$$v = \frac{dY}{dT} = \frac{d}{dT}\left[\pm a[1+\in\cos\{\frac{2\pi}{\lambda}(X - cT)\}]\right]$$

$$= \pm a\in\sin\{\frac{2\pi}{\lambda}(X - cT)\}.-\frac{2\pi}{\lambda}.c$$

$$= \pm\frac{2\pi ac\in}{\lambda}\sin\{\frac{2\pi}{\lambda}(X - cT)\}$$

We now introduce the dimensionless variables and parameters

$$x=\frac{X}{\lambda}=\frac{dis\tan ce}{wavelength}=\frac{L}{L}=1.$$

$$y=\frac{Y}{a}=\frac{dis\tan ce}{semi-width}=\frac{L}{L}=1.$$

$$t=\frac{cT}{\lambda}=\frac{velocity\times time}{wavelength}=\frac{LT^{-1}\times T}{L}=1.$$

$$\psi=\frac{\Psi}{ac}=\frac{areaper\sec ond}{semi-width\times velocity}=\frac{L^2T^{-1}}{L\times LT^{-1}}=1..........(8)$$

$$\delta=\frac{a}{\lambda}=\frac{semi-width}{wavelength}=\frac{L}{L}=1.$$

$$\mathrm{Re}=\frac{ac}{\nu}=\frac{semi-width\times velocity}{kinematicvis\cos ity}=\frac{ML^{-1}T^{-1}}{ML^{-1}T^{-1}}=1............(9)$$

So that equation (5) becomes, for this
We have

$$\nabla^2=\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}=\left[\frac{1}{\lambda^2}\frac{\partial^2}{\partial x^2}+\frac{1}{a^2}\frac{\partial^2}{\partial y^2}\right]$$

$$=\frac{\delta^2}{a^2}\frac{\partial^2}{\partial x^2}+\frac{1}{a^2}\frac{\partial^2}{\partial y^2}=\frac{1}{a^2}\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]$$

This implies that

$$\nu\nabla^4\Psi=\frac{ac}{\mathrm{Re}}\nabla^4 ac\psi$$

$$=\frac{a^2c^2}{\mathrm{Re}}\nabla^4\psi$$

$$=\frac{a^2c^2}{\mathrm{Re}}\left[\frac{1}{\lambda^2}\frac{\partial^2}{\partial x^2}+\frac{1}{a^2}\frac{\partial^2}{\partial y^2}\right]^2\psi.$$

$$=\frac{c^2}{a^2\,\mathrm{Re}}\left[\frac{a^2}{\lambda^2}.\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]^2\psi$$

$$=\frac{c^2}{a^2\,\mathrm{Re}}\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]^2\psi.$$

$$\frac{\partial \Psi}{\partial T} = \frac{\partial ac\psi}{\partial T} = \frac{\partial ac\psi}{\partial t}.\frac{\partial t}{dT} = ac\frac{\partial \psi}{\partial t}.\frac{\partial (c\frac{T}{\lambda})}{dT} = ac\frac{\partial \psi}{\partial t}.\frac{c}{\lambda}$$
$$= ac^2\frac{\partial \psi}{\partial t}.\frac{1}{\lambda} = ac^2\frac{\partial \psi}{\partial t}\frac{\delta}{a} = \delta c^2\frac{\partial \psi}{\partial t}$$
$$= \delta c^2 \psi_t.$$
$$\therefore \frac{\partial \Psi}{\partial T} = \delta c^2 \psi_t.$$

$$\Psi_Y = \frac{\partial \Psi}{\partial Y} = \frac{\partial ac\psi}{\partial Y} = ac\frac{\partial \psi}{\partial Y} = ac\frac{\partial \psi}{\partial y}.\frac{\partial y}{\partial Y}$$
$$= ac\frac{\partial \psi}{\partial y}.\frac{\partial (\frac{Y}{a})}{\partial Y} = ac\frac{\partial \psi}{\partial y}.\frac{1}{a} = c\frac{\partial \psi}{\partial y}$$
$$= c\psi_y$$
$$\therefore \Psi_Y = c\psi_y.$$

$$\Psi_X = \frac{\partial \psi}{\partial X} = \frac{\partial ac\psi}{\partial X} = ac\frac{\partial \psi}{\partial X} = ac\frac{\partial \psi}{\partial x}.\frac{\partial x}{\partial X}$$
$$= ac\frac{\partial \psi}{\partial x}.\frac{\partial (\frac{X}{\lambda})}{\partial X} = ac\frac{\partial \psi}{\partial x}.\frac{1}{\lambda}$$
$$= \frac{a}{\lambda}c\psi_x = \delta c\psi_x.$$
$$\therefore \Psi_X = \delta c\psi_x.$$

$$\nabla^2 \Psi_X = \left(\frac{1}{\lambda^2}\frac{\partial^2}{\partial x^2} + \frac{1}{a^2}\frac{\partial^2}{\partial y^2}\right)\delta c\psi_x$$
$$= \left(\frac{\delta^2}{a^2}\frac{\partial^2}{\partial x^2} + \frac{1}{a^2}\frac{\partial^2}{\partial y^2}\right)\delta c\psi_x$$
$$= \frac{1}{a^2}\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\delta c\psi_x.$$
$$\therefore \nabla^2 \Psi_X = \frac{1}{a^2}\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\delta c\psi_x.$$

And

$$\nabla^2\Psi_Y = \left(\frac{1}{\lambda^2}\frac{\partial^2}{\partial x^2}+\frac{1}{a^2}\frac{\partial^2}{\partial y^2}\right)c\psi_y$$

$$=\left(\frac{\delta^2}{a^2}\frac{\partial^2}{\partial x^2}+\frac{1}{a^2}\frac{\partial^2}{\partial y^2}\right)c\psi_y$$

$$=\frac{1}{a^2}\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)c\psi_y.$$

$$\therefore \nabla^2\Psi_Y = \frac{1}{a^2}\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)c\psi_y$$

Now equation

$$\nu\nabla^2\Psi = \nabla^2\Psi_T + \Psi_Y\nabla^2\Psi_X - \Psi_X\nabla^2\Psi_Y.\text{Reduces to}$$

$$\frac{c^2}{a^2\,\mathrm{Re}}\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]^2\psi = \frac{1}{a^2}\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\delta c^2\psi_t$$

$$+c\psi_y.\frac{1}{a^2}\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\delta c\psi_x - \delta c\psi_x\frac{1}{a^2}\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]c\psi_y.$$

$$\Rightarrow \frac{c^2}{a^2}.\frac{1}{\mathrm{Re}}\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]^2\psi = \delta\frac{c^2}{a^2}\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\psi_t$$

$$+\delta\frac{c^2}{a^2}\psi_y\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\psi_x$$

$$-\delta\frac{c^2}{a^2}\psi_x\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\psi_y.$$

$$\frac{c^2}{a^2}.\frac{1}{\mathrm{Re}}\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]^2\psi = \delta.\frac{c^2}{a^2}\left[\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\psi_t + \psi_y\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\psi_x - \psi_x\left[\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right]\psi_y\right]$$

$$\frac{1}{\delta\,\mathrm{Re}}\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2\psi = \left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_t + \psi_y\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_x$$

$$-\psi_x\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_y \quad\text{...............(10)}$$

The boundary conditions become

$\psi_y = 0[\because U = 0],$

$\psi_x = 2\pi \in \sin(x - \in)$..............(11)

$$\because \frac{\partial \psi}{\partial x} = \frac{\partial \frac{\Psi}{ac}}{\partial X} \times \frac{\partial X}{\partial x} = \frac{1}{ac} . \frac{2\pi ac \in}{\lambda} \sin\{\frac{2\pi}{\lambda}(x\lambda - t\lambda)\} \times \lambda$$

$= 2\pi \in \sin\{2\pi(x-t)\}.$

Thus the basic partial differential equations and the boundary condition together involve three dimensionless parameters:

i) The Reynolds number Re determined by the phase velocity, half the mean distance between the plates, and the kinematic viscosity.(This number is small if the distance between the walls is small or the phase velocity small or the kinematic viscosity is large.)

ii) The wave number δ which is small if the wave length is large as compared to the distance between the walls.

iii) The amplitude ratio which is small if the amplitude of the wave is small as compared to the distance between the walls.

In obtaining the equations for the stream function, the pressure gradient was eliminated. Hence there may arise a fourth dimensionless parameters, depending on the pressure gradient. Non-Newtonian fluids give rise to additional dimensionless parameters, depending on the parameters occurring in the constitutive equations of the fluids.

It is not possible to solve equation 2 for arbitrary values of δ, Re and, Therefore, this equation is solved under ,among others, the following alternatives sets of assumptions:

i) $\boldsymbol{\epsilon} << 1$, and stokes' assumption of slow motion so that inertial terms can be neglected.

ii) $\boldsymbol{\epsilon} << 1, \delta << 1.$

iii) $\delta << 1, Re << 1$, but $\boldsymbol{\epsilon}$ is arbitrary.

iv) $\boldsymbol{\epsilon} << 1, Re << 1$, but δ is arbitrary.

The initial flow may be taken as the Hagen – poiseuille flow [1].

2. Long- Wavelength Approximation to peristaltic Flow in a Tube

Let the equation of the tube surface be given by [1][2] [5]

$$h(Z,t) = a\left[1 + \boldsymbol{\epsilon sin}\left\{\left(\frac{2\pi}{\lambda}\right)(\boldsymbol{Z} - \boldsymbol{cT})\right\}\right]$$

(12)

Where a is the undisturbed radius of the tube, $\boldsymbol{\epsilon}$ the amplitude ratio, a $(1+\boldsymbol{\epsilon})$ and $a(1-\in)$ are the maximum and minimum disturbed radii and λ is the wave length, and c the phase velocity.

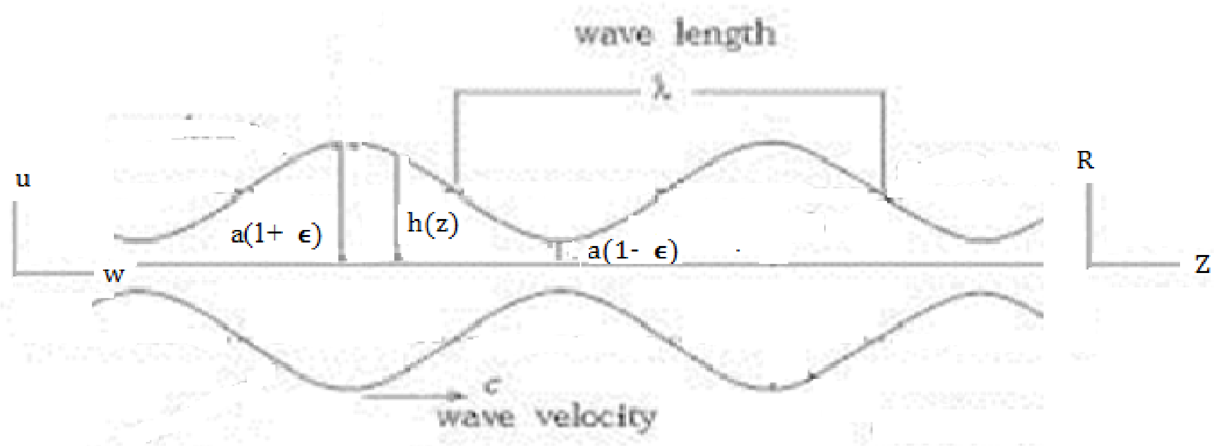


Fig. 1 Tube geometry.

Under the assumptions $\frac{a}{\lambda} << 1$. And $\frac{ac}{\nu} << 1$, we can conduct an order of magnitude study of the various terms in the equation of continuity and equations of motion in cylindrical polar coordinates to find

$$\frac{\partial P}{\partial R} << \frac{\partial P}{\partial Z} \qquad (13)$$

So that P is only weakly dependent on R and we can take

$$P = P(Z, t) \qquad (14)$$

Now it is convenient to use the moving coordinate system (r, Z) travelling with the wave so that

$$r = R, z = Z - ct \qquad (15)$$

In this system, P is a function of Z only. The equations of continuity and motion reduce respectively to

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rw) = 0 \qquad (16)$$

$$\frac{dp}{dz} = \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right) = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right), \qquad (17)$$

Where u and w are the velocity components for the motion of the fluids in relation to the moving coordinate system

The boundary conditions for solving (16) and (17) are

$$u = \frac{\partial h}{\partial t}, \quad w = -c \quad at\ r = h \qquad (18)$$

Integrating (17) at the constant z we obtain

i.e. $\frac{dp}{dz} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)$

$$\frac{dp}{dz}r = \mu\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right)$$

$$\frac{dp}{dz}\int rdr = \mu\int\partial\left(r\frac{\partial w}{\partial r}\right)$$

$\frac{dp}{dz}\frac{r^2}{2} = \mu\, r\,\frac{\partial w}{\partial r} + k$ (*)

at the boundary condition, $r = h,\ \ w = -c$

$$\frac{dp}{dz}\frac{h^2}{2} = \mu\, h\,\frac{d(-c)}{dh} + k$$

$$\frac{dp}{dz}\frac{h^2}{2} = k$$

Then (*) becomes

$$\frac{dp}{dz}\frac{r^2}{2} = \mu\, r\,\frac{\partial w}{\partial r} + \frac{dp}{dz}\frac{h^2}{2}$$

$$-\frac{dp}{dz}\frac{h^2}{2} + \frac{dp}{dz}\frac{r^2}{2} = \mu\, r\,\frac{\partial w}{\partial r}$$

$$-\frac{1}{2\mu}\frac{dp}{dz}\frac{(h^2 - r^2)}{r} = \frac{\partial w}{\partial r}$$

$$-\frac{1}{2\mu}\frac{dp}{dz}\frac{(h^2 - r^2)}{r}\partial r = \partial w$$

Again integrating,

$-\frac{1}{2\mu}\frac{dp}{dz}\left[h^2\log r - \frac{r^2}{2}\right] = w + k_1$ (*,*)

Again for the given boundary condition

$$-\frac{1}{2\mu}\frac{dp}{dz}\left[h^2\log h - \frac{h^2}{2}\right] = -c + k_1$$

Then (*,*) becomes

$$-\frac{1}{2\mu}\frac{dp}{dz}\left[h^2\log r - \frac{r^2}{2}\right] = w + c - \frac{1}{2\mu}\frac{dp}{dz}\left[h^2\log h - \frac{h^2}{2}\right]$$

$$-\frac{1}{2\mu}\frac{dp}{dz}\left[h^2\log r - \frac{r^2}{2}\right] + \frac{1}{2\mu}\frac{dp}{dz}\left[h^2\log h - \frac{h^2}{2}\right] = w + c$$

$$-\frac{1}{2\mu}\frac{dp}{dz}\left[h^2\log r - \frac{r^2}{2} - h^2\log h + \frac{h^2}{2}\right] = w + c$$

$$-\frac{1}{2\mu}\frac{dp}{dz}\frac{1}{2}(h^2 - r^2) = w + c$$

$w = -c - \frac{1}{4\mu}\frac{dp}{dz}(h^2 - r^2)$ (19)

To an observer moving with velocity *c* in the axial direction, the pressure and flow appear stationary. Hence the flow rate *q* measured in the moving coordinate system is a constant, independent of position and time [1] [5].

Now,

$q = 2\pi \int_0^h r\, w\, dr$ (20)

using (19) we have

$$q = 2\pi \int_0^h r \left[-c - \frac{1}{4\mu}\frac{dp}{dz}(h^2 - r^2) \right] dr$$

$$q = 2\pi \int_0^h \left[-rc - \frac{1}{4\mu}\frac{dp}{dz}(h^2 r - r^3) \right] dr$$

$$q = 2\pi \left[-c\int_0^h r dr - \frac{1}{4\mu}\frac{dp}{dz}\left\{ \int_0^h h^2 r dr - \int_0^h r^3 dr \right\} \right]$$

$$q = 2\pi \left[-c\frac{h^2}{2} - \frac{1}{4\mu}\frac{dp}{dz}\left\{ \frac{h^4}{2} - \frac{h^4}{4} \right\} \right]$$

$q = -\pi c h^2 - \frac{\pi h^4}{8\mu}\frac{dp}{dz}$ (21)

$$-\frac{\pi h^4}{8\mu}\frac{dp}{dz} = q + \pi c h^2$$

$\frac{dp}{dz} = -\frac{8\mu q}{\pi h^4} - \frac{8\mu c}{h^2}$ (22)

Substituting in (19) we get,

$$w = -c - \frac{1}{4\mu}\left[-\frac{8\mu q}{\pi h^4} - \frac{8\mu c}{h^2} \right](h^2 - r^2)$$

$$w = -c - \frac{8\mu}{4\mu}\left(\frac{q}{\pi h^4} + \frac{c}{h^2} \right)(h^2 - r^2)$$

$w = -c - 2\left(\frac{q}{\pi h^4} + \frac{c}{h^2} \right)(h^2 - r^2)$ (23)

To find the transverse velocity component *u*, we integrate the continuity equation (16) at the constant z. remembering that *u* = 0 at *r* = 0. We obtain

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rw) = 0$$

$$\partial(ru) = -\frac{\partial}{\partial z}(rw)dr$$

Integrating from 0 to r

$$\int_0^r (ru) = -\int_0^r \frac{\partial}{\partial z}(rw)dr$$

$$ru = -\int_0^r r\frac{\partial w}{\partial z}dr$$

$$u = -\frac{1}{r}\int_0^r r\frac{\partial w}{\partial z}dr$$

$$u = -\frac{1}{r}\int_0^r r\frac{\partial w}{\partial h}\frac{\partial h}{\partial z}dr$$

From (23), $w = -c - 2\left(\frac{q}{\pi h^4} + \frac{c}{h^2} \right)(h^2 - r^2)$

$$\frac{\partial w}{\partial h} = \frac{\partial(-c)}{\partial h} + 2\left(\frac{q}{\pi h^4} + \frac{c}{h^2}\right)2h + 2\,(h^2 - r^2)\left(-\frac{4q}{\pi h^5} - \frac{2c}{h^3}\right)$$

$$\frac{\partial w}{\partial h} = \frac{4q}{\pi h^3} + \frac{4c}{h} - 2\,(h^2 - r^2)\left(\frac{4q}{\pi h^5} + \frac{2c}{h^3}\right)$$

$$\frac{\partial w}{\partial h} = \frac{4q}{\pi h^3} + \frac{4c}{h} - \frac{8h^2q}{\pi h^5} - \frac{4h^2c}{h^3} - \frac{8r^2q}{\pi h^5} + \frac{4r^2c}{h^3}$$

$$\frac{\partial w}{\partial h} = \frac{4q}{\pi h^3} + \frac{4c}{h} - \frac{8q}{\pi h^3} + \frac{8r^2q}{\pi h^5} + \frac{4r^2c}{h^3} - \frac{4c}{h}$$

$$\frac{\partial w}{\partial h} = -\frac{4q}{\pi h^3} + \frac{8r^2q}{\pi h^5} + \frac{4r^2c}{h^3}$$

Now,

$$u = -\frac{1}{r}\frac{dh}{dz}\int_0^r r\left(-\frac{4q}{\pi h^3} + \frac{8r^2q}{\pi h^5} + \frac{4r^2c}{h^3}\right)dr$$

$$u = -\frac{1}{r}\frac{dh}{dz}\left[\int_0^r \left(-\frac{4rq}{\pi h^3} + \frac{8r^3q}{\pi h^5} + \frac{4r^3c}{h^3}\right)dr\right]$$

$$u = -\frac{1}{r}\frac{dh}{dz}\left[-\frac{4qr^2}{2\pi h^3} + \frac{8r^4q}{4\pi h^5} + \frac{4r^4c}{4h^3}\right]$$

$$u = -\frac{dh}{dz}\left[\frac{cr^3}{h^3} - \frac{2qr}{\pi h^3} + \frac{2qr^3}{\pi h^5}\right] \qquad (24)$$

We now revert to the stationary coordinate system with the coordinates R, Z, the velocity components U, W and the flow rate Q, so that

W = w + c, U = u (25)

$$Q = 2\pi\int_0^h WR\,dR$$

$$Q = 2\pi\int_0^h (w + c)R\,dR$$

$$Q = 2\pi(w + c)\int_0^h R\,dR$$

$$Q = 2\pi(w + c)\left[\frac{R^2}{2}\right]_0^h$$

$$Q = 2\pi(w + c)\frac{h^2}{2}$$

$$Q = \pi wh^2 + \pi ch^2$$

Using (20), since *w* is independent of *r*

$$Q = q + \pi ch^2$$

Let $\bar{Q}$ denote the time average of *Q* over a complete time period *T* for *h*, so that

$T = \frac{\lambda}{c}$ (26)

$$\bar{Q} = \frac{1}{T}\int_0^T Q\,dt$$

$$\bar{Q} = \frac{1}{T}(q + \pi c h^2)T$$

$$\bar{Q} = (q + \pi c h^2)$$

$$\bar{Q} = q + \pi c a^2\left(1 + \frac{1}{2}\epsilon^2\right) \qquad (27)$$

Again,

From (12) and (15)

$h(Z,t) = a\left[1 + \epsilon sin\left\{\left(\frac{2\pi}{\lambda}\right)(Z - cT)\right\}\right]$ and r= R, $z = Z - ct$

$$h(Z,t) = a\left[1 + \epsilon sin\left\{\left(\frac{2\pi}{\lambda}\right)(Z - cT)\right\}\right] = a\left[1 + \epsilon sin\left(\frac{2\pi}{\lambda}z\right)\right] \qquad (28)$$

$$h(Z) = a\left[1 + \epsilon sin\left(\frac{2\pi}{\lambda}z\right)\right]$$

$$\frac{dh}{dz} = a\varepsilon\frac{2\pi}{\lambda}\cos\left(\frac{2\pi}{\lambda}z\right)$$

$$\frac{dh}{dz} = \frac{2a\pi\varepsilon}{\lambda}\cos\left(\frac{2\pi}{\lambda}(Z - ct)\right) \qquad (29)$$

From (4), (12), (14) and U = u

$$U = -\frac{dh}{dz}\left[\frac{cr^3}{h^3} - \frac{2qr}{\pi h^3} + \frac{2qr^3}{\pi h^5}\right] \qquad (30)$$

From (15) and (29), equation (30) becomes

$$U = -\frac{2a\pi\varepsilon}{\lambda}\ cos\left\{\frac{2\pi}{\lambda}(Z - ct)\right\}\left[\frac{cR^3}{h^3} - \frac{2qR}{\pi h^3} + \frac{2qR^3}{\pi h^5}\right] \qquad (31)$$

We have

r = R ,$z = Z - ct$, W = w + c and U = u

$$W = w + c = -c + 2\left(\frac{q}{\pi h^4} + \frac{c}{h^2}\right)(h^2 - R^2) + c$$

$$W = 2\left(\frac{q}{\pi h^4} + \frac{c}{h^2}\right)(h^2 - R^2) \qquad (32)$$

Here, h is determined as a function of Z and t from (28), and q is known from (27) after $\bar{Q}$ is determined experimentally.

Conclusion

We obtained the long-wavelength approximations to peristaltic flow in a tube in this paper. The expression for the solution can be used to develop the model for the swallowing of food through the esophagus, vasomotion of small blood vessels, spermatic flows in the ductus efferentes of the male reproductive tract, embryo transport in the uterus, andtransport of urine through the ureter, among others. Such techniques are necessary for the approximations of models of flows in biofluids. Also we can apply the long-wave length approximation to peristaltic flow in a channel.

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