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Study on Effectiveness of Fuzzy Set Theory through its Practical Application

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Abstract

*This paper deals with the poverty as a main problem of our country Nepal. The Government wants to solve this problem by definite rule. The line of demarcation of defining the jurisdiction of the collection of poverty-stricken people does not exist. However, it is obvious that certain vague demarcation must exist in order to divide the resources as needed. In this paper, I have applied the fuzzy set of poverty-stricken Nepali as the function A: **N**→ [0, 1].*

Key Words: fuzzy set, membership function, characteristic function, etc.

Introduction

People from any discipline need a structure for existence on earth. Structure needs a theory, which is applied on a collection of well-defined objects, called set. People for such structures, encounter with many problems in their lives. To represent these problems, several mathematicians have introduced the concept of set in their own ways. These ways of representing problems are more rigid. The solutions using this concept are not so meaningful in many circumstances. This difficulty was overcome by the fuzzy concept, which was first introduced by eminent American cyberneticist Prof. L. A. Zadeh in 1965. Since then it has invaded almost all domains of human lives. The concepts of almost all branches of human knowledge have been redefined using fuzzy sets.

Fuzzy sets

The characteristic function of a classical set assigns a value of either 1 or 0 to each individual in the universal set, thereby discriminating between members and non-members of the set under consideration. But, in the case of fuzzy concept,this function is generalized in such a way that the values assigned to the elements of the universal set fall within a specified range of real numbers in the interval [0,1].

Definition

Let X be the universe of discourse and [0,1] be the closed interval of real numbers.

Then a mapping $A: X \rightarrow [0,1]$

is called a fuzy set A or fuzzy subset A of X for which

$$A(x) = \alpha,$$

where x is a member of X and a is a real number belonging to[0,1].

Every member x of X is a member of fuzzy set A with its grade α.

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It is denoted by

$x \in_\alpha A$ or by (x, a) ∈A ,

or by μA (x) = α.

In other words,

if X ={a,b,c} and A be a fuzzy set such that

A(a) = .5,

A(b) = .7 &

A(c)=1

then fuzzy set A is written as

A = {(a,.5) , (b,.7) , (c,1)}

Here A is called the membership function and A(x) is called the membership grade of x ∈ X.

We also write :

A = {(x, A(x) | x ∈X}.

Example:

Let X = {1,2,3,4,5} be the universal set and

A : X → [0,1] such that

A(1) = .3 i.e 1∈.3 A means 1∈A with gradation .3

A(2) = .1 i.e 2∈.1 A means 2∈A with gradation .1

A(3) = .7 i.e 3∈.7 A means 3∈A with gradation .7

A(4) = .9 i.e 4∈.9 A means 4∈A with gradation .9

A(5) = 1 i.e 5∈1 A means 5∈A with gradation 1.

Then the fuzzy set A is written as

A = {(1,.3), (2,.1), (3,.7), (4,.9), (5,1)}

Representation of a fuzzy set:

A fuzzy set on X can be represented by the following variousmethods:

M_1 . A = {(x, A(x)) | x ∈X}

M_2 . A = {(x, A(x))}

This is same as M 1 but the domain is notexplicitly specified.

M_3. A = Σ A(x)/x if the domain is finite.

If X = {a, b, c, d} and

A : X → [0, 1] s.t.

A(a) = 0

A(b) = .7

A(c) = .4

A(d) = 1

then

A = 0/a + .7/b + .4/c + 1/d.

M_4 . A = ∫ A(x)/x

if the domain is continuous.

M_5 . A = x_1 x_2 x_3 ……………. x_m

.5 .6 .7 ……………. .9

if the domain is finite and consists of n elements.

M_6 . By means of graph as is above figure.

Definitions.

Fuzzy Power Set :

The set of all fuzzy sets on X is called the fuzzy power set of X and is denoted by

PF(X).

i.e. PF(X) = {A|A : X → [0, 1]}.

A fuzzy set A is called :

I.. Universal set if A(x) = 1 for all x ∈X.

II. Null set if A(x) = 0 for all x ∈X.

III. Point or Singleton if it has a singleton support

say x of degree a, denoted by x a .

IV. Support of a fuzzy set:

The support of a fuzzy set A of X is the set that contains all the elements of X having non-zero membership grades in A. It is denoted by S(A) or Supp (A).

i.e. Supp (A) = {x∈X|A(x) > 0}

Ex. Let X = {a, b, c} and

A = {(a, 0), (b, .1), (c, .7)}

Supp (A) = {b, c}

V. Height of a fuzzy set:

The height of a fuzzy set A is the largest membership grade obtained by any element in the set. It is denoted by h(A).

$$h(A) = \sup_{x \in X} A(x)$$

= Sup (.1, .7, .9)

= .9

VI.Normal fuzzy Set:

A fuzzy set A of X is said to be normal h(A) = 1

Ex.Let X = {a, b, c} &

A = {(a, .1), (b, .7), (c, 1)}

Sinceh(A) = Sup (.1, .7, 1)

= 1

A is normal.

VII. α-cut set or α -Level set or cut worthy set:

The α-cut of a fuzzy set A of X is denoted by a A ,and defined by

^{a}A = { x∈X | A(x) ≥ α } where $_{\alpha}$∈[0, 1]

The strong a-cut of a fuzzy set A of X is denoted by a +A and defined by

α_{+A} = {x∈X | A(x) > }

whereα∈[0, 1].

Ex.Let X = {a, b, c, d} and

A = {(a, .2), (b, .3), (c, .6), (d. .9)}

Let α= .3

A(a) = .2 < .3

A(b) = .3 = .3

A(c) = .6 > .3

A(d) = .9 > .3

α_A = $.3_A$ = {b, c, d}

& $^{a+}$A= $.^{3+}$ A = {c, d}

VIII. Convex fuzzy Set:

The fuzzy set A of X is said to be convexif

A ((x_1) + (1-λ)x_2) ≥ min{A(x_1), A(x_2)}

for all x_1 , x_2∈ X

and λ∈[0, 1]

Ex.Let X = [0, 10] of real numbers.

Then

A = {x∈X | A(x) = x/(x+2)} is convex.

IX.Core of a fuzzy Set:

The Core of a fuzzy set A on X is denoted by Core (A) and defined by

Core (A) = {x∈X | A(x) = 1}

X.Cardinality of a fuzzy Set:

The cardinality of a fuzzy set A on X is denoted by |A| and defined by

|A| = ∑ x∈X (x)

Its relative cardinality is defined by

||A|| = |A|/10

Ex.Let X = {a, b, c, d} and

A = {(a, .2), (b, .4), (c, .6), (d, .9)}

Then |A| = $\sum_{x\in X} (x)$

= A(a) + A(b) + A(c) + A(d)

= .2 + .4 + .6 +.9

= 2.1

Relative Cardinality = ||A|| = 2.1/10 = .21

Criteria of Membership

The classical or ordinary or traditional set is defined as a collection of objects in a continuous divisions of the elements in some given universe of discourse (i.e. a set under consideration) into two disjoint groups: Members and Non members.

For example:

Let N be the collection of natural numbers 1, 2, 3, ... Let it be the given universe of discourse. We want to form a set A (say) by collecting those members of N which are less than 5. Clearly A = {1, 2, 3, 4}. The members of A areobviously 1, 2, 3 & 4 where as numbers like 5, 6, 7,.......are not membersof A.

A member of this traditional set A has only two grades 0 & 1. We write here $1 \in A$, $2 \in A$, $3 \in A$ & $4 \in A$ where as $5 \in A$, $6 \in A$, $7 \in A$,....etc. A sharp unambiguous distinction between these two groups of membership and non membership exists. However, many life situations describe sets that do not hold this characteristic.

Examples are:

* the set of poor people,
* the set of red flowers,
* the set of delicious dishes,
* the set of good cricketers,
* the set of very good books,
* the set of numbers much greater than one etc.

In all these sets, the adjectives ‘poor’, ‘red’, ‘delicious’, ‘good’, ‘very good’, and ‘much greater’, bring a situation of ambiguity or vagueness in the listener. All the above cited examples with adjectives are not sets in classical sense. The transition from membership to non-membership is abrupt in the classes. We understand these sets as having indefinite boundaries that make possible gradual transition from membership to non membership.

This is infact the basic concept of fuzzy set. Every member of the given universe of discourse is a member of the fuzzy set with different grade of membership.

To make it clear and more understandable we take the following example:

Example: 1.

Poverty is main problem of our state. The government wants to solve this problem by making a definite rule that “a person is poor if he earns Rs. 20,000 or less per annum and he will get aids/concessions from the government time to time.”

Suppose Mr. A earns Rs. 20,000 per annum and Mr. B earns Rs. 20,100 per annum. Then according to this given rule, A is poor and B is not. A is eligible for the concessions while B is not. This is an absurd conclusion. Though a clear line of demarcation defining the jurisdiction of the collection of the poor people does not exist, but it is obvious that some vague demarcation must exist. A person who earns Rs. 10,000 p.a is surely poor while a person earning Rs. 2,00000p.a is certainly not poor. So while moving from the financial status of the former person to later one, this line of demarcation must have been crossed somewhere.Such jurisdiction of the collection of poor people may start gradually. The above example is now represented in both classical and fuzzy way as follows.

Classical Representation of the problem

To solve the main problem of example (i), non fuzzy mathematicians have represented the collection of poor people in the following manner. Let I be the set of citizens of Nepal and A be the collection of poor Nepalese. Also let E(x) denotes the annual earning of a Nepalese x. Then the mathematical representation of the set A of poor Nepalese is viewed as a characteristic function A : I → {0, 1} such that if A(x) be the grade of membership of an Nepalese x then

A(x) = 1 if E(x) ≤ 20,000
= 0 if E(x) > 20,000

Here if E(Ram) = 19,000 then Ram is poor.
if E(Shyam) = 20,000 then Shyam is poor.
if E (Mohan) = 20,100 then Mohan is not poor.
if E(Sohan) = 20,500 then Sohan is not poor.
if E(Karim) = 21,000 then Karim is not poor.

In this representation, only those Nepalese are poor i.e. those Nepalese are members of A whose membership grade is 1 i.e. whose anual earning is Rs. 20,000 or less. But Nepalese with annual earnings Rs 20,100; 20,500; 21,000;

40,000;........, 2,00,000; and more are not poor. They all grade 0 and they are not members of the set A.

Fuzzy Representation of the Problem

The rigidity of the above classical representation of the problem can be viewed with the fact that one can easily understand about a person who earns Rs. 20,000 is poor but at the same moment how a person is not poor if he earns a slightly more amount than Rs. 20,000. No one government can succeed in solving this problem having such rigid representation. A group of some people having earnings like 20,100; 20,500; 21,000 even 22000, 23000 so on will not be benifitted from the financial aid/concession given by the government. Fuzzy Mathematicians have represented the fuzzy set A of poor Nepalese as the function

A : I → [0, 1] where

A(x) = 1 if E(x) ≤ 20,000 means Nepalese x is poor by all means.
= .99 if E(x) = 21,000 means Nepalese x is almost surely poor.
= .70 if E(x) = 40,000 means Nepalese x is more or less poor.
= .50 if E(x) = 50,000 means Nepalese x may or may not be poor.
= .30 if E(x) = 90,000 means Nepalese x is definitely not rich but it will be odd to call him poor.
= .01 if E(x) = 2,00,000 means Nepalese x is almost surely not poor.
= 0 if E(x) = 5,00,000 means Nepalese x is definitely not poor.

As earlier, we have seen that a member of a traditional set A has only two grades 0 & 1. If x ∈A then A(x) = 1 and if x ∉ A then A(x) = 0. But it does not happen in the case of fuzzy set. It has a whole range of grades (1, .99, .70, .50, .30, .01, ...0) between 0 & 1. This fuzzy set of poor Nepalese has indefinite boundaries. Every member of I i.e. every Nepalese is a member of the fuzzy set A i.e. every Nepalese is poor with different grade of membership.

Conclusion

We can see that the problem of poverty in Nepal can be solved in a better way by implementation of Fuzzy set than by classical set. Each and every Nepali will be benefitted from the above rule.

They will get appropriate aid according to their grade of poverty-stricken-ness:

If A(X) = 0.5 then, X will get 50% of the aid.

If A(Y) = 0.1 then, Y will get 10% of the aid and

If A (Z) = 1 then, Z will be get 100% of the aid announced by the government.

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