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New Concepts of T_o Separation Axioms in Fuzzy Topology

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Abstract

Fuzzy set was introduced by Zadeh in his classical paper of 1965. Three years later, Chang gave the definition of fuzzy topology, which is a family of fuzzy set satisfying the three classical axioms. In this paper, we have introduced and studied some new notions of T_o separation axioms in fuzzy topological spaces by using quasi-coincident relation for fuzzy set. Every ordinary (crisp) topological space vacuously satisfies condition of being quasi-To. In this paper concept of quasi coincident relation used to introduce and investigate some quasi separation axioms such as T_o, T_1, & T_2. Concerning quasi- T_o space in the general frame work of fuzzy topological spaces.

Keywords : Fuzzy topological space, quasi - coincidence, Fuzzy quasi-T_o, T_1 and T_2 space.

Introduction

The fundamental concept of Fuzzy set was introduced by L.A. Zadeh in 1965.[1] In 1968, C.L. Chang [2] introduced fuzzy topological space. In Chang's fuzzy topological space each fuzzy set is either open or not. Later, on Chang's idea was developed by Goguen [3]. He replaced the close interval $I = (0,1)$. In 1991, Ying [4] studied Hohles topology and called fuzzifying topology. This fuzzification opened for research, Zhang and Xu [5] established the neighborhood structure in fuzzifying topological spaces.

Separation is an essential part of fuzzy topology. In the framework of fuzzifying topology, Shen [6], Yue and Fang [7], Li & Shi [8] and Khedr et all [9] introduced some separation axioms and seprahen axioms are discussed on crisp points not on fuzzy points.

The present paper is organized in three sections. After introduction, section 2 is consisting to some preliminaries. In section 3, we introduce the notions of separation axioms such as To, T_1, & T_2 axiom with some properties and the relation between them in general framework of fuzzy topological space.

1. Preliminaries

Definition 2.1: (X, T) is called a fuzzy quasi- T_o space iff for every $x \in X$, and $\lambda \neq \mu; \lambda, \mu \in [0,1]$,

either, $X_{\lambda} \notin \underline{X}_{\mu}$
or, $\underline{X}_{\mu} \notin X_{\lambda}$.

Definition 2.2: (X, T) is called a fuzzy T_o- space iff, for any two fuzzy points e and d such that e $\neq$d,
either, $e \notin d$,
or, $d \notin \underline{e}$

Definition 2.3: (X, T) is called a fuzzy T_1- space iff every fuzzy point is a closed set. The following implication are obvious:

$$T_1 \Longrightarrow T_o \Rightarrow quasi - T_o$$

Every ordinary (crisp) topological space vacuously satisfies condition of being quasi- To and hence the quasi- T_o separation is a particularity in a fuzzy topology.

Let (X, T) be a quasi- T_o space,
Let, $x \in X$, and
$$\Lambda = (p_1, p_2)(0 \leq p_1 < p_2 < 1);$$

then there exists $B \in T$ such that $B(x) \in \Delta$

In fact,
Let $\lambda = 1 - p_1,\ \mu = 1 - p_2$
Then,
$$\lambda > p > 0$$
Since (X, T) is a quasi- T_o space,
$X_{\lambda} \notin \underline{X}_{\mu}$
Hence there exists some open Q-neighborhood;

$$b(B(x) > 1 - \lambda = p_1)$$

Which is not a quasi-coincident with x_{μ},

i.e. $(B(x) \leq 1 - \mu = p_2)$
Hence
$$B(x) \in \Delta$$
The following property concerning quasi-T_o space can be sharpened in the form of theorem as follows:
Theorem 1
(X, T) is called a fuzzy quasi- T_o space iff for every $x \in X$, and $p \in [0,1]$, there exist a $B \in T$ such that

$B(x) = p$

Proof

Necessity. When $p = 0$,

It suffices to take

$B = \phi$

When $0 < p < 1$, take a strictly monotonic increasing sequence of positive real number converging to p.

Let $\Delta_n = (p_n, p_{n+1})(n = 1, 2, \ldots)$;

From the property just proved above there exist $B_n \in T$ such that

$B^{(x)}_n \in \Delta_n$ for each n.

Therefore

$B = \cup_{n=1}^{\infty} B_n$ is open and

$B(x) = \rho$.

Sufficiency.

For two fuzzy points x_λ and x_μ, there exists from hypothesis an open set B such that

$B(x) = 1 - \mu > 1 - \lambda$.

It is evident that B is an open Q-neighborhood of x_λ but is not quasi-coincident with $\{x_\lambda\}$.

Hence it follows that

$x_\lambda \in \bar{x}_\mu$.

Theorem 2

(X, T) is a T_o space

iff (X, T) is quasi-T_o and for any two distinct points x, y in X and for any $\rho,\ v \in [0,1]$;

Then there exists $B \in T$ such that

$B(x) = \rho$ and

$B(y) > v$,

or $B(c) > \rho$ and

Proof:

Necessity.

When (X, T) is T_o, it is also quasi-T_o. For $x \neq y$ and

$\rho, v \in [0,1]$,

Putting $\lambda = 1 - \rho$ and

$\mu = 1 - v$,

We obtain two distinct fuzzy points x_λ and y_μ.

If $x_\lambda \notin \bar{y}_\mu$,

there exists an open Q-neighborhood

$B_1\ (B_1(x) > 1 - \lambda = \rho)$

Which is not equal quasi-coincident with $\{y_\mu\}$,

i.e., $B_1(y) \leq 1 - \mu = v$.

In view of Previous Theorem,

There is $B_2 \in T$ such that

$B_2(y) = \nu$.

Then the fuzzy open set

B = $B_1 \cup B_2$

is the required one.

If $y_\lambda \notin \{\bar{x}_\mu\}$, the argument can be carried out in a similar way.

Sufficiency.

Since (X, T) is quasi-T_o,

it suffices to consider the separation of two fuzzy points x_λ and y_μ with $x \neq q$.

Putting $\rho = 1 - \lambda$,

$\nu = 1 - \mu$,

from the hypothesis,

we may assume that there exists B ∈ T such that

$B(x) = \rho$ and

B(y) >ν.

Then B is a Q-neighborhood of y_μ which is not quasi-coincident with $\{x_\lambda\}$.

Hence $y_\mu \notin \bar{x}_\lambda$.

Theorem 3.

If (X, T) is both T_2 and quasi-T_o,

then it is also T_1.

Proof:

Let y_μ be an arbitrary fuzzy point. An accumulation point, if any, of y_μ is of the form

$$y_\lambda = (\lambda > \mu).$$

In the light of the property of (X, T) being T_o and Previous Theorem,

there exists a $B \in T$

such that

B(y) = $1 - \mu > 1 - \lambda$,

i.e., B is a Q-neighborhood of y_λ and is not quasi-coincident with y_μ.

Hence y_λ $(\lambda > \mu)$ cannot be an accumulation point of y_μ and therefore y_μ has no accumulation point.

y_μ is closed.

This means that (x, T) is T_1.

Since the derived set of every fuzzy point in a T_1 space is obviously ϕ,

we obtain the following result:

Theorem 5.

The derived set of every fuzzy set on a T_1space is closed.

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