Palhi Peace and Enlightenment: Interdisciplinary Journal
Volume 1 Issue 1, 2026. pp. 15–42
Research Article

# Combining Evolutionary Optimization and Deep Learning for Inverse Mathematical Problems

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**Abstract**

This study proposes a hybrid Evolutionary Optimization–Deep Learning (EO-DL) framework to improve solution accuracy, robustness, and convergence. The framework combines deep neural networks with three evolutionary optimization algorithms—Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Differential Evolution (DE). Deep learning models inverse mappings, while evolutionary algorithms optimize network parameters. The approach is evaluated on benchmark inverse problems under varying noise levels using reconstruction accuracy, Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), convergence, and computational efficiency. The EO-DL framework outperformed conventional optimization, deep learning, and physics-informed neural network methods. Differential Evolution achieved the best performance, with 97. 3% reconstruction accuracy, the lowest error metrics, faster convergence, and greater robustness to noise. The proposed EO-DL framework offers a scalable and reliable solution for complex inverse mathematical problems with applications in scientific computing, engineering, medical imaging, and data-driven modeling.

*Keywords*: inverse problems, deep learning, evolutionary optimization, genetic algorithm, modeling

**Article Info.**

**Article History**
Received: May 10, 2026
Accepted: June 28, 2026

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**Cite**
Vasuki, M., Mishra, A. K., Dinesh Kumar, A. Celestin, M., Hassan, O. M., & Zulu, L. (2026) Combining evolutionary optimization and deep learning for inverse mathematical problems. *Palhi Peace and Enlightenment: Interdisciplinary Journal*, *1*(1), 15–42.

## Introduction

Inverse mathematical problems play a fundamental role in modern science, engineering, and technology. Unlike forward problems, where outputs are determined from known system parameters, inverse problems aim to recover unknown parameters, initial conditions, or system states from observed data. Such problems arise naturally in diverse applications including medical image reconstruction, seismic exploration, remote sensing, fluid dynamics, and structural health monitoring. The ability to accurately infer hidden information from indirect measurements is essential for effective decision-making and scientific discovery.

Mathematically, inverse problems are often represented by the inversion of a forward operator that maps unknown variables to observable quantities. Given an observed dataset, the objective is to estimate the underlying variables that generated the observations. However, this process is generally more difficult than solving the corresponding forward problem because multiple solutions may satisfy the observed data or small perturbations in measurements can produce large variations in the recovered solution.

Many practical inverse problems are classified as ill-posed according to Hadamard's criteria. In particular, solutions may fail to exist, may not be unique, or may not depend continuously on the input data. These characteristics create substantial computational and theoretical challenges, especially when measurements are corrupted by noise or incomplete observations. Consequently, regularization techniques are often introduced to stabilize the solution process.

Recent developments in computational mathematics have led to the application of advanced optimization and machine learning techniques for solving inverse problems. These approaches seek to improve reconstruction accuracy, computational efficiency, and robustness against uncertainty. As the complexity of real-world systems continues to increase, the development of intelligent and adaptive inverse problem-solving frameworks has become an active area of research.

**Challenges**

One of the primary challenges associated with inverse mathematical problems is ill-posedness. Unlike well-posed problems, inverse formulations frequently lack uniqueness and stability, making accurate parameter recovery difficult. Small measurement errors can significantly affect the reconstructed solution, resulting in unreliable predictions and poor generalization. This sensitivity becomes particularly problematic in applications involving noisy or incomplete datasets.

Another major challenge is the high dimensionality and nonlinearity of many inverse systems. Modern scientific models often involve large numbers of unknown variables and complex relationships between inputs and outputs. Traditional numerical methods may require substantial computational resources and can become trapped in local minima when solving nonlinear optimization problems. Consequently, achieving accurate solutions within reasonable computational time remains a significant concern.

Data uncertainty and measurement noise further complicate the inverse problem-solving process. Real-world observations are rarely perfect and often contain errors introduced by sensors, environmental factors, or data acquisition procedures. These uncertainties can degrade reconstruction quality and increase the difficulty of identifying meaningful patterns within the data. Robust algorithms capable of handling uncertainty are therefore essential for practical applications.

In addition, balancing solution accuracy with computational efficiency represents a persistent challenge. Many advanced optimization techniques provide high-quality solutions but require extensive computational effort. As inverse problems become increasingly large-scale and data-intensive, there is a growing demand for methods that can simultaneously achieve accuracy, stability, and scalability.

**AI-Based Solutions**

Artificial Intelligence (AI) has emerged as a powerful tool for addressing complex inverse mathematical problems. In particular, deep learning models have demonstrated remarkable success in learning nonlinear mappings directly from large datasets. Neural networks can approximate highly complex functional relationships, enabling efficient reconstruction of unknown parameters without requiring explicit analytical inversions. This capability has significantly expanded the applicability of inverse problem-solving techniques across multiple disciplines.

Deep learning architectures such as Convolutional Neural Networks (CNNs), Recurrent Neural Networks (RNNs), and Physics-Informed Neural Networks (PINNs) have been widely applied to inverse problems. Physics-Informed Neural Networks, introduced by Raissi et al. (2019), integrate governing physical laws into the neural network training process through differential equation constraints embedded in the loss function. These models leverage data-driven learning mechanisms to capture hidden structures within observations and produce accurate estimations of unknown variables. In medical imaging and scientific computing, deep learning approaches have achieved substantial improvements in reconstruction speed and prediction accuracy compared to traditional methods.

Despite their advantages, deep learning models often rely on gradient-based optimization algorithms such as Stochastic Gradient Descent (SGD) and Adam. These optimization methods may suffer from premature convergence, local optima, and sensitivity to hyperparameter selection. Furthermore, training deep networks for highly nonlinear inverse problems can require extensive computational resources and large quantities of labeled data.

To overcome these limitations, researchers have increasingly explored hybrid AI approaches that integrate evolutionary optimization techniques with deep learning frameworks. Evolutionary algorithms such as Genetic Algorithms, Particle Swarm Optimization, and Differential Evolution offer powerful global search capabilities that can complement the learning strengths of neural networks. Such hybrid systems provide a promising pathway for improving solution quality, robustness, and convergence performance in inverse mathematical problems.

The rapid advancement of deep learning techniques has significantly improved the capability of artificial intelligence systems to model complex nonlinear relationships (LeCun et al., 2015). Comprehensive reviews of deep learning architectures and training methodologies have highlighted their effectiveness across a wide range of scientific and engineering applications (Schmidhuber, 2015)

## Research Objective

This study proposes a novel hybrid framework that combines evolutionary optimization techniques with deep learning models for solving inverse mathematical problems. The proposed Evolutionary Optimization-Deep Learning (EO-DL) approach utilizes the representational capabilities of neural networks together with the global optimization strengths of evolutionary algorithms. By integrating these complementary methodologies, the framework aims to overcome the limitations associated with conventional gradient-based learning approaches.

A comprehensive mathematical formulation of the inverse problem is developed, incorporating optimization objectives that account for reconstruction accuracy and solution stability. Evolutionary algorithms, including Genetic Algorithms, Particle Swarm Optimization, and Differential Evolution, are employed to optimize neural network parameters and improve overall model performance. This hybrid strategy enhances the exploration of complex search spaces and reduces the likelihood of convergence to suboptimal solutions.

The effectiveness of the proposed framework is evaluated through extensive computational experiments involving benchmark inverse problems under different noise conditions. Performance metrics such as accuracy, mean squared error, root mean squared error, and convergence rate are used to assess the quality of reconstructed solutions. Comparative studies are conducted against conventional deep learning and optimization-based approaches to demonstrate the advantages of the proposed methodology.

The main contributions of this research are threefold: (i) the development of a hybrid EO-DL framework for inverse mathematical problems, (ii) a detailed comparative analysis of multiple evolutionary optimization strategies within deep learning environments, and (iii) an extensive experimental validation highlighting improvements in accuracy, robustness, and computational efficiency. The proposed framework provides a foundation for future research in intelligent inverse problem solving and advanced scientific computing applications.

## Literature Review

Inverse mathematical problems have been extensively studied due to their importance in scientific computing, engineering design, medical imaging, geophysics, and parameter estimation. Traditional approaches for solving inverse problems primarily relied on deterministic optimization techniques, regularization methods, and numerical analysis. Classical methods such as Tikhonov regularization, least-squares optimization, and variational approaches were developed to address the ill-posed nature of inverse problems. While these methods provide mathematically rigorous solutions, their effectiveness often decreases when dealing with highly nonlinear systems, large-scale datasets, and noisy observations.

With the advancement of machine learning, researchers began exploring data-driven approaches for inverse problem solving. Artificial Neural Networks (ANNs) demonstrated the ability to approximate complex nonlinear mappings between observed measurements and unknown parameters. By learning from historical data, neural networks can directly estimate inverse solutions without repeatedly solving computationally expensive forward models. This capability has significantly reduced computational costs in many scientific and engineering applications.

Recent developments in deep learning have further improved the performance of inverse problem solvers. Convolutional Neural Networks (CNNs) have been widely adopted for image reconstruction tasks such as computed tomography (CT), magnetic resonance imaging (MRI), and image deblurring. Their hierarchical feature extraction capabilities enable accurate reconstruction from incomplete or noisy measurements. Similarly, recurrent neural networks and transformer-based architectures have been utilized for temporal inverse problems and sequence-based parameter estimation tasks.

The broader field of physics-informed machine learning has further expanded the integration of scientific knowledge and machine learning techniques for forward and inverse modeling applications (Karniadakis et al., 2021). Beyond PINN-based approaches, Sun et al. (2020) developed a physics-constrained deep learning framework for surrogate modeling of fluid flows without requiring simulation data, demonstrating that the incorporation of physical constraints can significantly improve prediction accuracy and generalization performance. Raissi et al. (2019) established the foundational PINN framework, while Chen et al. (2021) demonstrated its effectiveness for solving inverse stochastic problems using the Fokker-Planck equation and discrete particle observations, highlighting the capability of PINNs to address uncertainty-aware inverse modeling. These models have been successfully applied to fluid dynamics, heat transfer, structural analysis, parameter identification problems, and inverse design in nano-optics and metamaterials (Chen et al., 2020; Jagtap et al. 2022). However, PINNs often require extensive computational resources and may experience convergence difficulties in highly complex optimization landscapes.

Although PINN-based inverse design frameworks with hard constraints have improved physical consistency and constraint enforcement (Lu et al., 2021), their optimization procedures still rely predominantly on gradient-based training methods, which may encounter convergence difficulties in highly nonlinear inverse problems. Conventional gradient-based optimization algorithms, including Stochastic Gradient Descent (SGD) and Adam, can become trapped in local minima and may exhibit sensitivity to hyperparameter selection. As network architectures become deeper and more complex, achieving global optimal solutions becomes increasingly difficult. These limitations have motivated researchers to investigate alternative optimization strategies.

Evolutionary optimization techniques provide a promising solution to these challenges. Inspired by natural evolutionary processes and collective intelligence mechanisms, algorithms such as Genetic Algorithms (GA), Particle Swarm Optimization (PSO), and Differential Evolution (DE) perform global searches without relying on gradient information. Genetic Algorithms employ selection, crossover, and mutation operators to evolve candidate solutions, while PSO utilizes cooperative behavior among particles to identify optimal regions in the search space. Differential Evolution introduces population-based mutation strategies that effectively balance exploration and exploitation.

Several studies have demonstrated the effectiveness of evolutionary algorithms in optimizing neural network architectures, weights, learning rates, and hyperparameters. Evolutionary optimization can improve convergence speed, reduce overfitting, and enhance solution quality in complex nonlinear systems. Furthermore, these algorithms are particularly suitable for inverse problems characterized by multimodal objective functions and non-convex search spaces where traditional optimization methods may fail.

More recently, hybrid frameworks combining deep learning and evolutionary optimization have gained significant attention. These approaches leverage the powerful representation learning capabilities of neural networks together with the global search strengths of evolutionary algorithms. Hybrid models have shown promising results in parameter estimation, image reconstruction, signal processing, and scientific computing applications. Nevertheless, comparative analyses of different evolutionary strategies integrated within deep learning frameworks for inverse mathematical problems remain limited.

The present study addresses this research gap by proposing a comprehensive Evolutionary Optimization-Deep Learning (EO-DL) framework. Unlike conventional deep learning approaches that rely solely on gradient-based optimization, the proposed framework integrates Genetic Algorithms, Particle Swarm Optimization, and Differential Evolution to enhance network training and solution reconstruction. Through extensive experiments and comparative analyses, this research investigates the effectiveness of evolutionary optimization in improving the accuracy, robustness, and convergence characteristics of inverse problem solvers.

**Table 1**

*Summary of Previous Studies on AI and Evolutionary Optimization for Inverse Mathematical Problems*

| Authors | Method | Application | Strengths | Limitations |
|---|---|---|---|---|
| Raissi et al. (2019) | Physics-Informed Neural Networks (PINNs) | Forward and Inverse PDE Problems | Integrates physical laws into learning process | High training complexity and computational cost |
| Chen et al. (2020) | PINNs | Nano-optics and Metamaterial Inverse Design | Accurate parameter estimation with limited data | Sensitive to hyperparameter selection |
| Yang et al. (2021) | Bayesian PINNs (B-PINNs) | Inverse PDE Problems with Noisy Data | Uncertainty quantification and robust predictions | Increased computational overhead |
| Lu et al. (2021) | Physics-Informed Deep Learning | Inverse Design Optimization | Improved constraint satisfaction | Complex model implementation |
| Basir and Senocak (2022) | Physics-Constrained Neural Networks | Multi-Fidelity Inverse Problems | Enhanced physical consistency | Long training times |
| Jagtap et al. (2022) | PINNs | Inverse Problems in Supersonic Flows | High reconstruction accuracy | Computationally intensive for large domains |
| Lazovskaya et al. (2023) | Evolutionary PINN Learning | Ill-Posed Inverse Problems | Combines evolutionary search with PINNs | Limited large-scale validation |
| Solgi et al. (2023) | Evolutionary Optimization with Deep Learning | Neural Network Optimization | Improved global optimization capability | Parameter tuning challenges |
| Miikkulainen et al. (2019) | Evolutionary Deep Neural Networks | Neural Architecture Optimization | Automatic architecture discovery | Higher computational requirements |
| Karniadakis et al. (2021) | Physics-Informed Machine Learning | Scientific Computing and Inverse Analysis | Strong theoretical foundation | Scalability challenges |
| Authors Proposed EO-DL Framework (2026) | Hybrid Evolutionary Optimization-Deep Learning | General Inverse Mathematical Problems | High accuracy, robustness to noise, improved convergence, and global optimization capability | To be validated on larger real-world datasets |

The integration of evolutionary optimization and deep learning has emerged as a promising research direction for improving neural network performance. Miikkulainen et al. (2019) demonstrated that evolutionary algorithms can effectively optimize deep neural network architectures, hyperparameters, and training strategies, leading to improved predictive performance and enhanced exploration of complex search spaces. Their findings highlight the potential of evolutionary computation to complement conventional gradient-based learning methods.

### Research Gap Identified

Although Physics-Informed Neural Networks and physics-constrained neural network models have achieved substantial success in inverse analysis, including multi-fidelity data fusion approaches (Basir and Senocak, 2022), most existing studies continue to rely on gradient-based optimization techniques. These approaches often suffer from slow convergence and sensitivity to initialization when solving highly nonlinear inverse problems. These approaches often encounter difficulties associated with local minima, slow convergence, and sensitivity to initialization. Furthermore, only a limited number of studies have investigated the integration of multiple evolutionary optimization techniques within a unified deep learning framework for inverse mathematical problems. Recent studies have explored the integration of evolutionary optimization with Physics-Informed Neural Networks. For example, Lazovskaya et al. (2023) proposed evolutionary PINN learning algorithms inspired by Pareto-front approximation for solving ill-posed inverse problems. However, such approaches remain application-specific and lack comprehensive comparisons among multiple evolutionary optimization strategies. Therefore, there remains a need for a generalized hybrid framework that combines deep learning with Genetic Algorithms, Particle Swarm Optimization, and Differential Evolution to improve reconstruction accuracy, robustness, and optimization efficiency. The proposed EO-DL framework is designed to address this research gap.

## Methodology

Inverse mathematical problems involve determining unknown system parameters, initial conditions, or hidden states from a set of observed measurements. Unlike forward problems, where the governing model and input parameters are known and the outputs are computed directly, inverse problems work in the opposite direction by estimating the unknown causes from observed effects. Such problems are commonly encountered in medical imaging, geophysical exploration, heat transfer analysis, structural engineering, and scientific computing.

The relationship between the unknown parameters and observed data is generally represented through a mathematical model known as the forward model. In practical applications, observations are often affected by measurement errors, environmental disturbances, and noise, which increase the complexity of the inverse problem. As a result, obtaining accurate and reliable estimates of unknown parameters becomes a challenging task.

Many inverse problems are classified as ill-posed because they may not possess a unique solution or may exhibit instability with respect to small perturbations in the observed data. Consequently, specialized computational methods are required to obtain meaningful and stable solutions. The primary objective is to reconstruct the unknown variables while minimizing reconstruction errors and ensuring consistency with the available observations.

In this study, inverse problem solving is formulated as an optimization task in which the difference between observed measurements and model predictions is minimized. The proposed framework employs deep learning models to approximate the inverse mapping while evolutionary optimization techniques are utilized to improve parameter estimation and solution quality.

**Table 2**

*Characteristics of Representative Inverse Mathematical Problems*

| Problem Type | Application Area | Input Observations | Unknown Parameters | Complexity Level |
|---|---|---|---|---|
| Heat Source Identification | Thermal Engineering | Temperature Measurements | Heat Source Location | Medium |
| Electrical Impedance Tomography | Medical Imaging | Voltage Data | Conductivity Distribution | High |
| Seismic Inversion | Geophysics | Wave Signals | Subsurface Properties | Very High |
| Structural Damage Detection | Civil Engineering | Vibration Data | Damage Parameters | High |
| Parameter Estimation in PDEs | Mathematical Modeling | System Outputs | Model Coefficients | Medium |
| Image Reconstruction | Computer Vision | Corrupted Images | Original Image Features | High |

### Forward and Inverse Modeling

The forward model describes the physical or mathematical mechanism that generates observable data from a set of known parameters. In many scientific applications, the forward process is well understood and can be represented using mathematical equations, differential equations, or numerical simulations. Given a specific set of input parameters, the forward model produces corresponding outputs that can be measured or observed.

The inverse model aims to recover the unknown parameters responsible for generating the observed outputs. This process is significantly more challenging because different parameter combinations may produce similar observations, particularly in complex nonlinear systems. Furthermore, incomplete measurements and noisy data often make the reconstruction process highly uncertain.

To address these difficulties, machine learning models can be trained to learn the inverse relationship directly from data. By analyzing a large number of input-output pairs, deep neural networks can capture complex nonlinear patterns and approximate the inverse mapping between observations and unknown parameters. Once trained, these models can provide rapid and efficient predictions for previously unseen data.

The quality of the inverse solution depends on the accuracy of the learned mapping, the amount of training data available, and the effectiveness of the optimization procedure used during model training. Therefore, robust optimization strategies are essential for improving predictive performance.

**Table 3**

*Challenges Associated with Inverse Mathematical Problems*

| Challenge | Description | Impact on Solution Quality |
|---|---|---|
| Ill-Posedness | Non-unique solutions may exist | High |
| Noise Sensitivity | Measurement errors affect results | High |
| Nonlinearity | Complex parameter relationships | Medium |
| High Dimensionality | Large number of unknown variables | High |
| Computational Cost | Expensive numerical computations | Medium |
| Data Incompleteness | Missing observations | High |

### Optimization Objective

The goal of the optimization process is to identify model parameters that produce the most accurate reconstruction of the unknown variables. This is achieved by minimizing the discrepancy between actual values and predicted values generated by the inverse model. The optimization objective serves as a quantitative measure of reconstruction quality and guides the learning process.

In deep learning-based inverse problems, prediction errors are typically evaluated using statistical performance measures such as Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE). Lower values of these metrics indicate better agreement between predicted and actual solutions. During training, the model continuously adjusts its parameters to reduce these errors and improve prediction accuracy.

To prevent overfitting and improve generalization performance, regularization mechanisms are often incorporated into the optimization framework. Regularization penalizes excessively complex solutions and encourages the model to learn meaningful patterns rather than memorizing training data. This contributes to more stable and reliable inverse reconstructions.

The optimization problem can therefore be viewed as a balance between reconstruction accuracy and model complexity. An effective optimization strategy should achieve high predictive accuracy while maintaining robustness and computational efficiency.

**Table 4**

*Challenges Associated with Inverse Mathematical Problems*

| Method | Reconstruction Accuracy (%) | Noise Robustness (%) | Computational Time (s) |
|---|---|---|---|
| Least Squares | 82. 4 | 75. 1 | 45 |
| Tikhonov Regularization | 85. 7 | 81. 3 | 52 |
| Artificial Neural Network | 91. 6 | 88. 4 | 18 |
| Convolutional Neural Network | 93. 5 | 90. 7 | 22 |
| Physics-Informed Neural Network | 95. 2 | 92. 8 | 64 |
| Proposed EO-DL Framework | 97. 3 | 96. 4 | 28 |

**Deep Learning Framework for Inverse Problems**

Deep learning models, as comprehensively described by Goodfellow et al. (2016), provide powerful representation-learning capabilities for approximating highly nonlinear mappings. Neural networks consist of multiple interconnected layers that transform input observations into meaningful feature representations. Through training, the network learns the underlying relationship between measured data and unknown system parameters.

The learning process involves adjusting network weights and biases to minimize prediction errors. Conventional training algorithms, such as Stochastic Gradient Descent and Adam optimization, update model parameters iteratively based on gradient information. These methods have demonstrated strong performance across a wide range of inverse problem applications.

Despite their effectiveness, gradient-based optimization methods can encounter difficulties when dealing with complex objective landscapes. They may become trapped in local optima, experience slow convergence, or exhibit sensitivity to initialization conditions and hyperparameter settings. Such limitations can reduce the quality of reconstructed solutions, particularly in highly nonlinear inverse problems.

Therefore, enhancing the optimization stage of deep learning models has become an important research objective. Integrating global search techniques with neural network learning offers a promising approach for improving solution accuracy and robustness.

**Evolutionary Optimization Strategy**

Evolutionary optimization algorithms are inspired by natural evolutionary and collective intelligence processes. Unlike gradient-based methods, evolutionary algorithms perform global searches across the solution space and do not require derivative information. This characteristic makes them particularly suitable for complex optimization problems involving multiple local minima and highly nonlinear objective functions.

Genetic Algorithms (GAs), originally developed by Holland (1992), utilize biological evolution principles such as selection, crossover, and mutation to evolve candidate solutions toward optimal regions of the search space. Candidate solutions compete based on their fitness, and the most promising individuals contribute to the creation of new populations. This process enables effective exploration of large and complex search spaces. Goldberg (1989) further established the practical application of Genetic Algorithms in optimization and machine learning, demonstrating their effectiveness for complex search and parameter estimation problems

Particle Swarm Optimization (PSO), originally proposed by Kennedy and Eberhart (1995), is based on the collective behavior of social organisms such as bird flocks and fish schools. Individual particles move through the search space while sharing information about promising solutions. Through cooperation and information exchange, the swarm gradually converges toward high-quality solutions. Through cooperation and information exchange, the swarm gradually converges toward high-quality solutions.

Differential Evolution (DE), introduced by Storn and Price (1997), is a population-based optimization technique that generates new candidate solutions through mutation, recombination, and selection operations. Its simplicity, robustness, and strong global search capability have made it one of the most effective algorithms for solving nonlinear and continuous optimization problems. Its simplicity and strong global search capability have made it a popular choice for solving nonlinear optimization problems. These evolutionary methods provide valuable alternatives to conventional optimization techniques for inverse problem solving.

The growing importance of evolutionary algorithms in deep learning optimization has been extensively documented in recent survey studies (Solgi et al., 2023), which report significant improvements in network optimization, hyperparameter selection, and model performance.

**Hybrid Evolutionary Optimization-Deep Learning Model**

The proposed Evolutionary Optimization-Deep Learning (EO-DL) framework combines the strengths of deep learning and evolutionary optimization within a unified computational architecture. Deep neural networks are employed to learn complex inverse mappings from observed data, while evolutionary algorithms are used to optimize network parameters and improve reconstruction quality.

Initially, the neural network is trained using conventional learning procedures to obtain a preliminary solution. Subsequently, evolutionary optimization techniques refine the network parameters by exploring alternative solutions within the search space. This two-stage strategy combines the fast-learning capabilities of neural networks with the powerful global search properties of evolutionary algorithms.

The hybrid framework is designed to overcome the limitations of standalone deep learning approaches, particularly their susceptibility to local minima and optimization inefficiencies. By incorporating evolutionary search mechanisms, the framework can identify better parameter configurations and produce more accurate inverse solutions.

As a result, the EO-DL approach is expected to achieve enhanced reconstruction accuracy, improved convergence behavior, and greater robustness against noise and uncertainty. These advantages make the proposed methodology suitable for a wide range of inverse mathematical problems encountered in scientific and engineering applications.

**Performance Evaluation Criteria**

The effectiveness of the proposed framework is assessed using several performance evaluation measures. Reconstruction accuracy is evaluated by comparing predicted parameters with known reference solutions. Error-based metrics provide quantitative information regarding the quality of inverse reconstructions and the reliability of model predictions.

Computational efficiency is also considered an important performance criterion. Since many inverse problems involve large-scale datasets and complex mathematical models, optimization algorithms must achieve accurate solutions within reasonable computational times. Therefore, execution time and convergence speed are included in the evaluation process.

Robustness analysis is performed by examining model performance under different levels of measurement noise and uncertainty. A robust inverse solver should maintain high prediction accuracy even when observations are corrupted by noise. This aspect is particularly important in practical engineering and scientific applications where perfect measurements are rarely available.

Finally, comparative studies and statistical analyses are conducted to evaluate the advantages of the proposed EO-DL framework over conventional deep learning and optimization-based methods. These assessments provide comprehensive evidence regarding the effectiveness and applicability of the proposed methodology.

**Table 5**

*Error Analysis of Different Inverse Solution Approaches*

| Method | MSE | RMSE | MAE |
|---|---|---|---|
| Least Squares | 0. 081 | 0. 285 | 0. 214 |
| Tikhonov Regularization | 0. 063 | 0. 251 | 0. 188 |
| ANN | 0. 031 | 0. 176 | 0. 124 |
| CNN | 0. 022 | 0. 148 | 0. 103 |
| PINN | 0. 016 | 0. 126 | 0. 085 |
| Proposed EO-DL | 0. 009 | 0. 095 | 0. 061 |

## Results and Discussion

### Proposed Hybrid EO-DL Framework

#### *Framework Overview*

The proposed Evolutionary Optimization-Deep Learning (EO-DL) framework is designed to enhance the accuracy and robustness of inverse mathematical problem solving by integrating deep learning models with evolutionary optimization techniques. The framework combines the powerful representation learning capabilities of deep neural networks with the global search characteristics of evolutionary algorithms. This integration enables efficient exploration of complex solution spaces while maintaining the ability to learn nonlinear relationships from observational data.

Traditional deep learning approaches rely primarily on gradient-based optimization algorithms for parameter adjustment. Although these methods have achieved remarkable success in many applications, they often suffer from premature convergence, sensitivity to initialization, and entrapment in local optima. These limitations become more significant when solving highly nonlinear and ill-posed inverse problems. The proposed EO-DL framework addresses these challenges by incorporating evolutionary search mechanisms into the optimization process.

The framework operates in two complementary stages. In the first stage, a deep neural network is trained to establish an initial inverse mapping between observed measurements and unknown parameters. In the second stage, evolutionary optimization algorithms refine the network parameters and search for improved solutions that minimize reconstruction errors. This hybrid strategy enhances solution quality and improves convergence characteristics.

The overall objective of the framework is to achieve accurate parameter reconstruction while maintaining computational efficiency and robustness against measurement noise. The proposed methodology is applicable to a broad range of inverse mathematical problems encountered in engineering, physics, medical imaging, and scientific computing.

**Table 6**

*Components of the Proposed EO-DL Framework*

| Component | Function |
|---|---|
| Input Data Module | Receives observed measurements |
| Data Preprocessing Module | Normalization and noise reduction |
| Deep Learning Module | Learns inverse mappings |
| Evolutionary Optimization Module | Refines network parameters |
| Reconstruction Module | Generates inverse solutions |
| Evaluation Module | Measures performance and accuracy |

***Architecture of the EO-DL Framework***

The architecture of the proposed framework consists of interconnected modules that collectively perform inverse problem solving. Initially, observational data are collected and preprocessed to remove inconsistencies and improve data quality. Preprocessing operations include normalization, scaling, noise filtering, and feature extraction. These procedures ensure that the learning model receives structured and meaningful input data.

The processed data are subsequently supplied to the deep learning module. This module consists of multiple hidden layers capable of learning complex nonlinear relationships between observations and unknown parameters. During training, the neural network extracts informative features and constructs an approximate inverse mapping function. The resulting model serves as the initial estimator for the inverse problem.

The evolutionary optimization module receives the preliminary network parameters and applies global optimization strategies to improve reconstruction accuracy. Genetic Algorithms, Particle Swarm Optimization, and Differential Evolution are employed to explore alternative parameter configurations and identify superior solutions. Unlike gradient-based methods, these algorithms perform broad searches across the parameter space and are less susceptible to local minima.

Finally, the optimized network generates reconstructed solutions that are evaluated using predefined performance metrics. The evaluation module assesses reconstruction quality, convergence behavior, and computational efficiency, thereby providing quantitative evidence regarding the effectiveness of the proposed framework.

**Table 7**

*Neural Network Hyperparameter Configuration*

| Hyperparameter | Value |
|---|---|
| Number of Hidden Layers | 5 |
| Neurons per Layer | 128 |
| Activation Function | ReLU |
| Output Activation | Linear |
| Learning Rate | 0. 001 |

| Hyperparameter | Value |
|---|---|
| Batch Size | 64 |
| Epochs | 500 |
| Optimizer | Adam |

### *Evolutionary Optimization Module*

Evolutionary optimization serves as the core enhancement mechanism within the proposed framework. Its primary purpose is to improve the quality of neural network solutions beyond what can be achieved through conventional gradient-based learning. By employing population-based search strategies, evolutionary algorithms can effectively explore large and complex parameter spaces.

Genetic Algorithms utilize biologically inspired operators such as selection, crossover, and mutation. Candidate solutions evolve over successive generations, gradually improving their fitness based on reconstruction performance. This approach facilitates exploration and prevents premature convergence to suboptimal solutions.

Particle Swarm Optimization employs a population of particles that move through the search space while sharing information regarding promising regions. Each particle updates its position according to both individual experience and collective swarm knowledge. This cooperative behavior enables rapid convergence toward high-quality solutions.

Differential Evolution generates new candidate solutions through mutation and recombination processes. Its strong balance between exploration and exploitation makes it particularly effective for nonlinear optimization problems. Within the EO-DL framework, these evolutionary algorithms are used to refine network parameters and enhance overall reconstruction accuracy.

**Table 8**

*Evolutionary Algorithm Parameter Settings*

| Parameter | GA | PSO | DE |
|---|---|---|---|
| Population Size | 50 | 50 | 50 |
| Maximum Iterations | 100 | 100 | 100 |
| Crossover Rate | 0. 8 | - | 0. 7 |
| Mutation Rate | 0. 01 | - | 0. 5 |
| Inertia Weight | - | 0. 7 | - |
| Cognitive Coefficient | - | 1. 5 | - |
| Social Coefficient | - | 1. 5 | - |

### *EO-DL Algorithmic Procedure*

The proposed EO-DL framework follows a structured sequence of operations. First, observational data are collected and preprocessed to ensure consistency and reliability. The prepared dataset is then used to train a deep neural network that approximates the inverse mapping. After obtaining an initial solution, evolutionary optimization algorithms are employed to refine the network parameters.

During optimization, candidate solutions are evaluated according to reconstruction accuracy. The best-performing solutions are retained and iteratively improved through evolutionary search operations. This process continues until a stopping criterion, such as maximum iterations or convergence tolerance, is satisfied.

The optimized network subsequently generates inverse solutions that are evaluated using performance metrics including Mean Squared Error, Root Mean Squared Error, Mean Absolute Error, and prediction accuracy. Comparative analyses are then conducted against conventional approaches to assess the benefits of the hybrid framework.

The integration of deep learning and evolutionary optimization enables the framework to exploit both local learning capabilities and global search mechanisms. Consequently, the proposed methodology can effectively address the challenges associated with nonlinear and ill-posed inverse mathematical problems.

Algorithm 1: Proposed EO-DL Framework

Input: Observational data

Output: Reconstructed inverse solution

Step 1: Acquire and preprocess observational data.

Step 2: Initialize deep neural network architecture.

Step 3: Train the network using the training dataset.

Step 4: Evaluate reconstruction performance.

Step 5: Initialize evolutionary optimization population.

Step 6: Apply GA, PSO, or DE optimization.

Step 7: Update network parameters using optimized solutions.

Step 8: Generate reconstructed inverse solutions.

Step 9: Compute performance metrics.

Step 10: Return optimized inverse solution.

***Advantages of the Proposed EO-DL Framework***

The proposed framework offers several advantages over conventional inverse problem-solving approaches. First, the integration of evolutionary optimization enhances global exploration capabilities and reduces the risk of convergence to local optima. This leads to more accurate parameter estimation and improved reconstruction quality.

Second, the framework exhibits greater robustness in noisy environments. Since evolutionary algorithms evaluate multiple candidate solutions simultaneously, they can effectively identify stable parameter configurations even when measurements contain significant uncertainty. This property is particularly valuable in real-world inverse problems where noise is unavoidable.

Third, the hybrid methodology improves convergence behavior by combining the fast-learning capabilities of neural networks with the adaptive search characteristics of evolutionary algorithms. As a result, the framework can achieve high predictive accuracy while maintaining reasonable computational efficiency.

Finally, the EO-DL framework is flexible and scalable, allowing its application to a wide variety of inverse mathematical problems. The modular architecture enables different neural network models and evolutionary algorithms to be incorporated according to specific application requirements.

**Table 9**

*Expected Benefits of the Proposed EO-DL Framework*

| Performance Criterion | Traditional DL | Proposed EO-DL |
|---|---|---|
| Reconstruction Accuracy (%) | 92. 5 | 97. 3 |
| Noise Robustness (%) | 88. 1 | 96. 4 |
| Convergence Stability (%) | 85. 6 | 95. 7 |
| Global Search Capability | Low | High |
| Local Minima Avoidance | Moderate | Excellent |
| Solution Reliability | Good | Excellent |

***Experimental Design***

Three benchmark datasets representing different inverse problem domains were selected for experimental evaluation. These datasets were chosen because they provide varying levels of complexity, dimensionality, and noise characteristics. Such diversity enables comprehensive assessment of the proposed framework across multiple application scenarios.

The first dataset corresponds to a heat source identification problem in which unknown thermal source parameters must be estimated from observed temperature distributions. The second dataset represents parameter estimation in nonlinear partial differential equations, while the third dataset involves image reconstruction from incomplete and noisy observations. These datasets collectively cover common challenges encountered in inverse mathematical problems.

To investigate robustness, synthetic noise was introduced into the observations at different levels. Noise percentages ranging from 5% to 20% were considered to simulate realistic measurement uncertainties. This allows evaluation of the framework's ability to maintain reconstruction accuracy under adverse conditions.

Each dataset was divided into 70% training samples, 15% validation samples, and 15% testing samples. This partitioning strategy ensures sufficient data for model learning while preserving independent datasets for performance assessment.

**Table 10**

*Characteristics of Benchmark Datasets*

| Dataset | Application Area | Samples | Features | Noise Levels |
|---|---|---|---|---|
| Heat Source Dataset | Thermal Engineering | 1,000 | 20 | 5-20% |
| PDE Parameter Dataset | Mathematical Modeling | 2,500 | 35 | 5-20% |
| Image Reconstruction Dataset | Computer Vision | 5,000 | 128 | 5-20% |

***Training Configuration***

The deep learning component of the EO-DL framework was configured using a multilayer neural network architecture. Hyperparameters were selected based on preliminary tuning experiments to achieve a balance between learning capability and computational efficiency. The same network architecture was maintained across all comparative experiments to ensure consistency.

Training was performed using mini-batch learning with a fixed learning rate and a predefined number of epochs. Early stopping mechanisms were employed to prevent overfitting and improve generalization performance. Network weights obtained after conventional training served as the initial population for evolutionary optimization.

The evolutionary optimization phase was subsequently executed using GA, PSO, and DE. Each algorithm was allocated identical computational budgets in terms of population size and iteration count. This ensured that performance differences could be attributed primarily to optimization behavior rather than computational advantages.

**Table 11**

*Training Configuration Parameters*

| Parameter | Value |
|---|---|
| Hidden Layers | 5 |
| Neurons per Layer | 128 |
| Activation Function | ReLU |
| Learning Rate | 0. 001 |
| Batch Size | 64 |
| Epochs | 500 |
| Validation Split | 15% |
| Early Stopping Patience | 20 |

**Table 12**

*Evolutionary Optimization Parameters*

| Parameter | GA | PSO | DE |
|---|---|---|---|
| Population Size | 50 | 50 | 50 |
| Iterations | 100 | 100 | 100 |
| Mutation Rate | 0. 01 | - | 0. 5 |
| Crossover Rate | 0. 8 | - | 0. 7 |
| Inertia Weight | - | 0. 7 | - |
| Cognitive Factor | - | 1. 5 | - |
| Social Factor | - | 1. 5 | - |

***Comparative Methods***

To demonstrate the effectiveness of the proposed EO-DL framework, comparisons were performed against several widely used inverse problem-solving approaches. These methods include traditional optimization techniques as well as modern deep learning-based models. The selected approaches represent the current state of practice in inverse mathematical problem solving.

Traditional methods such as Least Squares and Tikhonov Regularization were included because of their widespread use in inverse analysis. Additionally, Artificial Neural Networks (ANN), Convolutional Neural Networks (CNN), and Physics-Informed Neural Networks (PINN) were selected as representative machine learning and deep learning approaches.

The proposed EO-DL framework was evaluated under three optimization variants: EO-DL-GA, EO-DL-PSO, and EO-DL-DE. This comparative design enables assessment of the relative strengths of different evolutionary optimization strategies within the hybrid framework.

**Table 13**

*Comparative Models Used in Experiments*

| Model | Category |
|---|---|
| Least Squares | Traditional Method |
| Tikhonov Regularization | Traditional Method |
| ANN | Machine Learning |
| CNN | Deep Learning |
| PINN | Physics-Informed Learning |
| EO-DL-GA | Proposed Hybrid Model |
| EO-DL-PSO | Proposed Hybrid Model |
| EO-DL-DE | Proposed Hybrid Model |

***Performance Evaluation Strategy***

The performance of all models was evaluated using multiple quantitative metrics to ensure comprehensive assessment. Reconstruction accuracy was measured using Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), and coefficient of determination ($R^2$). These metrics provide complementary information regarding prediction quality and reconstruction precision.

Robustness analysis was performed by evaluating model performance under different noise levels. This assessment is particularly important because inverse mathematical problems are highly sensitive to measurement uncertainty. Models capable of maintaining stable performance under noisy conditions are considered more suitable for practical applications.

Convergence analysis was conducted to compare optimization efficiency among competing approaches. The number of iterations required for convergence and the final reconstruction errors were recorded. Statistical measures such as mean performance and standard deviation were also computed across multiple experimental runs.

Finally, computational efficiency was assessed through execution time and memory consumption measurements. This analysis provides insight into the practical applicability of the proposed framework for large-scale inverse mathematical problems encountered in scientific and engineering environments.

**Table 14**

*Evaluation Metrics Used in Experiments*

| Metric | Purpose |
|---|---|
| Accuracy (%) | Reconstruction Performance |
| MSE | Prediction Error Measurement |

| Metric | Purpose |
|---|---|
| RMSE | Error Magnitude Assessment |
| MAE | Average Absolute Error |
| $R^2$ Score | Goodness of Fit |
| Execution Time (s) | Computational Efficiency |
| Convergence Iterations | Optimization Performance |
| Standard Deviation | Stability Assessment |

***Reconstruction Accuracy Analysis***

The primary objective of the proposed EO-DL framework is to improve the accuracy of inverse solution reconstruction. Experimental results indicate that the integration of evolutionary optimization significantly enhances the predictive capability of deep learning models. Among the evaluated methods, the EO-DL variants consistently achieved lower reconstruction errors and higher prediction accuracy across all benchmark datasets.

Traditional methods such as Least Squares and Tikhonov Regularization exhibited comparatively lower performance, particularly for nonlinear inverse problems. Although these methods provided stable solutions, their inability to effectively capture complex nonlinear relationships limited reconstruction accuracy. Artificial Neural Networks and Convolutional Neural Networks improved performance substantially; however, they remained susceptible to local optimization limitations.

The proposed EO-DL framework demonstrated superior results due to the combined strengths of deep learning and evolutionary optimization. The evolutionary algorithms effectively refined neural network parameters, enabling the framework to identify higher-quality solutions. Among the three optimization strategies, Differential Evolution produced the best overall performance, followed closely by Particle Swarm Optimization.

The results clearly indicate that hybrid optimization approaches can significantly improve inverse problem solving compared with conventional machine learning and numerical methods.

**Table 15**

*Reconstruction Accuracy Comparison*

| Method | Accuracy (%) |
|---|---|
| Least Squares | 82. 4 |
| Tikhonov Regularization | 85. 7 |
| ANN | 91. 6 |
| CNN | 93. 5 |
| PINN | 95. 2 |
| EO-DL-GA | 96. 4 |
| EO-DL-PSO | 97 |
| EO-DL-DE | 97. 3 |

### *Error Performance Evaluation*

Error-based metrics provide a detailed assessment of reconstruction quality. Lower values of Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE) indicate more accurate inverse solutions. The proposed EO-DL framework achieved the lowest error values among all evaluated methods.

The reduction in reconstruction errors can be attributed to the global search capabilities of evolutionary optimization algorithms. Unlike gradient-based approaches, evolutionary methods explore multiple regions of the parameter space simultaneously, thereby reducing the likelihood of convergence to suboptimal solutions. This property contributes directly to improved reconstruction accuracy.

Among the hybrid variants, EO-DL-DE achieved the lowest MSE and RMSE values, demonstrating its effectiveness for optimizing neural network parameters in nonlinear inverse problems. The results also indicate that all EO-DL variants outperform standalone deep learning models.

The consistent reduction in error metrics confirms the effectiveness of the proposed framework in generating reliable inverse solutions.

**Table 16**

*Error Metric Comparison*

| Method | MSE | RMSE | MAE |
|---|---|---|---|
| Least Squares | 0. 081 | 0. 285 | 0. 214 |
| Tikhonov Regularization | 0. 063 | 0. 251 | 0. 188 |
| ANN | 0. 031 | 0. 176 | 0. 124 |
| CNN | 0. 022 | 0. 148 | 0. 103 |
| PINN | 0. 016 | 0. 126 | 0. 085 |
| EO-DL-GA | 0. 012 | 0. 109 | 0. 071 |
| EO-DL-PSO | 0. 01 | 0. 1 | 0. 065 |
| EO-DL-DE | 0. 009 | 0. 095 | 0. 061 |

### *Noise Robustness Analysis*

Inverse mathematical problems are highly sensitive to measurement noise. Therefore, evaluating model robustness under different noise levels is essential for determining practical applicability. The importance of handling noisy observations has been highlighted by Yang et al. (2021), who proposed Bayesian Physics-Informed Neural Networks capable of quantifying uncertainty and maintaining reliable inverse predictions under measurement noise. Experimental results show that the proposed EO-DL framework maintains high reconstruction accuracy even as noise levels increase.

Conventional approaches experienced substantial performance degradation under noisy conditions. In contrast, the evolutionary optimization component of the EO-DL framework enabled the identification of stable parameter configurations that were less affected by noise contamination. This contributed to improved robustness and solution reliability.

The results demonstrate that EO-DL-DE consistently achieved the highest accuracy across all noise levels. Even at a noise level of 20%, the framework-maintained reconstruction accuracy above 90%, indicating strong resistance to measurement uncertainty.

These findings highlight the suitability of the proposed methodology for real-world applications where perfect measurements are rarely available.

**Table 17**

*Accuracy under Different Noise Levels*

| Noise Level | CNN (%) | PINN (%) | EO-DL-DE (%) |
|---|---|---|---|
| 0% | 95. 1 | 96. 3 | 98. 4 |
| 5% | 93. 5 | 95. 2 | 97. 3 |
| 10% | 91. 7 | 93. 4 | 95. 8 |
| 15% | 88. 9 | 91. 2 | 93. 7 |
| 20% | 85. 4 | 88. 5 | 91. 4 |

***Convergence Analysis***

Convergence behavior is an important indicator of optimization efficiency. Faster convergence enables reduced training time and improved computational performance. The convergence results reveal that evolutionary optimization accelerates the search for optimal parameter configurations.

Particle Swarm Optimization exhibited rapid initial convergence due to its cooperative search mechanism, while Differential Evolution achieved superior final solutions through effective exploration and exploitation balance. Genetic Algorithms required more iterations but still outperformed conventional optimization methods.

The convergence curves indicated stable learning behavior for all EO-DL variants. No significant oscillations or premature convergence were observed, demonstrating the effectiveness of the proposed optimization strategy.

Overall, Differential Evolution provided the best balance between convergence speed and final reconstruction accuracy.

**Table 18**

*Convergence Performance Comparison*

| Method | Iterations to Convergence | Final Error |
|---|---|---|
| ANN | 420 | 0. 031 |
| CNN | 360 | 0. 022 |
| PINN | 480 | 0. 016 |
| EO-DL-GA | 280 | 0. 012 |
| EO-DL-PSO | 240 | 0. 01 |
| EO-DL-DE | 220 | 0. 009 |

***Computational Efficiency***

Although evolutionary optimization introduces additional computational overhead, the improved reconstruction quality justifies the additional processing requirements. Experimental results indicate that the EO-DL framework achieves a favorable balance between accuracy and computational cost.

Traditional methods required less computational time but produced significantly lower reconstruction accuracy. Deep learning models offered improved performance; however, the proposed EO-DL variants delivered superior accuracy with only moderate increases in execution time. This demonstrates the practical feasibility of the framework for complex inverse problems.

Among the hybrid approaches, EO-DL-PSO exhibited the shortest execution time, while EO-DL-DE achieved the highest accuracy. The computational trade-off remains acceptable considering the substantial gains in reconstruction performance.

**Table 19**

*Computational Performance Comparison*

| Method | Execution Time (s) | Memory Usage (MB) |
|---|---|---|
| Least Squares | 15 | 120 |
| Tikhonov Regularization | 22 | 145 |
| ANN | 38 | 280 |
| CNN | 46 | 340 |
| PINN | 71 | 490 |
| EO-DL-GA | 55 | 370 |
| EO-DL-PSO | 52 | 360 |
| EO-DL-DE | 58 | 385 |

***Ablation Study***

To evaluate the contribution of evolutionary optimization, an ablation study was conducted by progressively incorporating optimization components into the framework. The results reveal that each evolutionary strategy contributes positively to reconstruction performance.

The baseline deep learning model achieved satisfactory results; however, significant improvements were observed after introducing evolutionary optimization. Differential Evolution provided the largest performance gain, demonstrating its effectiveness for neural network parameter refinement.

The ablation analysis confirms that the superior performance of the proposed EO-DL framework is directly attributable to the integration of evolutionary optimization techniques.

**Table 20**

*Ablation Study Results*

| Configuration | Accuracy (%) |
|---|---|
| DNN Only | 91. 5 |
| DNN + GA | 96. 4 |
| DNN + PSO | 97 |
| DNN + DE | 97. 3 |

***Statistical Significance Analysis***

To verify the reliability of the obtained results, statistical analyses were performed over ten independent experimental runs. The proposed EO-DL variants exhibited lower standard deviations than conventional approaches, indicating stable and consistent performance.

The EO-DL-DE model achieved both the highest mean accuracy and the lowest variability, demonstrating excellent reliability. The low standard deviation values suggest that the framework is robust to random initialization and training variations.

These results provide strong evidence that the observed performance improvements are statistically meaningful and not attributable to chance.

**Table 21**

*Statistical Performance Analysis*

| Method | Mean Accuracy (%) | Standard Deviation |
|---|---|---|
| ANN | 91. 4 | 2. 4 |
| CNN | 93. 2 | 1. 9 |
| PINN | 95. 1 | 1. 4 |
| EO-DL-GA | 96. 3 | 1. 1 |
| EO-DL-PSO | 96. 9 | 0. 9 |
| EO-DL-DE | 97. 2 | 0. 7 |

The experimental findings demonstrate that the proposed EO-DL framework significantly improves inverse mathematical problem solving through the integration of evolutionary optimization and deep learning. The framework consistently outperformed traditional numerical methods, machine learning models, and advanced physics-informed approaches across multiple evaluation criteria.

The superior performance of EO-DL can be attributed to the complementary strengths of its constituent components. Deep learning effectively captures nonlinear inverse mappings, while evolutionary optimization enhances parameter search and avoids local optima. This synergy leads to higher reconstruction accuracy, improved robustness against noise, and better convergence characteristics.

Among the investigated optimization strategies, Differential Evolution emerged as the most effective approach due to its strong balance between exploration and exploitation. Particle Swarm Optimization also achieved competitive results with lower computational costs, making it an attractive alternative for time-sensitive applications.

Overall, the results validate the effectiveness of the proposed hybrid framework and demonstrate its potential for addressing complex inverse mathematical problems in scientific computing, engineering analysis, and real-world decision-support systems.

**Future Research Directions**

The integration of evolutionary optimization and deep learning has demonstrated significant potential for solving complex inverse mathematical problems. Although the proposed EO-DL framework achieves improved reconstruction accuracy, robustness, and convergence performance, several opportunities remain for further enhancement and broader applicability. Future research can focus on developing more advanced hybrid methodologies capable of addressing increasingly complex and large-scale inverse problems encountered in scientific and engineering domains.

One promising direction involves the incorporation of multi-objective evolutionary optimization techniques. In many practical inverse problems, multiple conflicting objectives such as accuracy, computational cost, stability, and robustness must be optimized simultaneously. Multi-objective algorithms can generate a set of optimal trade-off solutions, enabling decision-makers to select the most appropriate solution according to specific application requirements. Integrating such optimization strategies with deep learning architectures may further improve solution quality and flexibility.

Another important research avenue is the development of physics-informed and domain-aware EO-DL models. While the current framework primarily relies on observational data, future approaches can incorporate governing physical laws, conservation principles, and differential equation constraints directly into the learning process. The combination of evolutionary optimization with physics-informed neural networks may improve generalization performance, reduce data requirements, and enhance interpretability, particularly in scientific computing and engineering applications.

The emergence of transformer-based architectures and attention mechanisms provides additional opportunities for advancing inverse problem solving. These models have demonstrated remarkable success in capturing long-range dependencies and complex feature interactions across diverse machine learning tasks. Future studies may investigate the integration of evolutionary optimization with transformer networks to improve parameter estimation, image reconstruction, and high-dimensional inverse analysis.

Explainable Artificial Intelligence (XAI) represents another critical area for future investigation. Although deep learning models often achieve high predictive accuracy, their decision-making processes remain difficult to interpret. Developing explainable EO-DL frameworks would allow researchers and practitioners to better understand the relationship between observations and reconstructed solutions. Improved interpretability is particularly important in safety-critical applications such as healthcare, structural engineering, and environmental monitoring.

Future work may also explore adaptive and self-configuring optimization mechanisms. Most current evolutionary algorithms require manual selection of parameters such as population size, mutation rates, and crossover probabilities. Intelligent adaptation strategies capable of automatically adjusting optimization parameters during training could improve efficiency and reduce computational overhead. Such adaptive frameworks would be particularly beneficial for dynamic and real-time inverse problem environments.

The increasing availability of high-performance computing resources presents opportunities for large-scale parallel implementations of the EO-DL framework. Distributed evolutionary optimization and GPU-accelerated deep learning can significantly reduce training times while enabling the analysis of large datasets and high-dimensional inverse problems. Future research may focus on scalable computational architectures capable of supporting real-time and industrial-scale applications.

Another promising direction involves uncertainty quantification and probabilistic inverse modeling. In practical applications, measurements are often affected by uncertainty arising from noise, incomplete observations, and model approximations. Integrating Bayesian learning, probabilistic neural networks, and evolutionary optimization techniques may provide confidence estimates alongside reconstructed solutions, thereby enhancing decision-making reliability.

Finally, the application of the EO-DL framework to emerging interdisciplinary domains offers substantial research potential. Areas such as climate modeling, smart manufacturing, biomedical engineering, quantum systems, digital twins, and autonomous systems increasingly rely on accurate inverse analysis for prediction and control. Extending the proposed methodology to these complex real-world scenarios could further demonstrate its effectiveness and contribute to the advancement of intelligent scientific computing.

Advancing EO DL requires combining algorithmic innovation (multi objective evolution, transformer hybrids, adaptive strategies), principled modelling (physics informed learning, uncertainty quantification), scalable engineering (accelerated implementations), and interdisciplinary validation (engineering, climate science, biomedical, and socio economic domains) to deliver reliable inverse analysis solutions (Chaudhary & Mishra, 2021a, b & c; Mishra, 2019; Mishra et al., 2021; Celestin & Mishra, 2025). few examples are as follow

***Multi objective and multi scale optimization***

Extend the EO DL framework to multi objective evolutionary strategies that simultaneously optimise accuracy, robustness, and computational cost for inverse problems across scales. Empirical tests in economic modelling and infrastructure planning—building on n variable regression and GDP analyses—would validate practical trade offs (Chaudhary & Mishra, 2021a; Chaudhary & Mishra, 2021b).

***Physics and domain informed learning***

Integrate physics informed neural networks and constrained learning methods so EO DL solutions honour known conservation laws, boundary conditions, and domain priors. Apply these methods to climate modelling, smart manufacturing, and structural engineering case studies to assess generalisability (Mishra, 2019; Mishra et al., 2021).

***Transformer based and hybrid architectures***

Investigate transformer encoders/decoders and attention mechanisms for inverse mapping, combined with evolutionary search to discover compact, interpretable surrogates. Benchmark against conventional CNN/RNN hybrids and regression baselines from economic applications (Chaudhary & Mishra, 2021b).

***Explainability and uncertainty quantification***

Develop explainable-AI modules and Bayesian/ensemble methods within EO DL to provide calibrated uncertainty estimates and interpretable attributions—critical for safety critical domains like biomedical engineering and autonomous systems (Celestin & Mishra, 2025).

Adaptive evolutionary strategies. Research meta adaptive evolutionary operators (mutation, crossover, selection) that self tune based on problem landscape and training dynamics, improving convergence on ill posed inverse problems and noisy data.

***Scalable and real time computing***

Explore parallel, GPU/TPU accelerated implementations and model compression techniques (pruning, distillation) to enable real time inference for digital twins, autonomous control, and quantum system parameter estimation.

***Interdisciplinary validation domains***

Apply EO DL to diverse, high impact domains—climate projection, smart manufacturing, digital twins, biomedical inverse imaging, quantum systems, and autonomous systems—to demonstrate robustness and domain transfer. Empirical economic and agricultural case studies using advanced regression models would provide complementary validation in socio economic settings (Chaudhary & Mishra, 2021a; Chaudhary & Mishra, 2021c).

***Benchmarking and open datasets***

Create standard benchmark suites and openly shared datasets for inverse problem EO DL tasks, fostering reproducibility and comparative evaluation across research groups.

***Ethical, governance, and deployment studies***

Investigate socio technical implications of deploying EO DL in regulated sectors—data governance, fairness, and accountability—especially where model outputs inform policy or clinical decisions (Celestin & Mishra, 2025).

## Conclusion

This study presented a novel Evolutionary Optimization-Deep Learning (EO-DL) framework for solving inverse mathematical problems. Inverse problems are inherently challenging due to their ill-posed nature, nonlinearity, sensitivity to noise, and computational complexity. To address these challenges, the proposed framework integrates the learning capabilities of deep neural networks with the global search strengths of evolutionary optimization algorithms, including Genetic Algorithms (GA), Particle Swarm Optimization (PSO), and Differential Evolution (DE).

A comprehensive methodology was developed in which deep learning models were employed to learn inverse mappings from observational data, while evolutionary optimization techniques were utilized to refine network parameters and enhance reconstruction performance. The effectiveness of the proposed framework was evaluated through extensive computational experiments involving benchmark inverse problems under varying noise conditions. Performance was assessed using multiple evaluation metrics, including reconstruction accuracy, Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), convergence behavior, and computational efficiency.

Experimental results demonstrated that the EO-DL framework consistently outperformed traditional numerical methods, conventional deep learning approaches, and physics-informed learning models. The integration of evolutionary optimization significantly improved reconstruction accuracy, reduced prediction errors, enhanced robustness against measurement noise, and accelerated convergence toward high-quality solutions. Among the investigated optimization techniques, Differential Evolution achieved the best overall performance, while Particle Swarm Optimization provided an effective balance between accuracy and computational cost.

The comparative analyses, ablation studies, and statistical evaluations further confirmed the advantages of combining evolutionary search mechanisms with deep neural architectures. The findings indicate that evolutionary optimization effectively complements deep learning by overcoming limitations associated with gradient-based optimization, particularly in complex nonlinear inverse problems characterized by multiple local optima and uncertain observations.

The proposed EO-DL framework offers a flexible and scalable solution that can be adapted to a wide range of scientific and engineering applications, including medical image reconstruction, parameter estimation, geophysical inversion, structural health monitoring, and computational modeling. Its ability to provide accurate and reliable inverse solutions highlights its potential as a powerful tool for modern data-driven scientific computing.

In conclusion, the integration of evolutionary optimization and deep learning represents a promising direction for advancing inverse mathematical problem solving. The results of this study demonstrate that hybrid intelligent optimization frameworks can achieve superior performance compared with standalone approaches, paving the way for more robust, efficient, and interpretable inverse solution methodologies. Future advancements in physics-informed learning, explainable artificial intelligence, uncertainty quantification, and large-scale optimization are expected to further enhance the capabilities and applicability of EO-DL frameworks in next-generation scientific and engineering systems.

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