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Models and Algorithms of Abstract Flows in Evacuation Planning

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Abstract

Flows over time generalize classical network flows by introducing a notion of time. Each arc is equipped with a transit time that specifies how long flow takes to traverse it, while flow rates may vary over time within the given edge capacities. Ford and Fulkerson's original 1956 max flow/min cut paper formulated max flow in terms of flows on paths. In 1974, Hoffman pointed out that Ford and Fulkerson's original proof was quite abstract, and applied to a wide range of max flow-like problems. In this abstract model we have capacitated elements and linearly ordered subsets of elements called paths that satisfy switching property. When two paths P and Q cross at an element (node) then there must be a path that is a subset of the first path up to the crossing element and a subset of the second path after the crossing element. Contraflowis a widely accepted solution approach that increases the flow and decreases the evacuation time making the traffic smooth during evacuation by reversing the required road directions from the risk areas to the safe places. In this paper, we integrate the abstract flow with contraflow, give mathematical formulations of these models and present efficient polynomial time algorithms for solving the abstract contraflow problems.

Key words: *Abstract flow, Contraflow, Algorithm, Lexicographically maximum abstract contraflow, Earliest arrival abstract contraflow.*

Introduction

A disaster is an uncertain disruption that causes massive loss of human, infrastructure and overall economy. After disasters, the process of removing residence as quickly and efficiently as possible from the disastrous areas to safer places is the evacuation planning problem. The disastrous areas and the safer places are considered as sources S and sinks D respectively, and the connection between these places are lanes or arcs. There may be number of intersections of street between the connections that are considered as nodes. Each lane has limited capacities and fixed travel times. The flow is considered as the group of evacuees passing through the network as a homogeneous group.

After disasters, in the course of placement of evacuees from the disastrous areas to safer places, the movement of large numbers vehicles causes high congestion on the roads. So, in this paper we deal with the transhipment of evacuees from the disastrous areas to safer places by reducing the crossing effect of the paths effectively.

In an abstract network, flow is transmitted along the given path system with switching property and it is not necessary to be used all elements, even the crossing element, in the switched path. The crossing effect is reduced by switching of the paths and the augmentation of flow on feasible paths. In abstract network, direction of paths from the common element are switched to the next path instead of crossing it.

Ford and Fulkerson (1956, 1962) introduced the maximum dynamic flow problems (MDFP),developed the first well known algorithm that sends maximum flow from the source to the sink by augmenting along s-d paths and proved the maximum amount of flow is equal to the total capacity of the arcs in minimum cut. After the development of maximal static flow and maximum dynamic flow models, different researchers have studied several network flow problems for evacuation planning. Some of them are illustrated here. The maximum dynamic flow problem is used to shift maximal amount of flow in a given time. The earliest arrival flow (Minieka, 1973) problem is used to maximize the number of evacuees in every possible time. The quickest flow problem is used for allocating the evacuees to a safer zone in minimum time. The lexicographically maximum dynamic flow problem is used to send maximum number of evacuees in given priority order within the given time period.

Hoffman (1974) introduced the concept of abstract flow. The abstract flow generalizes the concept of paths by replacing the underlying network configuration. This tactic makes the use of switching property that eliminates the crossing at intersections. The maximum abstract flow problem was introduced in (Hoffman, 1974) and a polynomial time algorithm for the solution of problem presented by McCormick (1996). He used the augmenting path structure satisfying the complementary slackness condition: every positive path meets the cut set exactly at one common element and every element of the cut is saturated. Based on the same algorithm, the maximum abstract contraflow algorithm has been developed in (Pyakurel, Dhamala & Dempe, 2017) for the path reversal maximum abstract contra flow.

The maximum weighted abstract flow model has been developed by Hoffman (1974) and solved by Martens and McCormick (2008). Kappmeier et al. (2014) have investigated the maximum dynamic abstract flow problem and its solution procedure. The existence of the lexicographically maximum abstract flow problem for prioritized terminals has been shown in Kappmeier (2015). He also introduced the earliest arrival abstract flow and solved the problem of maximization of the dynamic abstract flow from the source to the sink at every possible time. Zhao *et al.*(2016) deliberate the important lane-based routing strategy for reducing the interruptions that reduce (or eliminate) crossing and merging conflicts at nodes.

Different evacuation models with and/or without contraflow and their solution strategies can be found in Arulselvan (2009), Pyakurel and Dhamala (2016), Rebennack, Arulselvan, Elefteriadou, and Pardalos (2010). Moreover, Pyakurel *et al.*, (2017) introduced the abstract contraflow approach with path reversal capability. They presented a polynomial time algorithm to solve the abstract maximum dynamic contraflow in continuous time setting. Huertas and Van Hentenryck (2022) presented zone-based evacuation by assigning single evacuation path to the corresponding zone. They used convergent paths at intersections and non-preemptive schedules to ensure the evacuation process without interruptions.

The structure of the paper is as follows. In Section 2, we present the basic notations and prior works in abstract network flows. In Section 3, we propose maximum dynamic and earliest arrival abstract contraflow models with efficient solution procedures for two-terminal abstract networks. The lexicographically maximum contraflow model and algorithm for multiterminal abstract networks are also presented in Section 3. Section 4 concludes the paper.

Basic Notations and Models

In this section, we give basic mathematical notations that are used throughout the paper.

Definition. An abstract path system consists of a ground set E of elements and a family of paths $\mathcal{P} \subseteq 2^E$. For every path $P \in \mathcal{P}$ there is an order $<_P$ of the elements in P. A path system is an abstract network, if the switching property is fulfilled: For every paths P_1, $P_2 \in \mathcal{P}$ and every $e \in P_1 \cap P_2$, there is a path $P_1 \times_e P_2 \subseteq \{p \in P_1 : p \leq_{P_1} e\} \cup \{q \in P_2 : e \leq_{P_2} q\}$. Note that the definition above does not make any statement on the order of the elements in $P_1 \times_e P_2$. In particular, $<P_1 \times_e P_2$ does not depend on $<P_1$ or $<P_2$.

For multi-terminal evacuation network, let $N = (E, \mathcal{P}, b, \tau, S, D, T)$ where E and $\mathcal{P}$ represent the sets of elements and paths, respectively. Let $b: E \rightarrow Z^+$ be the capacity function and $\tau: E \rightarrow Z^+$ be the transit time function. The given non-negative time horizon T is symbolized by $T = \{0,1, \cdots, \mathbf{T}\}$ in discrete time setting, whereas it is denoted by $T = \{[0,1), \cdots, [\mathbf{T}, \mathbf{T}+1)\}$ in continuous time setting. For every path $P \in \mathcal{P}$ there is a linear order $< P$ of elements and the set of such paths. $\mathcal{P}$ satisfies the switching property in abstract network setting. A switching property requires that for each $P_1, P_2 \in \mathcal{P}$ and $e \in P_1 \cap P_2$, there exist paths $R \subseteq P_1 \times_e P_2 = \{a \in P_1: a \leq_{P1} e\} \cup \{a \in P_2: a \geq_{P2} e\}$ and $R' \subseteq P_2 \times_e P_1 = \{a \in P_2: a \leq_{P2} e\} \cup \{a \in P_1: a \geq P_1\ e\}$ where a is said to be left of e on P if $\leq P$ e and right of e if $a \geq_P e$. The before and after parts from e of the path P excluding e are denoted by $(P, e)=\{p \in P: p<_P e\}$ and $(e, P)=\{p \in P: p>_P e\}$, respectively. Let $f: \mathcal{P} \rightarrow R^+$ be the flow function. A network is considered as static or dynamic depending on time.

The consideration of time factor transforms the abstract flow model into dynamic abstract flow model. The dynamic abstract flow problems can be transformed into static network by constructing the corresponding time expended networks. The time expanded network can violate the switching property. So, Kappmeier *et al*. (2014) introduced the holdover of flow at intermediate nodes to construct an abstract time expanded network.

Here we present some lemmas and theorems mentioned in the work of Kappmeier *et al*. (2014) which are useful in our study:

Lemma: Let $P, Q \in P$, $e \in P \cap Q$, then there is a path $R \subseteq [P, e] \cup [e, Q]$ such that $a \in R \cap [P,e]$ and $b \in R \setminus [P,e]$ imply $a <_R b$.

Theorem-1: (Abstract max flow/min cut over time) The value of a maximum abstract flow over time equals the capacity of a minimum abstract cut over time.

Theorem-2: The value of an abstract flow over time with waiting at intermediate elements is no larger than the value of a maximum abstract flow over time without waiting.

The central idea behind the contraflow technique is to improve the outbound capacity by adopting the arc or path reversals toward the safer places keeping the same travel time in the evacuation network. As a result, the flow value is increased, evacuation time is decreased and traffic flow is made smooth. Let $\mathcal{P} = \{\vec{P}, \overleftarrow{P}\}$ be the set of all paths in contraflow abstract network N with capacities $b(\vec{P}) = \min\{b_{e:\, e \in \vec{P}}\}$ and $b(\overleftarrow{P}) =$ min in$\{b_{e:\, e \in \overleftarrow{P}}\}$ where *be* is the capacity of an edge. We define the undirected auxiliary. Network $\tilde{N}$ by adding the capacities on the corresponding two-way paths and keeping the transit time (if any) fixed. The set of elements and paths are dented by E and $\tilde{\mathcal{P}}$, where $\tilde{e} \in \tilde{E}$ and $\tilde{P} \in \tilde{\mathcal{P}}$. Then the capacity function is defined as $b(\tilde{P}) = \min\{b_{\tilde{e}:\, \tilde{e} \in \tilde{P}}\}$ while the travel times (if any)on paths remains the same. The auxiliary network resulting from path reversal holds order of elements for each $\tilde{P} \in \tilde{\mathcal{P}}$ and $\tilde{\mathcal{P}}$ satisfies the switching property.

Abstract Contraflow

In this section, we discuss the solution procedures to solve the evacuation abstract contraflow problems. The abstract contraflow means traversal of oppositely directed paths towards the destination element to improve the flow and reduce the evacuation time. This approach uses path reversals in abstract network at zero transit time between pair of elements along the oppositely directed paths without any switching costs.

Maximum Static Abstract Contraflow.

Let N = (G, b, s, z) be path reversible abstract network, where G = (E, *P*) and E represents the set of elements and $\mathcal{P} = \vec{P} \cup \overleftarrow{P}$ represents the set of two-way paths. Here $\vec{P}$ and $\overleftarrow{P}$ represent forward and backward source-sink paths, respectively. The maximum abstract contraflow doubles the flow value after contraflow reconfiguration if every element in a minimum abstract cut has symmetric capacity, (Pyakurel, Dhamala & Dempe, 2017). We define the maximum abstract contraflow (MACF) by integrating the contraflow model introduced in (Arulselvan, 2009, Rebennack et al., 2010) and the MAF problem solved in (McCormick, 1996). Here, we introduce a problem related to maximum abstract contraflow network.

Problem 1.

For a given path reversible static abstract network N = (G, b, s, z), where G = (E, *P*), the maximum abstract contraflow problem is to find the maximum flow leaving the source element that is to be sent to sink via s – z paths $\vec{P} \cup \overleftarrow{P}$.

To solve the problem, we first construct an auxiliary network by adding two-way movement capacities between two consecutive elements. The auxiliary network is denoted by $\tilde{N}$. For any two consecutive elements e and a with e $<_{\vec{P}}$ a and a $<_{\overleftarrow{P}}$ e, the movement capacity $\tilde{b}$ is defined by $\tilde{b}_e = b_{\text{e: e} \in \vec{P}} + b_{\text{a: a} \in \overleftarrow{P}}$ where $b_{\text{a: a} \in \overleftarrow{P}} = 0$ if a$\epsilon\overleftarrow{P}$.

We now present efficient algorithm to solve abstract contraflow problems. We first transform the given two-way network to an auxiliary network $\tilde{N}$ and construct cycle free paths on $\tilde{N}$. On these cycle free paths, we solve maximum static abstract flow problems and use Algorithm-1 to obtain an optimal solution to the corresponding contraflow problems.

Algorithm-1: Maximum Static Abstract Contraflow Algorithm

1. Given path reversible abstract network N = (G, b, s, z), where G = (E, $\mathcal{P}$).
2. Construct the auxiliary network, $\tilde{N} = (\tilde{G}, \tilde{b}, \tilde{s}, \tilde{z})$ with new capacity $\tilde{b}(\tilde{P}) = \text{b}(\vec{P}) + \text{b}(\overleftarrow{P})$.
3. Solve the maximum abstract network flow problem in $\tilde{N} = (\tilde{G}, \tilde{b}_e, \tilde{s}, \tilde{z})$ satisfying the switching property using (McCormick,1996) as follows:
   (a) Initialize $\tilde{f} = \tilde{f}_0$, if an initial solution is given, otherwise initialize as the zero flow.
   (b) While $\tilde{f}$ is not optimal:
   i. Compute an augmenting structure. If no such structure exists, return $\tilde{f}$.
   ii. Determine $\gamma \in \tilde{N}$ so that all paths in augmenting structure can be augmented by γ.
   iii. For each path $\tilde{P}^+$ in augmenting structure, set $\tilde{f}_{\tilde{P}^+} = \tilde{f}_{\tilde{P}^+} + \gamma$
   iv. For each path $\tilde{P}^-$ in augmenting structure, set $\tilde{f}_{\tilde{P}^-} = \tilde{f}_{\tilde{P}^-} - \gamma$

4. A path $\overleftarrow{P} \in \mathcal{P}$ is reversed if and only if the flow along $\vec{P} \in \mathcal{P}$ is greater than $b(\vec{P})$ or there is non-negative flow along the path $\vec{P} \notin \mathcal{P}$.

Maximum Dynamic Abstract Contraflow.

Let $N = (G, b, \tau, s, z, T)$ be path reversible abstract network, where, $G = (E, \mathcal{P})$ and E represents the set of elements and $\mathcal{P} = \vec{P} \cup \overleftarrow{P}$ represent the set of two-way paths. Here $\vec{P}$ and $\overleftarrow{P}$ represent forward and backward source-sink paths, respectively. Let τ be a symmetric] transit time between pair of consecutive elements e and a with $e <_{\vec{P}} a$ and $a <_{\overleftarrow{P}} e$, along a path, so that $\tau_{\vec{P}}$ (from element e to a) $= \tau_{\overleftarrow{P}}$ (from element a to e). Temporal component T represents the time horizon. The maximum dynamic abstract flow (MDAF) problem has been introduced and solved in (Kappmeier *et al.*, 2014). The maximum dynamic contraflow problem with arc reversal capability and its strongly polynomial time solution procedure are presented by (Rebennack *et al.*, 2010). We introduce and solve the maximum dynamic abstract contraflow (MDACF) problem with path reversal capability at time zero.

Problem 2.

For a given path reversible dynamic abstract network $N = (G, b, \tau, s, z, T)$, where $G = (E, P)$, the maximum dynamic abstract contraflow problem to find the maximum dynamic flow by reversing the direction of paths $\overleftarrow{P}$ at time zero, leaving the source element that is to be sent be sin k via $s - z$ paths $\vec{P} \cup \overleftarrow{P}$. To solve the problem, we construct an ausiliary network $\tilde{N}$. For any two consecutive elements e and a with $e <_{\vec{P}} a$ and $a <_{\overleftarrow{P}} e$, the movement capacity $\tilde{b}$ is defined by $\tilde{b}_e = b_{e:\, e \in \vec{P}} + b_{a:\, a \in \overleftarrow{P}}$ where $b_{a:\, a \in \overleftarrow{P}} = 0$ if $a \notin \overleftarrow{P}$ and transit time is

$$\tilde{\tau}_e = \begin{cases} \tau_{e:\, e \in \vec{P}} & if \quad e <_{\vec{P}} a \\ \tau_{a:\, a \in \overleftarrow{P}} & otherwise \end{cases}$$

We now present efficient algorithm to solve abstract contraflow problems. We first transform the given two-way network to an auxiliary network $\tilde{N}$ and construct cycle free paths on $\tilde{N}$. On these cycle free paths, we solve maximum dynamic abstract flow problems and use Algorithm-2 to obtain an optimal solution to the corresponding contraflow problems.

Algorithm-2: Maximum Dynamic Abstract Contraflow Algorithm

1. Given path reversible abstract network $N = (G, b, \tau, s, z, T)$, where $G = (E, \mathcal{P})$ and paths can be reverse without any cost.
2. Construct the auxiliary network, $\tilde{N} = (\tilde{G}, \tilde{b}_e, \tilde{\tau}, \tilde{s}, \tilde{z}, T)$ with new capacity and transit time functions $\tilde{b}(\tilde{P}) = b(\vec{P}) + b(\overleftarrow{P})$ and $\tilde{\tau}(\tilde{P}) = \sum_{\tilde{e} \in \tilde{P}} \tilde{\tau}(\tilde{e})$.

3. Generate a temporally repeated dynamic flows $\tilde{G}$ with capacity $\tilde{b}(\tilde{P})$and transit time $\tilde{\tau}$ $(\tilde{P})$, (Kappmeier *et al.*,2014).

4. A $\overleftarrow{P} \in \mathcal{P}$ is reversed if and only if the flow along $\vec{P} \in \mathcal{P}$ is greater than b $(\vec{P})$ or there is a non – negative flow along the path $\vec{P} \notin \mathcal{P}$.

We can solve problem-2 by using the above algorithm.

Lexicographically Maximum Abstract Contraflow.

Let N = (G, b, S, D) be path reversible abstract network, where G = (E, $\mathcal{P}$) and E represents the set of elements and $\mathcal{P} = \vec{P} \cup \overleftarrow{P}$ represents the set of two-way paths. Here, S and D represents sequence of sources $s_1, s_2, \cdots, s_k$ and sequence of sinks $z_1, z_2, \cdots, z_k$ respectively. If we have a given rank on the terminals with priorities, the flow is compared by its value in their rank ordering, referred to as lexicographically maximum flow. An existence and a polynomial solution of lexicographically maximum abstract flow (LMAF) problem have been presented in (Kappmeier, 2015). In his model, the order of terminals has to fulfil compatible property if more than one terminal node is contained in a path. Here we introduce a problem of lexicographically maximum abstract contraflow and solution procedure of this problem.

Problem 3.

Let N = (G, b, S, D) be an abstract network, where G = (E, $\mathcal{P}$) contains compatible set of sources S and sinks D. The lexicographically maximum abstract contraflow (LMACF) problem is to find a LMAF where paths can be reversed without any cost.

To solve the problem, we introduce the following algorithm:

Algorithm-3: Lexicographically Maximum Abstract Contraflow Algorithm

1. Given abstract network N = (G, b, S, D, $\omega \equiv 1$) with a compatible sequence of sources $s_1, s_2, \cdots, s_k$ or sinks $z_1, z_2, \cdots, z_k$ in S and D, respectively.
2. Construct auxiliary network, $\tilde{N} = (\tilde{G}, \tilde{b}_e, \tilde{s}, \tilde{z}, \omega \equiv 1)$ with capacity $\tilde{b}(\tilde{P}) = b(\vec{P}) + b(\overleftarrow{P})$.
3. Find solution in auxiliary network using Abstract Lexicographically Maximum Flow Algorithm, (Kappmeier,2015):
   (a) Set i = 0 and initialize $\tilde{f}^0 = 0$ as the zero flow on all paths.
   (b) Set i = i+1 and define the abstract networks $(\tilde{E}, \tilde{\mathcal{P}}^i_{\tilde{s}})$ (or $\tilde{E}, \tilde{\mathcal{P}}^i_{\tilde{z}}$) w.r.to the sinks.
   (c) Compute a flow $\tilde{f}^i$ using Step 3 of Algorithm 1 in $(\tilde{E}, \tilde{\mathcal{P}}^i_{\tilde{s}})$ starting with solution $\tilde{f}^{i-1}$.
   (d) If i = k return $\tilde{f}^k$, otherwise continue with Step 3(b).

4. A path $\overleftarrow{P}\in\mathcal{P}$ is reversed if and only if the flow along $\vec{P}\in\mathcal{P}$ is greater than $\mathrm{b}(\vec{P})$ of there is non-negative flow along the path $\vec{P}\notin\mathcal{P}$.

Earliest Arrival Abstract Contraflow.

Let N = (G, b, τ, s, z, T) be path reversible abstract network, where $\mathrm{G} = (\mathrm{E},\mathcal{P})$ and E represents the set of elements and $\mathcal{P} = \overrightarrow{P} \cup \overleftarrow{P}$ represents the set of two-way paths. The existence of earliest arrival abstract flow (EAAF) described by (Kappmeier, 2015) generalizes the earliest arrival flow which maximizes the EAF at each possible time point. Here, we introduce a problem of earlier arrival abstract contraflow and solution procedure of this problem.

Problem 4.

Let N = (G, b, s, z, T) be an abstract dynamic network, where G = (E, $\mathcal{P}$). The earliest arrival abstract contraflow (EAACF) problem is to find the EAAF with path reversal capability at time zero.

Algorithm- 4: Earliest Arrival Abstract Contraflow Algorithm

1. Given abstract network N = (G, b, τ, s, z, T) where $\mathrm{G} = (\mathrm{E},\mathcal{P})$ and paths can be reverse without any cost.
2. Construct the auxiliary network, $\widetilde{N} = (\tilde{G}, \tilde{b}_e, \tilde{\tau}, \tilde{s}, \tilde{z}, \mathrm{T})$with new capacity and transit time functions $\tilde{b}(\tilde{P}) = \mathrm{b}(\vec{P}) + \mathrm{b}(\overleftarrow{P})$ and $\tilde{\tau}\,(\tilde{P}) = \sum_{\tilde{e}\in\tilde{P}} \tilde{\tau}(\tilde{e})$ respectively.
3. Solve the problem in the auxiliary network using Step 3 of Algorithm 3 in corresponding $(\tilde{E}_{\mathrm{T}}, \tilde{\mathcal{P}}_{\mathrm{T}}^{\sigma})$ of $\widetilde{N}$.
4. A path $\overleftarrow{P}\in\mathcal{P}$ is reversed if and only if the flow along $\vec{P}\in\mathcal{P}$ is greater than $\mathrm{b}(\vec{P})$ or there is a non – negative flow along the path $\vec{P}\notin\mathcal{P}$.

Conclusion

In this paper, we discussed abstract flow, contraflow and abstract contraflow models from various perspectives. Integrating these models, we introduced abstract contraflow approach with discrete time settings. Through these models we came to know that the switching property is the most essential force behind abstract flow. In abstract flow model, some structural results of classical network are also valid such as, the maximum abstract flow and minimum abstract cut are strong dual to each other. We proposed efficient algorithms for maximum dynamic and earliest arrival abstract contraflow problems in two-terminal abstract networks. We also proposed polynomial time algorithm for lexicographically maximum abstract contraflowin multi-terminal abstract networks. Our results increase the flow values at every possible time by reducing crossing conflicts with arc reversals toward the safe destinations in evacuation planning.

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