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Research Article

The Divine Number System: Exploring the Fibonacci Numbers and Sequences

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ABSTRACT

The Fibonacci sequence has long been regarded as a divine number system due to its remarkable recurrence in mathematics, nature, art, architecture, and various other disciplines. Elements such as fruits, flowers, leaves, hills, valleys, the Himalayas, rivers, and springs demonstrate a rhythmic balance that makes them aesthetically pleasing. These ideas are of common interest not only for mathematicians but also to biologists, psychologists, architects, artists, musicians, designers, and many other professionals. This research presents an insight into the divine number system through an integrated analytical and exploratory approach to its fundamental patterns, recurrence relations, mathematical structures, identities, properties, and their aesthetic significance. It combines classical recurrence analysis, the golden ratio, Fibonacci ratio, Binet's formulae, Fibonacci-type sequences, Lucas numbers, Fibonacci matrices, and their generalizations to address the gap via library-based investigations for the broad horizon and area of interest in multidisciplinary research.

KEYWORDS: Fibonacci sequence, Golden ratio, recurrence relations

INTRODUCTION

An extraordinary ratio, such as the divine or golden ratio, which is abundantly found in nature, has captivated thinkers for centuries and is fundamental to ancient and modern architecture, design, art, and photography. The golden ratio reveals patterns inherent in nature and is closely associated with Fibonacci numbers and their ratios, which are like nature's numbering system. In the most famous temples and cultural heritage sites, Vastu Shastra, a traditional yet scientific architecture, also incorporates the idea of divine proportion based on mathematical principles to balance energy, visual harmony, and beauty. It is prominent in many religious and spiritual structures, such as Rangoli, Mandalas, and Yagya mandaps. Since Euclid, many have studied this fascinating concept in their works, such as Dunlap (1997).

Consider a line segment AB with a point X in it such that, $\frac{AB}{AX} = \frac{AX}{BX}$. Let AB = k , AX = 1, then $\frac{k}{1} = \frac{1}{k-1}$, which gives $k^2 - k - 1 = 0$. From this, the positive solution is named the golden ratio and is denoted by (phi) ϕ, and its conjugate is denoted by (psi) ψ. The number ϕ is irrational, approximately 1.618033..., called the golden ratio or divine proportion. Luca Pacioli compared it to God and mentioned the ratio is presented by God, as both are like themselves, Pacioli (1509).

Long before Fibonacci's era, a closely related numerical pattern was described in the Indian subcontinent, with early mention credited to the Sanskrit grammarian Pingala. Today, this sequence is recognized for its remarkable presence across the natural world and its role in describing spiral formations. It can be identified in tree branching, leaf arrangement, and the distribution of seeds in fruits such as raspberries. Except for the first few values, the quotients of successive terms in such a sequence approach the golden ratio, which has led to its reputation as a foundational pattern in nature and in the natural number system. For details, we refer to (Koshy, 2011; Livio, 2002; Posamentier & Lehman, 2012; Adhikari, 2023; Adhikari, 2024).

Consider a recurrence relation like $\xi(n+1) = \xi(n) + \xi(n+1)$, for n >1, with $\xi(1) = 1$ and $\xi(2) = 3$, then the formed sequence { $\xi(n)$} is called the Lucas sequence. For example, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123. Lucas and Fibonacci have some common mathematical relations; structural similarities closely connected in their recurrence relations and applications in theoretical and applied aspects, as mentioned in Koshy (2011).

An investigation by Miska et al. (2025) presented an innovative family of number sequences, the binary sequence, which expands the overall landscape of numerical patterns for sequence development and relationships. Lipati and Szalay (2024) presented the distribution generated by a random inhomogeneous Fibonacci sequence. Sharma et al. (2025) examined contemporary computer science and algorithm design in real-world applications.

This work presents the Fibonacci numbers and sequences as a divine number system, philosophically exploring their structures, representations, beauty, identities, and properties, analytically and computationally, to open the broader pathways for further theoretical and interdisciplinary research.

RESEARCH METHODS

This study adopts a mixed-methods approach, integrating analytical mathematical techniques with a technical insight to examine the Fibonacci sequence, recurrence patterns, the golden ratio, related mathematical structures, and the aesthetic significance of the Fibonacci sequence and related number systems. Deductive reasoning is employed to analyze recurrence relations, closed-form expressions, and generalizations such as Binet's formula and Lucas numbers, including the eigenvalues of such Fibonacci matrices, with their interconnection to the golden ratio and the conjugates. The methodology combines formal mathematical analysis with literature-based inquiry to provide both depth and breadth in understanding the "divine" nature of Fibonacci patterns across disciplines. The study does not involve experimental data collection but rather relies on rigorous examination of established theories, mathematical proofs, and the recurrence of Fibonacci patterns in natural and human-made structures and their comparative studies. Such a research design enables the discovery of connections, generalizations, and interpretive insights across mathematics, nature, and the arts. It is conducted ethically and responsibly with no external data or resources.

RESULTS AND DISCUSSION

Consider a hypothetical scenario where a pair of rabbits is introduced into a walled field. Suppose such a pair of rabbits can reproduce each month, with a new pair of offspring having the opposite sex and not dying within the year. Let it begin on January 1, 2025; the production cycle within the year is as in Table 1, as illustrated in Posamentier and Lehmann (2012).

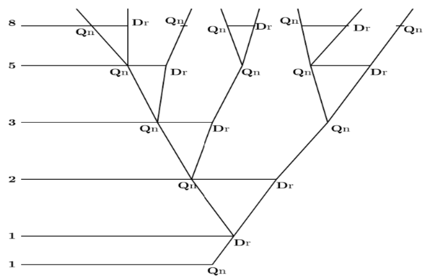
Table 1

Reproduction from a Pair of Rabbits for the Hypothetical Situation.

Date assumed	No. of adult pairs	No. of young pairs	Total
1st January 2025		1	1
1st February 2025	1	1	2
1st March 2025	2	1	3
1st April 2025	3	2	5
1st May 2025	5	3	8
1st June 2025	8	5	13
1st July 2025	13	8	21
1st August 2025	21	13	34
1st September 2025	34	21	55
1st October 2025	44	34	89
1st November 2025	89	55	144
1st December 2025	144	89	233

Figure 1

A Beehive Story



Now, consider a beehive with its story. A single female queen, denoted here by Q_n lays eggs. If it is not fertilized, it becomes a drone, a male bee denoted here by Dr. If it is fertilized, it becomes a female worker bee that doesn't lay eggs. But she will turn to the queen bee to struggle with the existing one if she is fed with royal jelly. If so, either she will be the queen or fly off from the hive and start a new hive; a new story begins. Hence, a drone has only one parent (a queen, with no father). A queen has a queen and a drone as its two parents. Its pattern is like 1, 1, 2. And it continues as 1, 1, 2, 3, 5, 8, ..., as illustrated in Figure 1. Here, the story is by the Fibonacci sequence denoted by zeta $\{\zeta_n\}$ as mentioned in Posamentier and Lehmann (2012).

Consider any two successive Fibonacci numbers, ζ_k and ζ_{k-1}. Then their ratio is ρ_n. The larger the numbers considered, the closer the ratio to ϕ, as in Table 2. Hence,

$$\phi = \frac{\zeta_k}{\zeta_{k-1}}.$$

Table 2

Fibonacci Sequence and the Value of ρ_n

S.No.	Fibonacci sequence	ρ_n
1	1	
2	1	1
3	2	2.00000000
4	3	1.50000000
5	5	1.66666667
6	8	1.60000000

7	13	1.62500000
8	21	1.61538462
9	34	1.61904762
10	55	1.61764706
11	89	1.61818182
12	144	1.61797753
13	233	1.61805556
14	377	1.61802575
15	610	1.61803714
16	987	1.61803279
17	1597	1.61803445
18	2584	1.61803381
19	4181	1.61803406
20	6765	1.61803396
21	10946	1.61803400
22	17711	1.61803399
23	28657	1.61803399

An ordinary Fibonacci can be obtained by raising the indices of ϕ to consecutive powers, like:

$\phi^1 = \phi$

$\phi^2 = 1.61803^2 = 2.61803 = 1 + 1.61803 = 1 + \phi.$

$\phi^3 = \phi(1+\phi) = \phi + \phi^2 = \phi + (1+\phi) = 1 + 2\phi.$

$\phi^4 = \phi(1+2\phi) = \phi + 2\phi^2 = \phi + 2(1+\phi) = 2 + 3\phi.$

$\phi^5 = \phi(2+3\phi) = 2\phi + 3\phi^2 = 2\phi + 3(1+\phi) = 3 + 5\phi.$

$\phi^6 = \phi(3+5\phi) = 3\phi + 5\phi^2 = 3\phi + 5(1+\phi) = 5 + 8\phi.$

$\phi^7 = \phi(5+8\phi) = 5\phi + 8\phi^2 = 5\phi + 8(1+\phi) = 8 + 13\phi.$

Here, the coefficients of ϕ are 1, 1, 2, 3, 5, 8, 13. The constant terms are 1, 1, 2, 3, 5, 8. Both will form the Fibonacci sequence. Hence, in general, $\phi^n = \zeta(n-1) + \phi.\zeta(n)$. Let ζ(n)=0 and $\zeta(1) = 1$. The sequence $\{\zeta(n)\}$ is simply a 2nd order difference equation, like $\zeta(n+2) - \zeta(n+1) - \zeta(n) = 0$ with a recurring formula $\zeta(n+2) = \zeta(n+1) + \zeta(n)$, as a Fibonacci.

Divine numbers and sequences transcend the boundaries of formal mathematics, like prime numbers, algebraic numbers, transcendental numbers, arithmetical sequences, and geometric sequences. They stimulate the fusion of philosophy, aesthetics, arts, and mathematical mysticism. These numbers are significant to the beauty and cosmic balance in nature, art, music, design, and architecture. Here, the word divine resonates with a universal appeal, bridging the realms of mathematics, nature, art, and spirituality. In this exploration, we will delve into the Fibonacci sequence and the golden ratio with their significance from a mathematical perspective.

Example 1

For $n \geq 2$ let $\{\zeta_n\}_0^\infty$ be

$\beta_0 = 0, \beta_1 = 1,\ \beta_n =$

$$\begin{cases} \alpha_1 p_{n-1} + p_{n-2}, & if\ n\ is\ even \\ \alpha_2 p_{n-1} + p_{n-2}, & if\ n\ is\ odd \end{cases}$$

Then $\alpha_1 = \alpha_2 = 1$ gives the ordinary Fibonacci sequence. But if $\alpha_1 = \alpha_2 = 2$ it provides Pell's number, then $\alpha_1 = \alpha_2 = k$ it becomes the k-Fibonacci sequence. In the ordinary Fibonacci sequence, the successive terms are inherently relatively prime, which means their greatest common divisor (gcd) is always one. This remarkable property highlights the unique nature of the Fibonacci sequence and underscores its fascinating mathematical significance. Hence,

Theorem 1

Gcd $(\zeta_n, \zeta_{n+1}) = 1, \forall\ n > 1$, for ζ_n and ζ_{n+1}, be any two successive terms of Fibonacci.

Proof: Let d>1. Assume that d divides both the terms, i.e., d | ζ_n and d | ζ_{n+1}. Then clearly, d | $\zeta_{n+1} - \zeta_n$. Moreover, $\zeta_n - \zeta_{n-1} = \zeta_{n-2}$, and hence d | ζ_{n-2}. By retracing this argument, we get, d | ζ_{n-3}. $d|\zeta_{n-4}, d|\zeta_{n-5}$ and of course, to $d|\zeta_1$. But, for $\zeta_1 = 1$ it is not divisible by any d>1, as a contradiction, and is sufficient for the proof.

By similar arguments, one can easily establish the following:

Theorem 2

Gcd of two Fibonacci numbers is again a Fibonacci, i.e., gcd $(\zeta_n, \zeta_m) = \zeta_d$ where d=gcd (m,n).

Proof: Here, we have an illustration for verification by calculating the gcd of any two randomly chosen Fibonacci numbers from Table 2, as 987 and 6567. From the Euclidean Algorithm,

6567= 6 × 987 + 645

987=1× 645+342

645=1 ×342+303

342=1× 303+ 39

303= 7 × 39+30

39= 1× 30+9

30 = 3× 9+3

9=3× 3+0.

Hence, their gcd is 3. Thus, gcd $(\zeta_{20}, \zeta_{16}) = 3 = \zeta_4$ =gcd (20, 16) as demanded by the theorem.

As a corollary, we can establish a result like:

Corollary

In a Fibonacci sequence, $\zeta_n |\ \zeta_m$ iff m | n for $m \geq n \geq 3$.

As mentioned above, the golden ratio and the Fibonacci sequence are associated with distinct beautiful properties (Dunlap, 1997; Gorbani, 2021). A few of them are like,

Example 2

Binet's Formula for the general term.

Consider a finite geometric series with the first term $t_1 = a = \left(\frac{1+\sqrt{\{5\}}}{2}\right)^{n-1}$ and standard common ratio $r = \frac{1-\sqrt{\{5\}}}{1+\sqrt{\{5\}}}$.

Then it's n^{th} term

$$\left(\frac{1+\sqrt{\{5\}}}{2}\right)^{n-1}\left[\frac{1-\left(\frac{1-\sqrt{\{5\}}}{1+\sqrt{\{5\}}}\right)^n}{1-\left(\frac{1-\sqrt{\{5\}}}{1+\sqrt{\{5\}}}\right)}\right] = \frac{\left(\frac{1+\sqrt{\{5\}}}{2}\right)^n-\left(\frac{1-\sqrt{\{5\}}}{2}\right)^n}{\sqrt{\{5\}}} = \frac{\phi^n-\psi^n}{\sqrt{\{5\}}} = \zeta_n.$$

Example 3

Apply Binet's formula to express $\zeta_{12} = 144$.

Numerically,

$$\zeta_{12} = \frac{\left(\frac{1+\sqrt{\{5\}}}{2}\right)^{12}-\left(\frac{1-\sqrt{\{5\}}}{2}\right)^{12}}{\sqrt{\{5\}}} = \frac{\phi^{12}-\psi^{12}}{\sqrt{\{5\}}}.$$

By simple calculation, we get,

(i) $\phi + \psi = 1$. (ii) $\phi \times \psi = -1$. (iii) $\psi + \frac{1}{\psi} = -\sqrt{5}$. (iv) $\phi + \frac{1}{\phi} = \sqrt{5}$. (v) $\phi^2 = \phi + 1$.

(vi) $\psi^2 = \psi + 1$. (vii) $\phi^{n+1} = \phi^n + \phi^{n-1}$.

Theorem 3

Based on these, $\xi_n = \phi^n + \psi^n, \forall\, n \geq 1$.

Proof: Clearly, $\phi + \psi = 1$. It is true for n=1. Let be true for n=2, 3, …, k. Then, by the recurrence relation,

$$\xi_{k+1} = \xi_k + \xi_{k-1} = \phi^k + \psi^k + \phi^{k-1} + \psi^{k-1} = \phi^k(1+\phi^{-1}) + \psi^k(1+\psi^{-1}) = \phi^k(1-\psi) + \psi^k(1-\phi) = \phi^k(1+\phi-1) + \psi^k(1+\psi-1) = \phi^{k+1} + \psi^{k+1} = \xi_{k+1}.$$

It is true for n=k+1. Hence, by the induction hypothesis,

$\xi_n = \phi^n + \psi^n, \forall\, n \geq 1$.

Theorem 4

For ϕ, ψ, and ζ_n are as usual, then $\psi^n = \psi.\zeta_n + \zeta_{n-1}$.

Proof: By using Binet's formula,

$$\psi.\zeta_n + \zeta_{n-1} = \psi.\frac{\phi^n-\psi^n}{\sqrt{5}} + \frac{\phi^{n-1}-\psi^{n-1}}{\sqrt{5}} = \frac{1}{\sqrt{5}}\left[\phi^n\left(\psi+\frac{1}{\phi}\right) - \psi^{n+1} - \psi^{n-1}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\phi^n\left(\frac{\phi.\psi+1}{\phi}\right) - \psi^n(\psi + \frac{1}{\psi})\right] = \frac{1}{\sqrt{5}}\left[\phi^n.0 + \psi^n.\sqrt{5}\right] = \psi^n.$$

Theorem 5

For the Fibonacci, (i) $\zeta_n \times \zeta_{n+1} = \sum_{i=1}^{n} \zeta_i^2$ and verify (ii) $\sum_{i=1}^{n} \zeta_i = \zeta_{n+2} - 1$.

Proof: For (i),

$$\zeta_n \times \zeta_{n+1} = \zeta_n.[\zeta_n + \zeta_{n-1}] = \zeta_n^2 + \zeta_{n-1}.\zeta_n = \zeta_n^2 + \zeta_{n-1}.[\zeta_{n-1} + \zeta_{n-2}\} = \zeta_n^2 + \zeta_{n-1}^2 + \zeta_{n-1}.\zeta_{n-2} = \cdots \ldots = \zeta_n^2 + \zeta_{n-1}^2 + \cdots + \zeta_2.[\zeta_2 + \zeta_1] = \zeta_n^2 + \zeta_{n-1}^2 + \cdots + \zeta_n^2 + \zeta_2.\zeta_1 = \zeta_n^2 + \zeta_{n-1}^2 + \cdots + \zeta_n^2 + \zeta_1^2 = \sum_{i=1}^{n} \zeta_i^2.$$

For (ii), let $i \in Z^+$ and $i < 12$. Then, $\{\zeta_i\}$ becomes 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144. Then, $\sum_{i=1}^{10} \zeta_i = 1+1+2+3+5+8+13+21+34+55 = 143 = 144 - 1 = \zeta_{n+2} - 1$.
Similarly, for Lucas, we can establish,

Theorem 6

$(i)\ \xi_n \times \xi_{n+1} = \sum_{i=1}^{n} \xi_i^2 + 2$, and (ii) $\sum_{i=1}^{n} \xi_i = \xi_{n+2} - 3$.

Some other numbers that are similar to Fibonacci are like Tribonacci, $\zeta_{n+1} = \zeta_n + \zeta_{n-1} + \zeta_{n-2}$, for $n \geq 2$ with $\zeta_0 = 0$, $\zeta_1 = 1$, and $\zeta_2 = 1$. Then, such a sequence is 0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 564, 927, ... Different higher-order types of sequences, like Tetranacci, Pentacci, Hexanacci, Heptacci, etc., are also developed similarly, as mentioned in Feinberg (1963).

In matrix representation, the recurrence formula related to Fibonacci is equivalent to,

$\begin{bmatrix} \zeta_{n+1} \\ \zeta_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \zeta_n \\ \zeta_{n-1} \end{bmatrix}$, for n=1,2, … Thus, if $y_{n+1} = \begin{bmatrix} \zeta_{n+1} \\ \zeta_n \end{bmatrix}$ and $\zeta = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ for n=1,2, … , it gives, $y_{n+1} = \zeta^n \cdot y_n$ for n=0, 1, 2. By repeatedly applying the recurrence,

$y_{n+1} = \zeta^n . y_1 = \begin{bmatrix} \zeta_{n+1} \\ \zeta_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

, for n=0, 1, 2… For details, we refer to Ahamad and Prasher (2022).

Example 4

The positive eigenvalue of a Fibonacci matrix is the golden ratio.

By the recurrence formula, $\begin{bmatrix} \zeta_{n+1} \\ \zeta_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \zeta_n \\ \zeta_{n-1} \end{bmatrix}$, for n=1,2. By repeatedly applying the recurrence, it yields, $\begin{bmatrix} \zeta_{n+1} \\ \zeta_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} \zeta_n \\ \zeta_{n-1} \end{bmatrix}$. The base matrix for the Fibonacci is $\zeta = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. For its eigenvalue, the characteristic equation is $\begin{vmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0$. It gives, $\lambda^2 - \lambda - 1 = 0 \Rightarrow \lambda = \frac{1 \pm \sqrt{5}}{2}$. Here, the positive eigenvalue is $\lambda_1 = \frac{1+\sqrt{5}}{2} = \phi$, whereas $\lambda_2 = \frac{1-\sqrt{5}}{2} = \psi$.

Example 5

Binet's formula can be expressed in terms of the eigenvalues of the Fibonacci matrix, i.e.,

$$\zeta_n = \frac{\left(\frac{1+\sqrt{\{5\}}}{2}\right)^n - \left(\frac{1-\sqrt{\{5\}}}{2}\right)^n}{\sqrt{\{5\}}} = \frac{\phi^n - \psi^n}{\sqrt{\{5\}}} = \frac{\lambda_1^n - \lambda_2^n}{\sqrt{\{5\}}}.$$

For $\lambda_1 = \frac{1+\sqrt{5}}{2} = \phi$ & $\lambda_2 = \frac{1-\sqrt{5}}{2} = \psi$.

Example 6

A beautiful consequence of a simple mathematical conversion of distances. Consider any two consecutive Fibonacci numbers, like 13 and 21 or 5 and 8. Approximately, a larger number in Km = a smaller number in Miles, and a smaller number in Miles = a larger number in Km. As 21 Km = 13.05 Miles ≈ 13 Miles. Likewise, 5 Miles = 8.047 Km ≈ 8 Km. For distances not in Fibonacci values, split such a number into two or more Fibonacci values, i.e., 34 = 21 + 8 + 5, then, 21 Km ≈13 Mile; 8 Km ≈5 Mile; and 5 Km ≈3 Mile. ∴ 34 km ≈ 13+5+3= 21 Miles.

Example 7

A beautiful consequence of the area of a square and a seemingly equivalent rectangle by using Fibonacci numbers.

Let us choose three consecutive Fibonacci numbers, 3, 5, and 8, as ζ_{n-1}, ζ_n, and ζ_{n+1}, respectively. Then the area of the square formed on the base $\zeta_{n-1} + \zeta_n$ is equivalent to the area of the rectangle formed by the sides ζ_n and $\zeta_{n+1} + \zeta_n$. Symbolically, $[\zeta_{n-1} + \zeta_n]^2 \approx \zeta_n \times [\zeta_{n-1} + \zeta_n]$. However, a slight variation, an error, can be observed in the increment of an extra and small slice-type parallelogram in the formed rectangle by 1 unit.

Figure 2

A Fibonacci Square with sides 3+5 and 8 units. It is divided into 4 different plane figures: A, B, C, and D. The first two are right-angled triangles, and the next two are trapeziums.

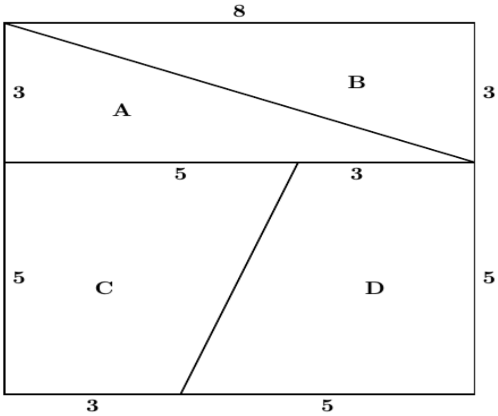
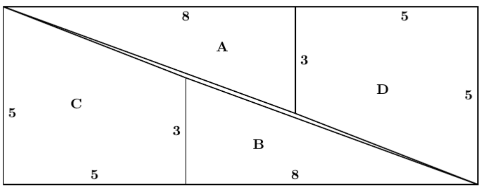


Figure 3

An equivalent Fibonacci rectangle to Figure 2, having sides 5 and 13 units.



There are various applications of the Golden and Fibonacci-type ratios, mainly for natural beauty, harmony, pleasure, design, and perfection. These are nature-friendly numbers. There is no field in non-life or life sciences, mathematical or management sciences, untouched by such sequences and numbers. Most natural things around us follow such patterns, structures, and designs, even in ancient and modern arts, including photography, music, and stock market analysis. It is so in the universe, galaxies, and cyclones. It is a surprising fact that most human organs, DNA coding, and even the growth of a body in the Womb consist of a natural phenomenon of the Fibonacci-type sequence. There has been a fair amount of work on its natural existence and its real-life applications in Literature. For details, we refer to Adhikari and Kattel (2023); Adhikari (2023); Adhikari (2024); Adhikari (2025); Hemenway (2005), and the references therein.

CONCLUSION

The Fibonacci sequence is crucial for understanding nature's patterns, serving as a natural numbering system closely linked to the golden ratio. These concepts have captivated attention for centuries, revealing important connections between mathematics, art, and the natural world. These are the world-famous, astonishing topics of great interest to ancient human civilizations.

This research presents an insight into the divine number system through an integrated analytical and exploratory approach to its fundamental patterns, recurrence relations, mathematical structures, identities, properties, and their aesthetic significance. It combines the golden ratio with the Fibonacci ratio, Binet's formulae, Fibonacci-type sequences, Lucas numbers, Fibonacci matrices, and their generalizations. It opens a broad horizon and area of interest for further research in a multidisciplinary approach.

CONFLICTS OF INTEREST

The authors declare that there is no conflict of interest.

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