Review of Greek and Lichhavi Numeral Systems

Eka Ratna Acharya (PhD and Post Doc)
(ekaratna.acharya@tucded.edu.np)
Central Department of Education, Tribhuvan University, Nepal
Received Date: February 20, 2025; **Reviewed Date:** March 25, 2025
Accepted Date: April 28, 2025

ABSTRACT

The aim of this paper is to express the review of Greek and Lichhavi numeral systems then interpret and explain the similarities and differences between these two systems. It is well known assumptions that numerals, counting system and pattern of the numerals are the most important phenomenon in mathematical approach to every scientific and social events. Here, the revisions on these numeral systems are used to explore on the basis of document analysis and historical practices. Greece and Nepal have indigenous mathematical practices with their own numeral and counting systems. In early period Greek has used Acrophonic and Alphabetical numeral systems and in context of Nepal, we have used Lichhavi numeral system. In my study I found that both of these two systems ligatures are used to form the large numbers.

Key words: Review, Greek, Lichhavi Numeral systems, ligatures.

Background

Since in teaching-learning activities in mathematics generally, we found and listen the statement that 'the word or letter is used or derived from Greek or Latin and Italic language like 'epsilon, delta, lambda, Π, Γ, Δ, etc.'. So, I feel does it is really always true! And I am interested to visit Greece and Italy to realize what is the truth; fortunately, I got an ERASMUS scholarship to visit University of Crete, Greece for my Post Doctor program in 2017/018. So, this opportunity reinforces me to learn history of Greek mathematics and to connect my Doctoral study in classical Nepali mathematics and its history. In this context, I thought to prepare a revision note between these two countries' numeral systems and this paper is as the byproducts of my experiences of different libraries and museums in Nepal, Greece and Italy. And, I also convinced that all the mathematical developments are not limited only few countries or continents; it is common developments from different communities, civilizations, and countries. But we do not forget the indigenous developments of any countries and these indigenous developments give the scenarios of the countries and civilizations.

Introduction

Revision between or among the things is not an easy task as it regarded but it is a good practise of study of new subject matter. Now, it is trying to explore a brief comparative information of these two countries-Nepal and Greece with numeral and counting systems. Both of these beautiful countries have bio-diversity in natural resources, like water resources, vegetation, land, handicrafts, animals, natural heritage, monuments, culture and natural caves (Nepal tourism Board, 2016). These two countries have diplomatic bilateral relations on 2nd of February 196o. Greece offered search and rescue teams to Nepal in the devastating earthquakes that struck Nepal in 2015. So, Nepal and Greece have good relations in different situations as neighbors. Cajori (1894, 1909) states that the mathematician finds the geometry of the Greeks and the arithmetic of the Hindoos as useful and admirable as any research of to-day in history of mathematics. This statement also supports to blend the review of these two different numeral systems that practices in different communities.

Here, our concern is reviews on numeral as mathematical developments of these countries. Numbers and counting are the part of everyone's life, and understanding of numbers and their structure is fundamental to progress in mathematics (Nataraj & Thomas, 2009). So, this paper tries to tie up these two countries by numeral and counting systems as an academic work.

Objective

The objective of this paper is to express the review of the numeral systems in context of Greece and Nepal in mathematical practices, i.e. it expresses the comparisons between Greek and Lichhavi numeral systems. This task supports to tie up the academic relation and enhance the research on both countries on classical mathematical developments, heritages, cultures and indigenous mathematics with modern scientific developments.

Methods

This research paper would comprise a historical document analysis of the different stages in the use of Greek and Lichhavi numeral systems that developed in Greece and Nepal in civilizations followed by an application and principles of development of numeration system.

Literature Reviews

Nepal and Greece have their own indigenous mathematics; like Tally Marks, sticks, notches, heap of beans/rice/corn, rock arts, artifacts, *Shruti* and *Smriti* (Remembering/listening, etc.) and early periods' mathematics developments (Pant, 2004, Acharya, 2015, 2018b). Thus, indigenous mathematics are the main scenario of the civilization of a community and any country or continent.

Either community of people, the principle of one-to-one correspondence was used in earliest counting systems. The shepherd may have carried them with a bag of pebbles, one pebble for each of their sheep. At the end of the day, they could match each pebble to each sheep to make sure that they all are returned. Related marking systems including tying knots in a rope and making notches on a stick. Tally marks and counting on one's fingers are one-to-one correspondence systems that used today (Cycleback, 2014).

The following list of numerals were used for counting in very early human civilization at different communities in Nepal-*Eka* (1), *daśa* (10), sata (100), *sahasra* (1000), *ayuta* (10,000), *niyuta* (100,000), *prayuta* (1,000,000), *arbuda* (10,000,000), *nyarbuda* (100,000,000), *samudra* (1,000,000,000), *madhya* (10,000,000,000), *anta* (100,000,000,000), *parárdha* (1,000,000,000,000) (Nataraj & Thomas, 2009). *Shruti-smriti* were the elementary ways that practiced local mathematics and counting (Acharya, 2015).

Greek mathematics based in geometry and it has indigenous features. *Squaring the circle*, *doubling the cube* and *trisection of an angle*, etc., were as the antique problems in Greek mathematics which open the various concept on mathematics. The early Greek mathematics is famous by *Hellenistic mathematics* (Cooke, 2013). In Nepal, many antique local mathematics were practiced in early periods (Acharya, 2016).

The Ancient Greeks had a 10-base system using the letters of the Greek alphabet. They didn't have a zero, so needed a different symbol for 10, 20, 30, etc., (Cycleback, 2014). There were many different schools; Ionian, Pythagorean, Eleatic, Sophist, Platonic, Eudoxus and School of Aristotle to practice mathematics and philosophies (Allen, 2000). In Nepal schools were based on Sanskrit and Jyoutisha Ganita at first without such divisions for education like *Guru Kula* a kind of earliest school system.

In Nepali language the word '*Ganita'* is used for mathematics whereas the word '*Mathema' was used in Greek* (Heath, 1921). Numerals are a system of writing numbers using the letters of the alphabet or using any other symbols. The Greek

numeral systems are based on Greek alphabets. These alphabetic numerals are also known as *Ionian* numerals, *Milesian* numerals, or *Alexandrian* numerals. There were two main systems of numerical notation in classical times of Greece 'Acrophonic' and 'Alphabetical'. The Acrophonic also known as the *Attic* system and used for cardinal numbers exclusively, consists of the set of signs somewhat absurdly called '*Herodianic'* because they are described in a fragment attributed to *Herodian*, a grammarian of the latter half of the second century AD (Heath, 1921, digital version 2014).

These signs used there cannot claim to be numerals in the proper sense; they are mere compendia or abbreviations; for, except in the case of the stroke 'I' representing a unit, the signs are the first letters of the full words for the numbers, and all numbers up to 50,000 were represented by combinations of these signs. The stroke 'I', representing the unit, may be repeated up to four times (Heath, 1921). The letter 'Γ' is used to denote the first letter to stand for 5 (□□□□□□□□□□□□□□□in early period, the word □□□□ is used for five. The letter 'Δ' is used to denote the first letter of (δέκα) for 10. H for representing (Εκατό) 100. Similarly, X for 1000, and M for 10,000 and so on (https://www.google.gr). These signs were illustrated in different sources which are illustrated as below.

Γ	Δ	H	X	M
Pente	Deka	Hekaton	Khilioi	Murioi
Πεντε	Δεκα	Ηεκατον	Χιλιοι	Μυριοι
5	10	100	1000	10000

Fig. 1: ***Greek Acrophonic Numeral System***

Also, there were seen hieroglyphs in Crete. The Cretan hieroglyphic or pictographic script was first described by British archaeologist Sir Arthur Evans based on his work at Knossos. Cretan hieroglyphic form of writing was developed in the period from about 2000 to 1600 BC (Ifrah, 1998). It is now generally thought to have been arisen at about the same time as, or slightly later than, the Linear A script. Its use is generally thought to have lasted from 1750 to 1600 BC. Although Cretan hieroglyphic writing has still not been deciphered, probably used by a bureaucracy that arose in the earliest palaces of Minoan civilization. They were probably accounting device giving details of deliveries or exchange (Subedi, 2017). The Six types of Cretan hieroglyphs-*Syllabogrammes*, *Logogrammes*, *Klasmatogrammes*, *Aritmogrammes*, *Stiktogrammes* and *Ligatures* are illustrated below (Karnava, 2014).

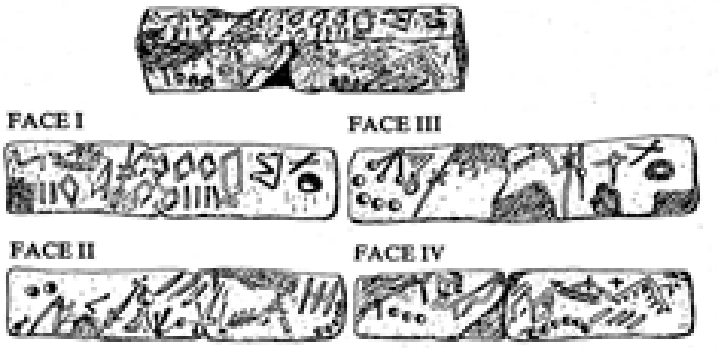
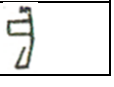


Fig. 2: ***Counting Devise in Minoan Civilization***

Now, a days we have used the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 to do any calculations or mathematical works. But this situation was not in ancient period. In Babylonian, Chinese, Egypt, Greek, Mayan, Roman civilization there were not used the symbols of zero and they used distinct symbols to denote the separate numbers (Pant, 2015). In context of Nepal, Murray Samrat Asoka's two archaeological records were the very oldest records. These two oldest archaeological records were found at Lumbini of Rupandehi district and Niglihawa of Kapilvastu district (Pant, 1986). However, the original Lichhavi numerals are illustrated in the following table (Acharya, 2018a, 2018c, 2017).

Table 1: ***Lichhavi Numeral System***

One	Two	Three	Four	Five	Six	Seen	Eight	Nine
								
Ten	Twenty	Thirty	Forty	Fifty	Sixty	Seventy	Eighty	Ninety
								
Hundred	Two Hundred	One Thousand						
								

In writing and typography, a ligature occurs where two or more graphemes or letters are joined. The origin of typographical ligatures comes from the invention

of writing with a stylus on fibrous material like paper or clay. The 'ligature' is a short-legged sign that used to represent the increasing magnitude of the numbers. Here, can be seen a small-legged sign right side of the symbol ५ on the symbol of 100 to form the symbol ५ for 200 (Acharya, 2018a). In Nepali (Lichhavi numeral system), ligatures were used to form the large numbers. Some attachment of two numbers found in Greek numeral system; such pattern is like as ligatures in the numbers 50 and 500 as shown above in fig.1. Similarly, the ligatures are used to form large numbers in Lichhavi numerals system (Acharya, 2018a). For example, a number is expressed in the following figure.

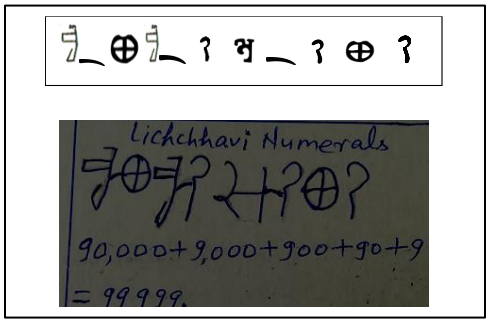


Fig. 3: Construction of 99999 by using Lichhavi Numerals

1. Contrast between Greek and Lichhavi numeral systems

On the basis of the evidences as described above, the differences between Greek and Lichhavi numeral systems can be summarized as below.

Table 2: Differences between Greek Numeral System and Nepali (Lichhavi) Numeral System

S. N	Greek Numeral System	Lichhavi Numeral System
1	It was used as early as 2nd millennium BC	It was developed in 300 BC-200CE and practices 6th to 10th CE.
2	It has major two numeral systems: i. Acrophonic ii. Alphabetical	It has no other systems. It is based in Brahmi numeral system.
3	In Acrophonic numeral system six (6) major symbols were used. This system also called Attic or Herodianic system.	In Lichhavi numeral system 18 symbols were used.
4	It is non-positional and sign value with base 10 system (Swanson, M. (n. D.), Bunt, L. H. and et al. (1976).	It is non-positional and sign value system

5	It has 10 as primary base and 5 as secondary bases. Symbols were assigned for 5 and powers of 10.	It has no secondary base. Ligatures are used for forming large numbers.
6	It has no symbol of zero and absence of symbol of zero no hurdle on this system.	It has no symbol of zero but at Lichhavi period concept of zero is used. Here also no hurdle.
7	It has used the numeric symbols in left to right decreasing magnitude order like: 108 = ΗΓΙΙΙ, 256 = Η Η 𐅂Γ Ι.	It has used the numeric symbols in left to right decreasing magnitude order like:108 =  , 211=
8	In Greek Alphabetical numeral system, there were 27 symbols (24 alphabetical letter, 3 obsolete letters for 100 and 1000 and 10,000) (Ball, W.W. Rouse (1908 4th ed. 1960).	Lichhavi numeral system has no another alphabetical notation.
9	It has used multiple bases of 5 and 10 for large number, X is used for 1000 and in early Cretan system  is used for 1000.	It has no multiple bases; ligature is used to form large number & symbol  is used for 1000.
10	Greek alphabetical numeric symbol used algebraic notation.	Lichhavi numerical symbols are unary and it has no algebraic notation.
11	It is related to Hebrew numeral system.	It is related to Brahmi numeral system.
12	It has additive grouping system (Hardegree, G. (n. d).	It has additive grouping system.
13	Nowadays Greek people used Hindu Arabic numerals for representing the numeric value and for word numeral they used Greek letter.	Nowadays Nepali people used either Devanagari of Hindu Arabic numeral symbols to denote the numeric value and for word numeral they used Devanagari letter.

Let's to observe some examples of counting system in Greek and Nepali with related languages for comparing to first letters of the word numeral and pronunciation. It is tabulated below and denoted by 'S' if near to similarity; 'N' if not near to similarity and by 'SN' somewhat near to each other as the decision.

Table 3: ***Pattern in Counting System in Greek and Nepali Language***

Number	Greek	Nepali	First Letter	Decision
1	Ένα (éna)	Eka	Same	S
2	Δύο (dýo)	Dui	Same	S
3	τρία (tría)	Teena	Same	S
4	Τέσσερα (téssera)	Charaha	Different	SN
5	Πέντε (pénte)	Pancha	Same	S
6	Έξι (éxi)	Xa	Same	S
7	Επτά (eptá)	Saata	Different	SN
8	Οκτώ (októ)	Aatha	Different	SN
9	Εννιά (enniá)	Nawa	Different	SN
10	Δέκα (déka)	Dasha	Same	S

By observing the first letters used by these two languages and their pronunciations, mostly we get nearness. And the law of evolution (Gert Schubring, 16th Sep., 2017) is used in development of elements of mathematics (Lisle, 16th Sep. 2017). It depends upon the practices. So, this shows that these two languages were initially originated from the same practice. It shows these both Greek and Nepali numerals systems are near to Sanskrit language. Let's observe the pattern in counting system of the numbers greater than 10 and less than 20 in Greek and Nepali systems.

Table 4: ***Counting System in Greek, Nepali and English Languages***

Number	Greek	Nepali	English	Pattern change
11	Έντεκα (énteka)	Yagharaha	Eleven	(Deka) is used back of the number in Greek.
12	Δώδεκα (dódeka)	Baraha	Twelve	
13	δέκα τρία (déka tría)	Teraha	Thirteen	(Deka) is used front of the name of number in Greek and (Teen) is used for ten in English.
14	δεκατέσσερα (dekatéssera)	Chaudha	Fourteen	
15	δεκαπέντε (dekapénte)	Pandraha	Fifteen	
16	δεκαέξι (dekaéxi)	Soraha	Sixteen	
17	δεκαεπτά (dekaeptá)	Satraha	Seventeen	
18	δεκαοχτώ (dekaochtó)	Atharaha	Eighteen	
19	δεκαεννέα (dekaennéa)	Unnaisaha	Nineteen	
20	Είκοσι (Ecosi)	Bisha	Twenty	

Here the counting pattern is after 12; that is from 13 to 19 is similar in Greek. There is an interesting pattern after 20 to 29, after 30 to 39 and 41 to 49, 51 to 59, 61 to 69, 71 to 79, 81 to 89 and 91 to 99. Like; ikosi éna, eíkosi dýo, eíkosi tría, eíkosi téssera, eíkosi pénte, eikosi exi, eíkosieftá, eíkosi ochtó, eíkosi ennéa. In Nepali 11 to 19 is alike to each other numbers.

For more than 100 is like as 101 = Ekató éna (Εκατόν ένα) 102 = Ekató dýo (Εκατό δύο) and so on up to 110. Now, 111 = Ekató éndeka (Εκατό έντεκα), 112 = Ekatón dódeka (Εκατόν δώδεκα), 113 = Ekatón dekatría (Εκατόν δεκατρία). Same pattern holds for other numbers. 120 = Ekatón eíkosi (Εκατόν είκοσι), 121 = Ekató eíkosi éna (Εκατό είκοσι ένα), 122 = Ekatón eíkosi dýo (Εκατόν είκοσι δύο).

In Devanagari or in Hindu Arabic or in English for One hundred (100) we used two words but in Greek language we use a single word 'Ekató (Εκατό)'. Similarly, for two hundred (200) there are two words in Devanagari, Hindu Arabic or English language but in Greek language there is a single word Diakósia (Διακόσια). So, it looks like shorter than other languages. But if we pluralized it as two hundred then in Greek also used two words as Dýo ekatontádes (Δύο εκατοντάδες). We know that every language has their own peculiar properties, but Greek language has interesting and very fundamental properties which have not seen in other languages. It will become an interesting part to linguist and anthropologist and sociologists.

In Nepali system, the pattern after 10 and after 20 and so on are same but in Greek and English have some changes the pattern of counting after 12 and similarity among Greek and English. This also shows the initial phase to be near to each other between Greek and English. Perhaps this pattern indicates their origin or initial phase is more or less same.

Conclusions

On the basis of above revisions; Greek people used Hindu Arabic numerals for representing the number but in word numeral, they used Greek letters. Different pattern for counting after 20 to 29 and 30 to 39, 41 to 49, 51 to 59, 61 to 69, 71 to 79, 81 to 89 and 91 to 99 than the pattern between the patterns 11 to 19. In Nepali either used Devanagari or Hindu Arabic symbols to denote the numbers and for word numerals they used Devanagari letter or English Roman. The pattern as changes Greek 21 to 29 and so on has not seen in Nepali counting system. Using

ligatures in number notation is found in Lichhavi (Nepali) and Greek numeral systems, but it is very popularly used in Lichhavi numeral system.

Acknowledgements

I would like to express sincere thanks Prof. Krishna Kumar Shrestha who initiated continuous encouragements to me for working and publishing the antique & numismatic mathematics, that is indigenous/classical mathematics in context of Nepal by providing space to me for an article 'Ancient Nepali Mathematics' in a popular Journal of Mathematics, '*Yeti*' (2012) and so on. Similarly, Prof. Shankar Raj Pant, Prof. Dinesh Raj Pant, Dr. Mahesh Raj Pant, Prof. Hari Prasad Upadhyay, Prof. Siddhi Prasad Koirala, Prof. Santosh Man Maskey and Prof. Binod Prasad Dhakal. Also, I extend sincere thanks to Prof. Chet Raj Bhatta for his constructive suggestion and inspiration. I remember faith love and cooperation from Prof. Christos Kourouniotis, Prof. Maria Antonio from Greece at University of Crete when I was as a Post Doc fellow researcher at this university. This paper is dedicated to Prof. late Ram Man Shreshtha for his continuous encouragement to me in such historical fields of mathematics. At last, but not least, I sincerely acknowledged to the entire team of Nepal Professors Association, Unit Committee, University Campus, Kirtipur, T.U.

References

Acharya, ER (2018d). *Naya Raj Pantaka Ganitiya Kriti*, Kathmandu: Khilsharma-Rajeevlochanjoshismarakpratishthan.

Acharya, ER (2018c). *Historical Information of Mathematics in Nepal*, Butwal: Champa Acharya.

Acharya, ER (2018b). *Numeral Systems in Nepal*, Butwal: Champa Acharya.

Acharya, ER (2018a). *A Report Submitted to the University of Crete (ERASMUS MUNDUS/LEADER) Post-Doc Fellowship in Mathematics* (Unpublished research report).

Acharya, ER (2017). Lichhavi Numeral Systems in Nepal: *J. of Ramanujan Society of Math. and Math. Sc.* ISSN: 2319-1023 Vol.6, No.2 (2017), pp. 69-80.

Acharya, ER (2016). Antiquity of Nepali Mathematics, *American Research Journal of History and Culture* (ISSN-online- 2378-9026).

Allen GD (2000). *The Origin of Greek Mathematics*: http://www.math.tamu.edu/~dallen/masters/Greek/greekorg.pdf (Retrieved-29th Aug., 2017).

Ball, WW et al (1908 4th ed. 1960). *A Short Account of the History of Mathematics*. New York: Dover Publications, INC.

Bunt, LH et al (1976). *The Historical Roots of Elementary Mathematics*. New York: Dover Publications, INC.

Cajori, F (1894, reprint 1909). *A History of Mathematics*. Macmillan And Co., London.

Cooke, RL (2013). *The History of Mathematics*, Burlington, VT: Wiley.

Cycle back, DR (2014). A Brief Introduction to Ancient Counting Systems for Non-Mathematicians (https://cycleback.files.wordpress.com/2014/05/cycleback_countingsystems.pdf (Retrieved 16th Sep., 2017).

Ifrah, G (1998). *The Universal History of Numbers: From Prehistory to the Invention of the Computer*. Translated by David Bellos. E. F. Harding. Sophie Wood and Ian Monk. New York: John Wiley and Sons.

Heath, ST (1921, digitally printed version 2014). *A History of Greek Mathematics Vol I* http://www.archive.org/details/cu31924008704219 (Retrieved 22nd Sep., 2017).

Karnava, A (2014). Cretan Hieroglyphic Script. Retrieved 22nd Sep., 2017, hhtps://www.researchgate.net

Lisle, J (Retrieved 16th Sep. 2017). *Evolutionary Math?* http://www.icr.org/article

Nataraj, MS & Thomas, MOJ (2009). Developing Understanding of Number System Structure from the History of Mathematics. *Mathematics Education Research Journal* 2009, Vol. 21 (2): 96-115.

Nepal tourism Board (2016). *Nepal at a Glance*, Kathmandu: Nepal Tourism Board.

Pant DR (2015), Writings of Numerals System in Nepal in Ancient Times. *Journal of Mathematics Education* (ISSN 2467-9364): 63-72.

Pant NR (1986). *Lichhavi Sambatko Nirnaya*, Kathmandu: Nepal Rajkiya Pragya-Pratisthan.

Pant, SR. (2004). Mathematics in Nepal: A Historical Analytical. *Scientific World*, Government of Nepal, Environment, Science and Technology, B.P. Koirala Memorial Planetarium Observatory and Science Museum Development Committee, 3(3): 30-35.

Schubring, G (Retrieved 16th Sep., 2017). *Conceptions for Relating the Evolution of Mathematical concepts to Mathematics Learning -Epistemology, History, and Semiotics Interacting, Germany*: gert.schubring@uni-bielefeld.de

Subedi, N (2017). *The Development of Numeral System of Ancient Nepal*. Unpublished PhD Dissertation, Nepal Sanskrit University, Beljhundi, Dang.

https://www.google.gr/search?sa=G&hl=el&q=old+greek+number (Retrieved 15th Aug., 2017).

https://www.google.gr/search?dcr=0&biw=1236, (Retrieved 19th Sep., 2017)