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Research Article

A Study of Sequence Spaces Defined by Statistical Convergence of Fuzzy Numbers

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Abstract

Sequence spaces are mathematical structures that play a pivotal role in studying functional analysis, topology, and sequence theory. These spaces consist of sequences of elements from a given set, typically the set of real or complex numbers, equipped with specific topological or algebraic properties. It explores sequence spaces defined by the statistical convergence of fuzzy numbers, focusing on the development and analysis of new sequence spaces that extend classical sequence spaces in the context of fuzzy set theory. Employing a difference operator, furthermore provides a sequence space of fuzzy numbers, $F_{(c)}I(S)$ and $F_{(c)}I(S)_0$ determined via I-statistical convergence. Research investigates the basic algebraic and topological features of these spaces, offering a thorough examination of their structural features. Additionally, it explores crucial links related to these spaces, including symmetry, solidity, and convergence-free features, and it establishes several significant inclusion outcomes. The research advances knowledge of I-statistical convergence in fuzzy number sequence space by expanding on traditional ideas and providing guidance on using them in fuzzy set theory and uncertainty-related fields.

Keywords: Sequence Spaces, Statistical Convergence, Fuzzy Numbers, I-statistical Convergence .

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1 Introduction

Classical and fuzzy evaluations need that almost every series component satisfies the convergence requirement, such as for classical convergence, it must fall within a small neighborhood

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of the limit [13]. Statistical convergence aims to reduce the requirement for convergence condition accuracy for the majority of items, focusing on large numbers and the majority as a stand-in for all components. Statistical analysis uses finite populations and examples, while mathematical theory focuses on infinite sets [9]. Statistical convergence has been addressed in many regions, such as Banach spaces, Fourier estimates, trigonometric series theory, summation of matrices, and series theory, which remains the focus of investigations [10]. Additionally, it demonstrated the I-statistical convergence of the function definition sequences and the I-density of a subset of real numbers. [4]. In general, a fuzzy number was used to indicate the uncertainty of the information and data used in a particular mathematical procedure. The statistical convergence for a series of fuzzy integers was demonstrated using the concept of characterization theorems [6]. The definitions demonstrated that statistical convergence sequence will be obtained by summarizing the normal convergence for a series and a sequence of fuzzy numbers with low natural densities [14]. Statistically bounded sequences are subject to fuzzy approximations of the superior and inferior limit qualities for real numbers using the statistical limits notion. A positively consistent summability matrix to determine the convergence of a set of fuzzy numbers numerically establishes the magnitude of continuous procedures with fuzzy numerical values [1]. Considering the fuzzy modulus of consistency, it calculated the likelihood of statistical fuzzy convergence according to a positive frequently summable matrix of operation and presented the estimation theorem for fuzzy positively linear amplifiers. Investigation has been conducted using fuzzy random factors as a generalization and complement to traditional possibility concept, such as apprehension and incorporation, because of measurement errors or the generally acknowledged uncertainty of the information itself [20]. Following the initial introduction of fuzzy sets and their operations, several individuals have addressed various aspects of fuzzy set theory and applications [23]. Bounded and convergent fuzzy number sequences were examined a few of their characteristics and demonstrated that each of the sequences were bounded. The real, complex, and fuzzy number sequences and difference sequences have a wide range of applications. For instance, numerical sequences have amazing and useful applications in a variety of fields, including acoustical [24]. In this paper we will see that the sequence spaces formed by statistical convergence of fuzzy numbers are examined, with an emphasis on generating and analyzing novel sequence spaces that expand on classical sequence spaces within the conceptual framework of fuzzy set theory. It uses a difference operator to build a sequence space of fuzzy numbers, $F_cI(S)$ and $F_cI(S)_0$, which can be explained by I-statistical convergence. The paper is separated into 7 phases, such as Phase II describes general definitions and preliminaries, Phase III explains fuzzy sequences spaces algebraic properties, Phase IV represents topological properties, Phase V provides results, Phase VI presents the discussion, and finally, the statistical convergence of fuzzy numbers, which is defined by sequences spaces are concluded in Phase VII.

2 Preliminaries and definition:

Sequence Spaces: Any vector space with infinite real or complex numbers or a function space with operations from natural numbers to the domain of real or complex numbers is called a sequence space. Symbols identifiers like C_0 and ℓ_p are typically used to define it. To create topological vector spaces with a linear topology, sequence spaces must be closed under scalar multiplication and coordinate-wise additions. In the context of coordinate-wise multiplication, as well sequence algebras that are also closed.

Fuzzy Numbers: It is a particular type of real numbers used to model uncertainty and to represent a range of potential values. It is characterized by a membership function $\mu(x)$, which assigns a value between 0 and 1 to each element of R. The α-cut of a fuzzy number, corresponding to a given level of membership α, forms a convex set. The membership function $\mu(x)$ is typically normalized such that the maximum value at the center x_0 is 1. The function $\mu(x)$ is upper semi-continuous if, for every $\varepsilon > 0$, there exists a $\delta > 0$ such that for all x within δ –distance from x_0, the difference $|\mu(x) - \mu(x_0)| < \varepsilon$. Finally, the support of the fuzzy number defied as$\{x \in R : \mu(x) > 0\}$, is compact, meaning it is closed and bounded.

I-statistical Convergence of fuzzy numbers: A set $\mathcal{L} = \{n_1 < n_2 < n_3 \ldots . < n_l < \cdots\} \subset \mathbb{N}$ needs to exists for a sequence $x = (x_n) \in L(\mathbb{S})$ to be I-statistically convergent with respect to y_o and for each $\epsilon > 0$, have $\lim_{n\to\infty} \frac{1}{m} |\{n_l < m : \bar{c}\,(Y_{nl}, Y_0) < \epsilon\} \in F(I)| = 1$. The operator $I^* - st - \lim_{n\to\infty} Y_m = Y_0$ represents y_0, the I-statistical limitation of y_m.

Example 1. The sequence $Y = (Y_m)$, which is defined as follows in equation 2.1.

$$y = (y_m) = \begin{cases} 0 & \text{for } m = l^2 \text{ where } l \in \mathbb{N} \\ & \frac{1}{m} \text{ otherwise} \end{cases} \tag{2.1}$$

Where in I-statistical convergence is zero. Let $\mathcal{L} = \{n_1 < n_2 < n_3 \ldots . < n_l < \cdots\} \subset \mathbb{N}$, where the nonperfect squares values are where the non-perfect squares values are $n_1, n_2, n_3, \ldots\ldots\ldots n_l, \ldots$. Next, for every value $\varepsilon > 0$. Then, $\lim_{n\to\infty} \frac{1}{m} |\{n_l < m : \bar{c}\,(Y_{nl}, 0) < \varepsilon\} \in \mathcal{L}| = 1$. It is easy to demonstrate that I is an optimum if it is assortment of divisions of $Y = \{m \in \mathbb{N} : m = l^2\}$. It suggests that $\mathcal{L} \in F(I)$ consequently, $\lim_{n\to\infty} \frac{1}{m} |\{n_l < m : \bar{c}\,(Y_{nl}, 0) < \varepsilon\} \in F(I)| = 1$.

3 Algebraic properties of fuzzy sequence spaces:

Definition 3.1. Let $\tilde{y}, \tilde{x} \in \mathcal{F}$. Let $\tilde{y}$ is equivalent to $\tilde{x}$ and write $\tilde{y} \sim \tilde{x}$ if there occur symmetric fuzzy numbers $\tilde{t_1}, \tilde{t_2} \in \mathcal{S}$ such that displayed in equation 3.1.

$$\tilde{y} + \tilde{t_1} = \tilde{x} + \tilde{t_2} \tag{3.1}$$

Example 2: Consider two fuzzy integers along with level sets, $\tilde{y}$ and $\tilde{x}$. $[\tilde{y}]^\alpha = [\tilde{y}_K(\alpha), \tilde{y}_R(\alpha)] = \left[\frac{\alpha+1}{2}, \frac{5-3\alpha}{2}\right]$ and $[\tilde{x}]^\alpha = [\tilde{x}_K(\alpha), \tilde{x}_R(\alpha)] = [\alpha, 3-2\alpha]$, using level sets, two symmetrical fuzzy numbers: $\tilde{t_1}$ and $\tilde{t_2}$ for each $\alpha \in [0,1]$, correspondingly, were generated.

$$\left[\tilde{t_1}\right]^\alpha = \left[\frac{3(\alpha-1)}{2}, \frac{3(1-\alpha)}{2}\right]$$

And $\left[\tilde{t_2}\right]^\alpha = [\alpha-1, 1-\alpha]$, for every $\alpha \in [0,1]$, respectively, then get that $[\tilde{y}]^\alpha + \left[\tilde{t_1}\right]^\alpha = [\tilde{x}]^\alpha + \left[\tilde{t_2}\right]^\alpha = [2\alpha-1, 4-3\alpha]$, which implies $\tilde{y} + \tilde{t_1} = \tilde{x} + \tilde{t_2}$. Hence we get $\tilde{y} \sim \tilde{x}$.

Theorem 3.2. *As previously stated, the equivalency connection is transitive, symmetric, and reflexive.*

Let $\mathcal{F}/\mathcal{S}$ represent the collection of equivalence classes and let $\langle\tilde{y}\rangle$ represent the equivalence class that contains the element $\tilde{y}$. The following lemmas can be readily proved using level establish predictions for fuzzy numbers.

Lemma 3.3. *For any* $\tilde{y} \in \mathcal{F}, \tilde{y} - \tilde{y} \in \mathcal{S}$.

Lemma 3.4. $\tilde{t} \in \mathcal{S}$ *If and only if* $\forall\alpha \in [0,1]$, *Crisp symmetric sets are those in the* α*-level set* $[\tilde{t}]^\alpha$ *i. e.* $\tilde{t}_K(\alpha) = -\tilde{t}_R(\alpha)$.

Remark 1: In relation to the axis $y = 0$, the fuzzy numerical category $\mathcal{S}$ membership function is symmetrical.

4 Topological Properties of Sequence Spaces Defined by Fuzzy Numbers

Theorem 4.1. *All of* $\mathcal{F}/S$ *'s elements are closed sets of* $(\mathcal{F}, c_\infty)$.

Proof. For any sequence $\{\tilde{y}_j\}_{j=1}^\infty \subseteq \langle\tilde{y}\rangle$, assume that $\lim_{j\to\infty} c_\infty(\tilde{y}_j, \tilde{x}) = 0$ for some $\tilde{x} \in \mathcal{F}$. Since, $\lim_{j\to\infty} c_\infty(\tilde{y}, \tilde{x}) = \lim_{j\to\infty} \sup_{0\le\alpha\le1} \max\{|\tilde{y}_{jK}(\alpha) - \tilde{x}_K(\alpha)|, |\tilde{y}_{jR}(\alpha) - \tilde{x}_R(\alpha)|\} = 0$. The sequences $\{\tilde{y}_{jK}\}_{j=1}^\infty$ and $\{\tilde{y}_{jR}\}_{j=1}^\infty$ to converge uniformly to $\tilde{y}_K$ and $\tilde{y}_R$, respectively. Thus, for all $\alpha \in [0,1]$, the value is computed in equation 4.1.

$$\tilde{y}_N(\alpha) = \frac{\lim_{j\to\infty} \tilde{y}_{jK}(\alpha) + \lim_{j\to\infty} \tilde{y}_{jR}(\alpha)}{2} = \frac{\lim_{j\to\infty} \tilde{y}_{jK}(\alpha) + \tilde{y}_{jR}(\alpha)}{2} = \tilde{y}_N(\alpha) \tag{4.1}$$

which implies that $\tilde{x} \in \langle\tilde{y}\rangle$.

□

Theorem 4.2. *Every element of $\mathcal{F}/\mathcal{S}$ is a convex set of $(\mathcal{F}, c_\infty)$.*

Proof. Let $\tilde{x}, \widetilde{w} \in \langle \tilde{y} \rangle$ and $\lambda \in (0,1)$. Where calculating it from Equations 4.2 4.3 4.4 4.5 4.6

$$[\lambda\tilde{x} + (1-\lambda)\widetilde{w}]^\alpha = \lambda[\tilde{x}]^\alpha + (1-\lambda)[\tilde{w}]^\alpha \tag{4.2}$$

$$= \lambda\tilde{y}_K(\alpha) + (1-\lambda)\widetilde{w}_K(\alpha), \lambda\tilde{x}_R(\alpha) + (1-\lambda)\widetilde{w}_R(\alpha) \tag{4.3}$$

which implies,

$$(\lambda\tilde{x} + (1-\lambda)\widetilde{w})_N(\alpha) = \frac{\lambda\tilde{y}_K(\alpha) + (1-\lambda)\tilde{w}_K(\alpha), \lambda\tilde{x}_R(\alpha) + (1-\lambda)\tilde{w}_R(\alpha)}{2} \tag{4.4}$$

$$= \lambda\tilde{x}_N(\alpha) + (1-\lambda)\tilde{w}_N(\alpha) \tag{4.5}$$

$$= \tilde{y}_N(\alpha) \tag{4.6}$$

Hence $\lambda\tilde{x} + (1-\lambda)\widetilde{w} \in \langle \tilde{y} \rangle$.

□

Theorem 4.3. *Let the sequences $\{\tilde{x}_j\}_{j=1}^\infty$ and $\{\widetilde{w}_j\}_{j=1}^\infty$ converge to $\tilde{x}$ and $\widetilde{w}$, respectively, If $\tilde{x}_j \sim \widetilde{w}_j$ for all $j \in \mathbb{N}$, then $\tilde{x} \sim \widetilde{w}$.*

Proof. Arguments equivalent to the proof in 4.1 can be used to determine that the sequences, $\{\tilde{y}_{jK}\}_{j=1}^\infty, \{\tilde{y}_{jR}\}_{j=1}^\infty, \{\widetilde{w}_{jK}\}_{j=1}^\infty$, and $\{\widetilde{w}_{jR}\}_{j=1}^\infty$ converge uniformly to $\tilde{y}_K, \tilde{y}_R, \widetilde{w}_K$, and $\widetilde{w}_R$, respectively. Thus, the sequences $\{\tilde{y}_{jN}\}_{j=1}^\infty$, and $\{\widetilde{w}_{jN}\}_{j=1}^\infty$, converge uniformly to $\tilde{y}_N$ and $\widetilde{w}_N$, respectively. Consequently, since $\tilde{x}_j \sim \widetilde{w}_j$, i.e., $\tilde{x}_{jN} = \widetilde{w}_{jN}$, for all $j \in \mathbb{N}$, get $\widetilde{x}_N = \widetilde{w}_N$.

□

5 Main Results

The subsequent unique sequences spaces of fuzzy will be presented throughout this part, along with an analysis of some of their attributes.

Assume that I represent an appropriate ideal of N and Let $W = (W_l)$ be a sequence of fuzzy numbers. The next fuzzy-number sequence spaces are defined by following equations 5.1 - 5.5,

$$F^{C^{I(S)}}(\Delta) = \{W = (W_l) : I - \text{ st lim } \Delta W_l = W_0\} \tag{5.1}$$

$$F^{c_0^{I(s)}}(\Delta) = \left\{W = (W_l) : I- \text{ st lim } \Delta W_l = \overline{0}\right\} \tag{5.2}$$

$$F^{k_\infty}(\Delta) = \left\{W = (W_l) : \sup_l \bar{c}\left(\Delta W_l, \overline{0}\right) < \infty\right\} \tag{5.3}$$

$$F^{m^{I(S)}} = F^{C^{I(S)}}(\Delta) \cap F^{k_\infty}(\Delta) \tag{5.4}$$

$$F^{c_0^{I(s)}}(\Delta) = F^{c_0^{I(s)}}(\Delta) \cap F^{k_\infty}(\Delta) \tag{5.5}$$

From the definition, it is obvious that $F^{c_0^{I(S)}}(\Delta) \subset F^{C^{I(S)}}(\Delta) \subset F^{k_\infty}(\Delta)$.

Example 3: Assume that the fuzzy number sequence $W = (W_l)$ is defined by equations 5.6 - 5.10:

$$W_l(s) = \left\{\begin{array}{l} (s-l) \text{ for } s \in [l, l+1] \\ (-s+l+2) \text{ for } s \in [l+1, l+2] \\ 0, \text{ otherwise} \end{array}\right\}, \text{ if } l = 3^m (m = 0, 1, 2, , \ldots). \tag{5.6}$$

$$W_l(s) = \left\{\begin{array}{l} (s+3) \text{ for } s \in [-3, -2] \\ (-s-1) \text{ for } s \in [-2, -1] \\ 0, \text{ otherwise} \end{array}\right\}, \text{ if } l \neq 3^m \text{ and } l \text{ is odd.} \tag{5.7}$$

$$\left\{\begin{array}{l} (s-3) \text{ for } s \in [6, 7] \\ (-s+8) \text{ for } s \in [7, 8] \\ 0, \text{ otherwise} \end{array}\right\}, \text{ if } l \neq 3^m \text{ and } l \text{ is even.} \tag{5.8}$$

Then for $\alpha \in [0, 1]$, the α-level set of W_l and ΔW_l are respectively.

$$[W_l]^\alpha = \begin{cases} l+\alpha, l+2-\alpha, \text{ for } l = 3^m \\ -3+\alpha, -1-\alpha, \text{ for } l \neq 3^m \text{ and } l \text{ is odd} \\ 6+\alpha, 8-\alpha, \text{ for } l \neq 3^m \text{ and } l \text{ is even} \end{cases} \tag{5.9}$$

$$[\Delta W_l]^\alpha = \begin{cases} l-8+2\alpha, l-4-2\alpha, \text{ for } l = 3^m \\ -l+3+2\alpha, -l+7-2\alpha, \text{ for } l+1 = 3^m \\ -11+2\alpha, -7-2\alpha \text{ for } l \neq 3^m, l+1 \neq 3^m \text{ and } l \text{ is odd} \\ 7+2\alpha, 11-2\alpha, \text{ for } l \neq 3^m, l+1 \neq 3^m \text{ and } l \text{ is even} \end{cases} \tag{5.10}$$

An example of a series that is Δ-statistically bounded yet not Δ-statistically convergent is (W_ℓ), which is I_Δ-statistically convergent if assume $I = I_\delta$.

Remark: If $I = I_f$ then the sequence spaces $F^{C_0^{I(S)}}(\Delta)$ and $F^{C^{I(S)}}(\Delta)$ coincide with the sequence spaces $F^{C_0}(\Delta)$ and $F^C(\Delta)$.

Theorem 5.1. *The spaces* $F^{C_0^{I(S)}}(\Delta)$ *and* $F^{C^{I(S)}}(\Delta)$ *are linear spaces*

Proof. Firstly we show that $F^{C_0^{I(S)}}(\Delta)$ is linear space.
Let $W = (W_l), Z = (Z_l)$ be any two elements of $F^{C_0^{I(S)}}(\Delta)$, and α, β be any scalar. Then below equations (5.11-5.13).

$$B(\varepsilon) = \left\{m \in N : \frac{1}{m}\left|\left\{l \leq m : \bar{c}\left(\Delta W_l, \overline{0}\right) \geq \frac{\varepsilon}{2}\right\}\right| \geq \delta\right\} \in J \tag{5.11}$$

$$A(\varepsilon) = \left\{m \in N : \frac{1}{m}\left|\left\{l \leq m : \bar{c}\left(\Delta Z_l, \overline{0}\right) \geq \frac{\varepsilon}{2}\right\}\right| \geq \delta\right\} \in J \tag{5.12}$$

Now,

$$D(\varepsilon) = \left\{m \in N : \frac{1}{m}\left|\left\{l \leq m : \bar{c}\left(\Delta\left(\alpha W_l \oplus \beta Z_l\right), \overline{0}\right) \geq \frac{\varepsilon}{2}\right\}\right| \geq \delta\right\} \in J \subseteq \left\{m \in N : \frac{1}{m}\right| \{l \leq \tag{5.13}$$

$$m : |\alpha|\bar{c}\left(\Delta W_l, \overline{0}\right) \geq \frac{\varepsilon}{2}\right\} | \geq \delta\Big\} \cup \left\{m \in N : \frac{1}{m}\left|\left\{l \leq m : |\beta|\bar{c}\left(\Delta Z_l, \overline{0}\right) \geq \frac{\varepsilon}{2}\right\}\right| \geq \delta\right\}$$

i.e., $D(\varepsilon) \subseteq B\left(\frac{2}{2|\alpha|}\right) \cup A\left(\frac{2}{2|\beta|}\right)$
i.e., $D(\varepsilon) \in J$.

Therefore, the space $F^{c_0^{I(s)}}(\Delta)$ is linear. In the same approach, we can show that $F^{C^{I(s)}}(\Delta)$ is a linear space.

□

Theorem 5.2. *: The following spaces, which is* $F^{C_0^{I(S)}}(\Delta)$ *and* $F^{C^{I(S)}}(\Delta)$*, are normal and monotone.*

Proof. Let $W = (W_l)$ be any elements of $F^{c_0^{I(S)}}(\Delta)$ and $Z = (Z_l)$ be any sequence such that $\bar{c}\left(\Delta W_l, \overline{0}\right) \geq \bar{c}\left(\Delta Z_l, \overline{0}\right). \forall l \in N$. Then $\forall > 0$,
$\left\{m \in N : \frac{1}{m}\left|\left\{l \leq m : \bar{c}\left(\Delta W_l, \overline{0}\right) \geq \frac{\varepsilon}{2}\right\}\right| \geq \delta\right\} \supseteq \left\{m \in N : \frac{1}{m}\left|\left\{l \leq m : \bar{c}\left(\Delta Z_l, \overline{0}\right) \geq \frac{\varepsilon}{2}\right\}\right| \geq \delta\right\} \in I$
Hence $Z = (Z_l) \in, F^{C_0^{I(s)}}(\Delta)$. Consequently, the spaces $F^{c_0^{I(s)}}(\Delta)$ and,$F^{m_0^{I(s)}}(\Delta)$, are normal and monotone. □

Theorem 5.3. *In cases that I does not represent the maximal ideal, the space* $F^{C^{I(S)}}(\Delta)$ *is neither monotonic nor normal.*

Example 4: Consider a sequence of fuzzy numbers.

$$W_l(s) = \begin{cases} \frac{1+s}{2} \text{ for } s \in [-1, 1] \\ \frac{3-s}{2} \text{ for } s \in [1, 3] \\ 0 \text{ otherwise} \end{cases} \tag{5.14}$$

Then $(W_l) \in F^{c^{I(S)}}(\Delta)$. According to lemma 3.4 , it has a subset L of N such that $L \notin I$ and $N - L \notin I$. Since I is or maximum. The sequence $Z = (Z_l)$ can be defined by equations 5.14 and 5.16.

$$Z_l = \begin{cases} W_l, & l \in L \\ \overline{0}, & \text{otherwise} \end{cases} \tag{5.15}$$

It implies that (Z_l) is a classical pre-image of the l-step spaces of $F^{C^{I(S)}}(\Delta)$, but $(Z_l) \notin F^{C^{I(S)}}(\Delta)$. For this reason, $F^{c^{I(S)}}(\Delta)$ cannot be monotonic. Lemma 3.3 consequently states that $F^{c^{I(S)}}(\Delta)$ fails to be normal.

Theorem 5.4. *The spaces* $F^{C^{I(S)}}(\Delta), F^{C_0^{I(S)}}(\Delta), F^{m^{I(S)}}(\Delta)$, *and* $F^{m_0^{I(S)}}(\Delta)$ *are sequence algebra.*

Proof. Let $(W_l), (Z_l) \in F^{c_0^{I(S)}}(\Delta)$ and $0 < \varepsilon < 1$. The subsequent incorporation context then yields the outcome that follows:
$\left\{m \in N : \frac{1}{m}\left|\left\{l \leq m : \bar{c}\left(\Delta\left(\alpha W_l \oplus \beta Z_l\right), \overline{0}\right) \geq \varepsilon\right\}\right| \geq \delta\right\} \supset \left\{m \in N : \frac{1}{m}\right| \left\{l \leq \ m : \bar{c}\left(\Delta W_l, \overline{0}\right) \geq \varepsilon\right\} \left|\geq \delta\right\}$
$\supseteq \left\{m \in N : \frac{1}{m}\left|\left\{l \leq m : \bar{c}\left(\Delta Z_l, \overline{0}\right) \geq \varepsilon\right\}\right| \geq \delta\right\}$. □

Theorem 5.5. $F^{c^{I(S)}}(\Delta)$, *and* $F^{c_0^{I(s)}}(\Delta)$ *are not convergence free domains.*

Proof. Let's consider a sequence of fuzzy numbers from equation 5.16.

$$W_l(s) = \begin{cases} \frac{2+s}{4}, -2 \leq s \leq 2 \\ \frac{6-s}{4}, 2 \leq s \leq 6 \\ 0, \ \text{otherwise} \end{cases} \tag{5.16}$$

Then $W_l(s) \in F^{C^{I(s)}}(\Delta)$. Let $Z_l(s) = \frac{1}{l} \forall l \in N$. Then $Z_l(s) \in F^{C^{I(s)}}(\Delta)$. However, $X_l = \overline{0}$ is not implied by $Y_l = \overline{0}.F^{C^{I(S)}}(\Delta)$ and $F^{C_0^{I(S)}}(\Delta)$ are therefore not convergence-free.

□

Theorem 5.6. *:* $F^{n^{I(S)}}(\Delta)$ *and* $F^{n_o^{I(S)}}(\Delta)$ *were entire metric spaces with respect to the metric that has been identified as* $\bar{c}(Y, X) = \bar{c}\left(Y_1, X_1\right) + \text{Sup}_l\, \bar{c}\left(\Delta Y_l, \Delta X_l\right)$, *where* $(Y_l), (X_l) \in F^{n^{I(S)}}(\Delta)$.

6 Conclusion

Mathematical frameworks referred to as series spaces are critical to the analysis of spaces, concept, topological construction, and features. This paper employed a difference operator to build a sequence space of fuzzy numbers, $F_c\ I(S)$ and $F_c\ I(S)_0$, which is described by means of I-statistical convergence. It also offered a detailed evaluation of the structural factors of these areas at the same time as examining their essential algebraic and topological traits. It additionally established numerous critical inclusive outcomes and investigates crucial connections associated with these areas, such as symmetry, solidity, and convergence-unfastened traits. By constructing on established ideas and imparting pointers for the utility in fuzzy set indication and uncertainty-associated domains, the research contributes to the know-how of I-statistical convergence in fuzzy range sequences area. I-statistical convergence of fuzzy numbers in sequences is the focus of the research, other convergence approaches, which could provide numerous insights into the behavior of fuzzy range sequences, were not endangered. Future studies endeavors might amplify those series spaces to encompass additional types of convergence, investigate multidimensional fuzzy range sequences, and utilize those ideas in an increasing number of difficult real-world situations incorporating fuzzy good judgment and uncertainty.

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