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ANALYSIS OF EFFECT OF HEMODYNAMIC PARAMETERS ON TWO-LAYERED BLOOD FLOW IN A MILD STENOSED ARTERY

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Abstract: Arterial stenosis is the lessening of the arterial wall due to the growth of aberrant tissues that prevent adequate blood flow in the human circulatory system and induces cardiovascular diseases. Mild stenosis, over time, can lead to serious and permanent damage if it remains uncured. Navier-Stokes equation in cylindrical polar coordinates system has been extended by two-layered blood flow along the axial direction with appropriate boundary conditions. A steady flow through a stenosed artery has been investigated extensively using the analytical approach in the case of two-layered blood flow. Mathematical expressions for two-layer hemodynamic parameters such as velocity profile, volumetric flow rate, and effect of stenosis progression on parameters with the variation of core and peripheral layered coefficient of viscosity are derived. Moreover, pressure drop, and shear stress have been calculated analytically in an artery with and without stenosis. Peripheral viscosity has less contribution to varying velocity distribution than the core and is proportional to stenosis size. The volumetric flow rate decreases with an increasing viscosity coefficient. Pressure drop and shear stress attain maximum value in the region stenosis occur maximum height in core layer. The present work could serve as a model in biomedical engineering for the cure of vascular-related diseases and has the potential in designing the devices of this field.
Key Words: Mild stenosis; viscosity coefficient; velocity profile; volumetric flow rate; pressure drop; shear stress; two-layered model

1. Introduction

An anomalous biological response results from unusual hemodynamic circumstances, cardiovascular diseases are intimately related to flow in the blood vessels [2, 6]. The human circulatory system is a closed cardiovascular network of arteries, veins, and capillaries [8, 15]. Arteries are the vascular vessels that transport oxygen-rich blood to various parts of the body, hence cardiovascular network plays a vital role in sustaining life [14, 18]. Any type of intrusion of fat into the arterial wall blocks the way of blood flow and causes a deficiency of oxygen supply [13]. Intravascular atherosclerotic plaque that develops across the inner wall of an artery due to fat deposition, calcification, and other abnormal growth of tissues is known as stenosis [23, 17]. It locates in several parts of the circulatory system and protrudes into the lumen of the arteries narrowing them, and causing arterial disorders under diseased conditions [28, 36]. With time, constriction might deteriorate and this condition has the potential to cause both arterial thrombosis and bleeding [16, 28]. Gautam et al. [21] have studied the effect of increasing stenosis on flow parameters taking blood as a non-Newtonian fluid. The flow disturbance further influences the incidence of diseases namely heart attacks, strokes, angina pectoris, and cerebral stroke [11, 13]. According to Hiatt et al. [12], if the stenosis increases for an extended period of time, it can result variety of medical disorders, including vision loss, heart and brain hemorrhages. Circulatory problems are recognized to be the cause of seventy-five

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percent of all fatalities and stenosis is one of the most occurring cardiovascular diseases [23].

Halder [9] examined the flow of blood through a confined tube while treating the blood as a Newtonian fluid. Ku [18] pointed out that the strong wall shear stress at the throat of stenosis can activate platelets and cause thrombosis, which can completely obstruct the flow. Misra et al. [25] have discovered analytical expressions for velocity distribution, pressure gradient, total angular velocity, wall shear stress, flow rate, and resistance to fluid motion. Pokharel et al. [27] evaluated the pressure, pressure drop against the wall, shear stress on the wall, the ratio of the maximum to the minimum shear stress, and pressure drop with and without stenosis. Further Kafle et al. [17] have considered curved arteries and derived velocity profile and volumetric flow rate with a variation of curvature assuming axial symmetric one-layered laminar flow.

In the literature above, flow has been described as one-layered, However; Bulgliarello and Sevilla [19] experimentally demonstrated that the blood moving in small capillaries is composed of a core region of suspended erythrocytes and outer layer of cell-free plasma. Srivastava and Saxena [33] found plasma thickness slows down resistance to flow and increases shear stress on the wall. Chaturani and Kaloni [32] described the two-layer model with shear stress at the core layer taking blood as incompressible fluid. Halder and Andersson [10] has considered, flow as laminar, steady, axially symmetric and fully developed and discussed pressure drop and wall friction in two layer blood flow model. Ponalagusamy [26] further considered the two-layer model of blood flow to determine the apparent viscosity and examined the analytical and numerical impact of mild stenosis on the blood flow characteristic in a two-layer model. Joshi et al. [16] have used a two-layered model to examine how peripheral plasma viscosity affects the flow characteristic. Ponalagusamy and Manchi [28] employed a mathematical model to analyze steady flow in six distinct types of stenoses, and found that the two-layered model considerably reduced the axial variance of wall shear stress and flow resistance.

In view of the above literature, it is evident that in-depth investigations into blood flow with vascular disorders have been conducted to determine the factor associated with it. Material structure is different for peripheral and core layer which affects all flow parameters in core and peripheral layer. Therefore, study of arterial two layer model in presence of mild stenosis yields more accurate and realistic results. In this article, the study focuses on two-layer model of steady, laminar, and axi-symmetric flow in an artery with mild stenosis which may contribute a better understanding of blood flow in an artery.

2. Two-layered Blood Flow Model

We have assumed blood is incompressible and Newtonian in the stenosed artery. Blood flow in arteries can be modeled by using Navier-Stokes equation [7]. Suppose a uniform, cylindrical, axially symmetric, laminar blood flow that is steady and completely developed through the artery of radius R_0 in the presence of mild stenosis. Let r be the radial velocity function and p be the pressure. Consider three components v^r, v^θ, and v^z are velocities along the radius vector, perpendicular to the radius vector, and along the axial direction cylindrical shaped artery. Then, the continuity equation is given by [7]

$$\frac{1}{r}\frac{\partial}{\partial r}(rv^r) + \frac{\partial}{\partial z}(v^z) = 0 \tag{2.1}$$

Navier-Stoke's equation along radial and z-axis is,

$$\rho\left(\frac{\partial v^r}{\partial t} + v^r\frac{\partial v^r}{\partial r} + v^z\frac{\partial v^r}{\partial z}\right) = -\frac{\partial p}{\partial r} + \mu\left(\frac{\partial^2 v^r}{\partial r^2} + \frac{\partial^2 v^r}{\partial z^2} + \frac{1}{r}\frac{\partial v^r}{\partial r} - \frac{v^r}{r^2}\right) \tag{2.2}$$

$$\rho\left(\frac{\partial v^z}{\partial t} + v^r\frac{\partial v^z}{\partial r} + v^z\frac{\partial v^z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 v^z}{\partial r^2} + \frac{\partial^2 v^z}{\partial z^2} + \frac{1}{r}\frac{\partial v^z}{\partial r}\right) \tag{2.3}$$

In axi-symmetric flow, $v^\theta = 0$ and v^r, v^z and p are independent of θ. Let us consider the steady flow and let ρ be the density. The velocity component parallel to the z-axis is $v^z = v(r)$ and $v^r = 0$, $v^\theta = 0$ then equation (3) reduces to

$$0 = -\frac{\partial p}{\partial z} + \mu\left\{\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right\} \tag{2.4}$$

where, μ is the coefficient of viscosity of the blood.

Here blood is represented by two-layer, inner core and outer peripheral layer with different homodynamic parameters. Suppose R_s^p is radius of the artery in peripheral layer from center of radius, R_s^c is the radius of artery in the core layer in presence of stenosis such that

$$R_s^c = \beta R_s^p, \quad R_0^c = \beta R_0^p$$

whereas R_0^p and R_0^c are the radius of peripheral (i.e., radius of the normal artery) and core layer of normal artery respectively and β is the ratio of core radius to normal artery radius which is called *scaling model parameter*. For two-layered model, we divide whole region into peripheral layer as $R_s^c(z) \le r \le R_s^p(z)$ and core region as $R_s^c(z) \le r \le R_s^p(z)$.

Assume μ_p and μ_c are viscosity of peripheral and core layer respectively. More precisely two-layered viscosity (μ) is defined as

$$\mu(r) = \begin{cases} \mu_p & \text{for} \quad R_s^c(z) \le r \le R_s^p(z) \\ \mu_c & \text{for} \quad 0 \le r \le R_s^c(z) \end{cases}$$

Suppose δ_p is the maximum height of the stenosis in the peripheral layer and δ_c same as in core layer such that

$$\delta_c = \beta \delta_p$$

The geometry of the stenosis in peripheral layer is given by [5]

$$R_s^p(z) = \begin{cases} R_0^p - \frac{\delta_p}{2}\left(1 + \cos \pi \frac{z}{z_0}\right) & \text{for} \quad |z| \le z_0 \\ R_0^p & \text{for} \quad |z| \ge z_0 \end{cases} \tag{2.5}$$

Similarly, the geometry of the stenosis in core layer can be expressed as [5]

$$R_s^c(z) = \begin{cases} R_0^c - \frac{\delta_c}{2}\left(1 + \cos \pi \frac{z}{z_0}\right) & \text{for} \quad |z| \le z_0 \\ R_0^c & \text{for} \quad |z| \ge z_0 \end{cases} \tag{2.6}$$

The equation 2.4 for peripheral and core layer respectively can be expressed as

$$0 = -\frac{\partial p}{\partial z} + \mu_p \left\{ \frac{\partial^2 v_p}{\partial r^2} + \frac{1}{r}\frac{\partial v_p}{\partial r} \right\} \tag{2.7}$$

$$0 = -\frac{\partial p}{\partial z} + \mu_c \left\{ \frac{\partial^2 v_c}{\partial r^2} + \frac{1}{r}\frac{\partial v_c}{\partial r} \right\} \tag{2.8}$$

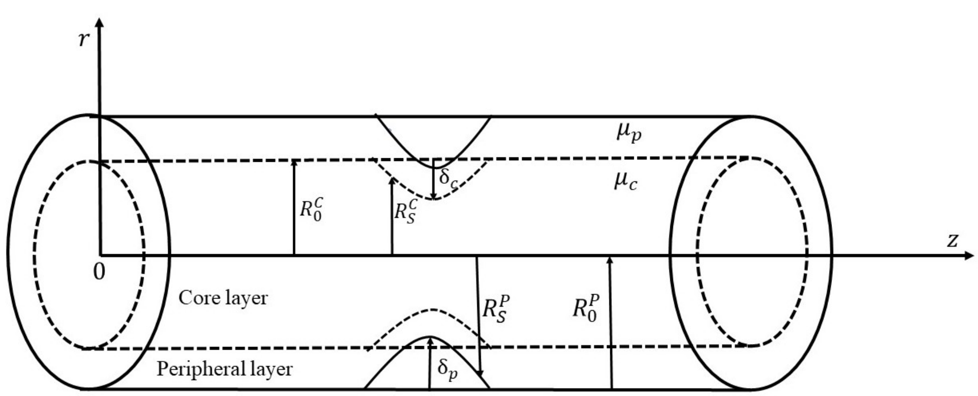


Figure 1. Geometry of stenosis in two layer model [16].

with boundary conditions (BCs):

$$v_p = \begin{cases} v_c & at \quad r = R_s^c(z) \\ 0 & at \quad r = R_s^p(z) \end{cases} \tag{2.9}$$

and

$$\frac{\partial v_p}{\partial r} = 0, \quad \frac{\partial v_c}{\partial r} = 0 \qquad at \quad r = 0 \tag{2.10}$$

3. Analytical Solution of Two-layered Blood Flow Model

3.1. **Two-layered velocity profile of blood flow. Peripheral layered velocity:** Let v_p be the velocity peripheral layer [i.e., region $R_s^c(z) \le r \le R_s^p(z)$]. Then, equation 2.7 become

$$-P(z)\frac{r}{\mu_p} = \frac{\partial}{\partial r}\left(r\frac{\partial v_p}{\partial r}\right)$$

On integration

$$r\frac{\partial v_p}{\partial r} = -P(z)\frac{r^2}{2\mu_p} + A(z)$$

Using boundary condition $\frac{\partial v_p}{\partial r} = 0$ when $r = 0$

$$\frac{\partial v_p}{\partial r} = -P(z)\frac{r}{2\mu_p} \tag{3.1}$$

Again, integration

$$v_p = -\frac{P(z)}{2\mu_p}\left(\frac{r^2}{2}\right) + B(z)$$

Using boundary condition $v_p = 0$ at $r = R_s^p$, we have $B(z) = \frac{P\left(R_s^p\right)^2}{4\mu_p}$ and thus peripheral layered velocity is

$$v_p = \frac{P}{4\mu_p}\left((R_s^p)^2 - r^2\right) \qquad R_s^c \le r \le R_s^p \tag{3.2}$$

Core layered velocity: Next, suppose v_c be velocity of core layer [i.e., region $0 \le r \le R_s^c(z)$]. Then, from equation 2.8

$$-P(z)\frac{r}{\mu_c} = \frac{\partial}{\partial r}\left(r\frac{\partial v_c}{\partial r}\right)$$

After integration,

$$-P(z)\frac{r^2}{2\mu_c} = r\frac{\partial v_c}{\partial r} + C(z)$$

Using boundary condition $\frac{\partial v_c}{\partial r} = 0$ at $r = 0$, gives $C(z) = 0$. Then

$$r\frac{\partial v_c}{\partial r} = -P(z)\frac{r^2}{2\mu_c}$$

Integrating again

$$v_c = -\frac{P(z)}{2\mu_c}\left(\frac{r^2}{2}\right) + D(z)$$

Using boundary condition $v_p = v_c$ at $r = R_s^c$, we have

$$\frac{P}{4\mu_p}\left((R_s^p)^2 - (R_s^c)^2\right) = -P(z)\frac{(R_s^c)^2}{4\mu_c} + D(z)$$

Therefore, core layered velocity [i.e., in the region $0 \le r \le R_s^c(z)$] is

$$v_c = \frac{P}{4\mu_p}\left[\left((R_s^p)^2 - (R_s^c)^2\right) + \frac{\mu_p}{\mu_c}\left((R_s^c)^2 - r^2\right)\right] \tag{3.3}$$

3.2. **Two-layered Volumetric Flow Rate. Peripheral layered volumetric flow rate:** Let us consider volumetric flow rate in peripheral layer be Q_p and obtained as

$$Q_p = \int_{R_s^c}^{R_s^p} 2\pi r v_p dr = 2\pi \frac{P(z)}{4\mu_p} \int_{R_s^c}^{R_s^p} \left(r(R_s^p)^2 - r.r^2 \right) dr = \frac{\pi}{8\mu_p} P(z) \left((R_s^p)^2 - (R_s^c)^2 \right)^2 \tag{3.4}$$

Core layered volumetric flow rate: Similarly, volumetric flow rate in core region Q_c is obtained as

$$Q_c = \int_0^{R_s^c} 2\pi r v_c dr \ = 2\pi \frac{P(z)}{4\mu_p} \int_0^{R_s^c} \left[(r(R_s^p)^2 - r(R_s^c)^2) + \overline{\mu} \left(r.(R_s^c)^2 - r.r^2 \right) \right] dr$$

$$Q_c = \frac{\pi}{8\mu_p} P(z) \left[2(R_s^c)^2 \left[(R_s^p)^2 - \left(1 - \frac{\overline{\mu}}{2} \right) (R_s^c)^2 \right] \right] \tag{3.5}$$

where $\overline{\mu} = \frac{\mu_p}{\mu_c}$.

Total volumetric rate: The total volumetric rate (Q) is sum of volumetric rate of peripheral layer Q_p and volumetric rate of core layer Q_c. Then, total volumetric rate is

$$Q = Q_p + Q_c = \frac{\pi}{8\mu_p} P(z) [(R_s^p)^4 - (1 - \overline{\mu}) (R_s^c)^4] \tag{3.6}$$

3.3. **Two-Layered Pressure Gradient Across the Stenosis Surface. Peripheral layered pressure gradient across the stenosis surface:** Assume pressure drop across stenosis in peripheral region is $(\Delta P)_s^p$. Then,

$$(\Delta P)_s^p = \int_{-z_0}^{z_0} \left(-\frac{dp}{dz} \right) dz = \frac{8\mu_p Q_p}{\pi (R_0^p)^4} \int_{-z_0}^{z_0} \frac{dz}{\left[\left(\frac{R_s^p}{R_0^p} \right)^2 - \left(\frac{R_s^c}{R_0^p} \right)^2 \right]^2} = \frac{8\mu_p Q_p}{\pi (R_0^p)^4} \int_{-z_0}^{z_0} \frac{dz}{\left[\left(\frac{R_s^p}{R_0^p} \right)^2 - \beta^2 \left(\frac{R_s^p}{R_0^p} \right)^2 \right]^2}$$

Using equation 2.5

$$(\Delta P)_s^p = \frac{8\mu_p Q_p}{\pi (R_0^p)^4} \frac{1}{(1-\beta^2)^2} \int_{-z_0}^{z_0} \frac{dz}{\left[1 - \frac{\delta_p}{2R_0^p} \left(1 + \cos \pi \frac{z}{z_0} \right) \right]^4}$$

Using binomial expansion, we get

$$(\Delta P)_s^p = \frac{8\mu_p Q_p}{\pi (R_0^p)^4} \frac{1}{(1-\beta^2)^2} \int_{-z_0}^{z_0} \left[1 + \frac{4\delta_p}{2R_0^p} \left(1 + \cos \pi \frac{z}{z_0} \right) + \frac{5}{2} \frac{\delta_p^2}{(R_0^p)^2} \left(1 + \cos \pi \frac{z}{z_0} \right)^2 + o(\delta_p)^3 \right] dz$$

Neglecting higher order $o(\delta_p)^3$ and integrating, Therefore, pressure gradient across the stenosis surface in peripheral layer is

$$(\Delta P)_s^p = \frac{8\mu_p Q_p}{\pi (R_0^p)^4} \frac{1}{(1-\beta^2)^2} \left(1 + \frac{2\delta_p}{R_0^p} + \frac{15\delta_p^2}{4(R_0^p)^2} \right) 2z_0 \tag{3.7}$$

This is the pressure drop in stenosed region of outer peripheral layer. For the condition when there is no stenosis (i.e., $\delta_p = 0$), $R_s^p = R_0^p$ which implies $\frac{R_s^p}{R_0^P} = 1$. Then, pressure drop in peripheral layer in the case of without stenosis, i.e., $(\Delta P)_0^p$ is

$$(\Delta P)_0^p = \frac{8\mu_p Q_p}{\pi (R_0^P)^4} \frac{1}{(1-\beta^2)^2} \quad 2z_0 \tag{3.8}$$

Core layered pressure gradient across the stenosis surface: Let pressure drop across stenosis in core region is $(\Delta P)_s^c$. Then

$$(\Delta P)_s^c = \int_{-z_0}^{z_0} \left(-\frac{dp}{dz} \right) dz = \frac{8\mu_p Q_c}{\pi} \int_{-z_0}^{z_0} \frac{dz}{2(R_s^c)^2 \left[(R_s^p)^2 - \left(1 - \frac{\overline{\mu}}{2} \right) (R_s^c)^2 \right]}$$

$$= \frac{4\mu_p Q_c}{\pi (R_0^c)^4} \int_{-z_0}^{z_0} \frac{dz}{\left(\frac{R_s^c}{R_0^c} \right)^2 \left[\left(\frac{1}{\beta} \frac{R_s^c}{R_0^c} \right)^2 - \left(1 - \frac{\overline{\mu}}{2} \right) \left(\frac{R_s^c}{R_0^c} \right)^2 \right]}$$

This is pressure drop across stenosis in core layer. When there is no stenosis ($\delta_c = 0$), $\frac{R_s^c}{R_s^p} = 1$. Let pressure drop in this region without stenosis be $(\Delta P)_0^c$. Then,

$$(\Delta P)_0^c = \frac{4\mu_p Q_c \quad 2z_0}{\pi (R_0^c)^4 \left(\frac{1}{\beta^2} - \left(1 - \frac{\overline{\mu}}{2} \right) \right)} \tag{3.9}$$

3.4. **Two-layered Shear Stress on the Stenosis Surface. Peripheral layered shear stress on the stenosis surface:** The shear stress on surface of peripheral layer across stenosis at $r = R_s^p$ is denoted by τ_s^p. Then,

$$\tau_s^p = \left[-\mu_p \frac{\partial v_p}{\partial r}\right]_{r=R_s^p} = \left[(-\mu_p)\,(-P(z))\,\frac{r}{2\mu_p}\right]_{r=R_s^p} = \left[P(z)\frac{R_s^p}{2}\right]$$

Substituting value of $P(z)$, we get

$$\tau_s^p = \frac{4\mu_p Q_p R_s^p}{\pi}\left[\frac{1}{\left((R_s^p)^2-(R_s^c)^2\right)^2}\right] = \frac{4\mu_p Q_p R_s^p}{\pi (R_0^p)^4}\frac{1}{\left[\left(\frac{R_s^p}{R_0^p}\right)^2-\left(\frac{R_s^c}{R_0^p}\right)^2\right]^2}$$

Using equation 2.5,

$$\tau_s^p = \frac{4\mu_p Q_p}{\pi (R_0^p)^3\,(1-\beta^2)^2}\frac{1}{\left[1-\frac{\delta_p}{2R_0^p}\left(1+\cos\pi\frac{z}{z_0}\right)\right]^3} = \frac{4\mu_p Q_p}{\pi (R_0^p)^3}\frac{\left(1-\frac{\delta_p}{R_0^P}\right)^{-3}}{(1-\beta^2)^2}$$

Using binomial expansion

$$\tau_s^p = \frac{4\mu_p Q_p}{\pi (R_0^p)^3}\frac{1}{(1-\beta^2)^2}\left(1+3\frac{\delta_p}{R_0^p}+6\frac{{\delta_p}^2}{(R_0^p)^2}\right) \tag{3.10}$$

For the condition when there is no stenosis (i.e., $\delta_p = 0$) on the peripheral layer, we have

$$\tau_0^p = \frac{4\mu_p Q_p}{\pi (R_0^p)^3}\frac{1}{(1-\beta^2)^2} \tag{3.11}$$

Core layered shear stress on the stenosis surface: Shear stress on the stenosis at $r = R_s^c$ is denoted by τ_s^c and obtained as

$$\tau_s^c = \left[-\mu_c \frac{\partial v_c}{\partial r}\right]_{r=R_s^c} = \left[-\mu_c\,(-P(z))\,\frac{r}{2\mu_c}\right]_{r=R_s^c} = \left[P(z)\frac{R_s^c}{2}\right]$$

After substituting the value of $P(z)$, we have

$$\tau_s^c = \frac{2\mu_p Q_c R_s^c}{\pi}\left[\frac{1}{(R_s^c)^2\left[(R_s^p)^2-\left(1-\frac{\overline{\mu}}{2}\right)(R_s^c)^2\right]}\right] = \frac{2\mu_p Q_c}{\pi (R_0^c)^3}\frac{1}{\left[\frac{1}{\beta^2}-\left(1-\frac{\overline{\mu}}{2}\right)\right]\left(\frac{R_s^c}{R_0^c}\right)^3}$$

Using equation 2.6

$$\tau_s^c = \frac{2\mu_p Q_c}{\pi (R_0^c)^3\left(\frac{1}{\beta^2}-\left(1-\frac{\overline{\mu}}{2}\right)\right)}\frac{1}{\left[1-\frac{\delta_c}{2R_0^c}\left(1+\cos\pi\frac{z}{z_0}\right)\right]^3} = \frac{2\mu_p Q_c}{\pi (R_0^c)^3}\frac{\left[1-\frac{\delta_c}{R_0^c}\right]^{-3}}{\left(\frac{1}{\beta^2}-\left(1-\frac{\overline{\mu}}{2}\right)\right)}$$

Using binomial expansion

$$\tau_s^c = \frac{2\mu_p Q_c}{\pi (R_0^c)^3}\frac{1}{\left(\frac{1}{\beta^2}-\left(1-\frac{\overline{\mu}}{2}\right)\right)}\left(1+3\frac{\delta_c}{R_0^c}+6\frac{{\delta_c}^2}{(R_0^c)^2}\right) \tag{3.12}$$

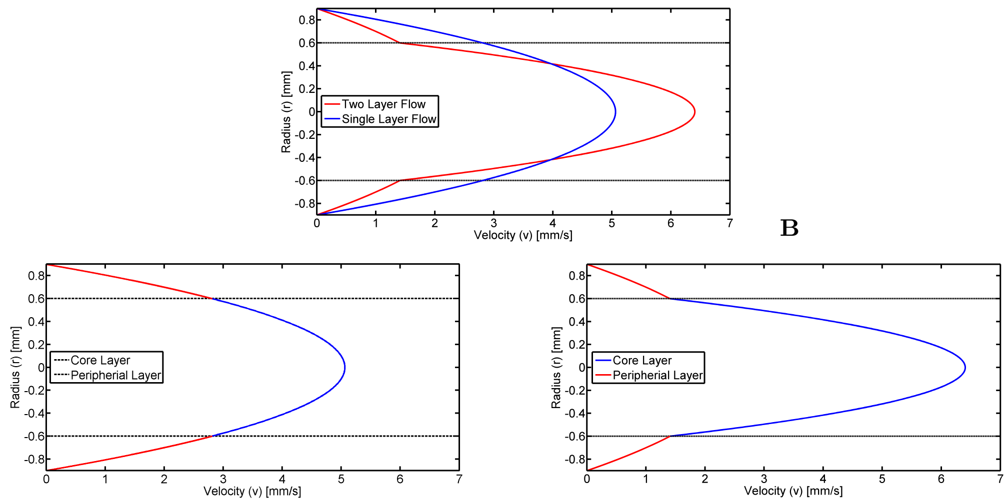
Moreover, shear stress on the surface of core layer in the case of no stenosis, i.e., $\delta_c = 0$ is

$$\tau_0^c = \frac{2\mu_p Q_c}{\pi (R_0^c)^3}\frac{1}{\left(\frac{1}{\beta^2}-\left(1-\frac{\overline{\mu}}{2}\right)\right)} \tag{3.13}$$

4. Result and Discussion

4.1. **Two-Layered Velocity Profile of Blood Flow through a Stenotic Artery. Comparison of velocity profile between single and two-layered:** Figure (2)A illustrates a comparison of velocity profiles in two-layered and single-layered blood flow for different values of radial distances $(0.0 - 0.8)$ mm. At radial distance 0.6 mm velocity is 1.5 mm/s in two-layered that of 2.75 mm/s approximately in single-layer. Velocity is 4 mm/s in both cases at 0.4 mm. At radial distance 0.2 mm, velocity is 6 mm/s and 5 mm/s approximately in both layers respectively. In the center, it is 6.5 mm/s and 5.25 mm/s for two-layered and single. It is observed that velocity distribution is higher near the arterial wall in the single-layered but quite a different phenomenon is seen, velocity is maximum in the center for two-layered as result the peak value of velocity in the two-layered occurs forward to the peak value of velocity in single-layered. Figure 2B

A



B

C

FIGURE 2. Velocity distribution **A:** Single-layered verses two-layered model. **B:** Single-layered model. **C:** Two-layered Model.

shows single-layered blood flow with various values of radial distances $(0.0 - 0.8)$ mm. Velocity is 2.75 mm/s at value 0.6 mm, 4 mm/s at value 4 mm, 5 mm/s at value 0.2 mm and 5.25 mm/s at center. It is revealed that maximum velocity 5.25 mm/s occurs at the center of the artery whereas minimum velocity 2.75 mm/s is near the wall of the artery. Figure 2C describes velocity distribution in two-layered blood flow for a radial distance of values $(0.0 - 0.8)$ mm. At radial distance 0.6 mm, velocity is1.5 mm/s, at 0.4 mm, velocity is 4 mm/s and at 0.2 mm, it is 6 mm/s. It is seen that velocity is maximum in a core layer at the center with a value of 6.5 mm/s. Velocity attains the least value 1.5 mm/s near the arterial wall in the peripheral layer. It is concluded, velocity increases with increasing stenosis thickness in both layers but higher velocities were observed for two-layered blood flow when compared with single-layered.

Two-layered velocity profile with variation of viscosity coefficients: Figure 3A shows peripheral velocity distribution with radial distance r for various values of peripheral layer viscosity. Viscosity coefficient μ_p takes values (0.2, 0.4, 0.6, 0.8) gm mm^{-1}s^{-1}. Radii in the region of stenosis along the core layer and peripheral layer have been taken the value of $R_s^c = 0.6$ mm and $R_s^p = 0.9$ mm respectively. It is observed that velocity distribution in the peripheral layer is the maximum at least value of viscosity coefficient in the peripheral layer. Velocity in peripheral layer, v_p is maximum for the value of viscosity coefficient $\mu_p = 0.20$ gm mm^{-1}s^{-1} and velocity is minimum for value of viscosity coefficient $\mu_p = 0.80$ gm mm^{-1}s^{-1}. It is also noted that, as R_s^p decreases, the coefficient of viscosity μ_p decreases, and R_s^p increases with the coefficient of viscosity μ_p increases. Figure 3B describes the relation between radial distance (r) and core velocity (v_c) for varying values of peripheral viscosity (μ_p). For peripheral viscosity values 0.20 gm mm^{-1}s^{-1}, 0.4 gm mm^{-1}s^{-1}, 0.6 gm mm^{-1}s^{-1}, 0.8 gm mm^{-1}s^{-1} corresponding core velocities are 7.4 mm s^{-1}, 5.8 mm s^{-1}, 5.4 mm s^{-1}, 5.1 mm s^{-1} approximately. Because of the different values of peripheral viscosity, the initial points of the core velocity are different in the boundary between the core and peripheral layers. Flow is axisymmetric and the velocity increases gradually towards center which can be seen in the figure. These data indicate that the maximum velocity at the center decreases as the peripheral viscosity (μ_p) increases, which shows the plasma viscosity affects the core velocity.

Figure 3C describes the relation between radial distance (r) and core velocity (v_c) for different values of core viscosity (μ_c). In this case, we have taken the peripheral viscosity (μ_p) constant, so the initial point at the boundary is the same for all values of core viscosity (μ_c). For the core viscosity with different values 0.09 gm mm^{-1}s^{-1}, 0.1 gm mm^{-1}s^{-1}, 0.11 gm mm^{-1}s^{-1}, 0.12 gm mm^{-1}s^{-1} corresponding core velocities are 5.9 mm s^{-1}, 5.3 mm s^{-1}, 4.9 mm s^{-1}, 4.5 mm s^{-1} approximately. The effect of the core viscosity (μ_c)

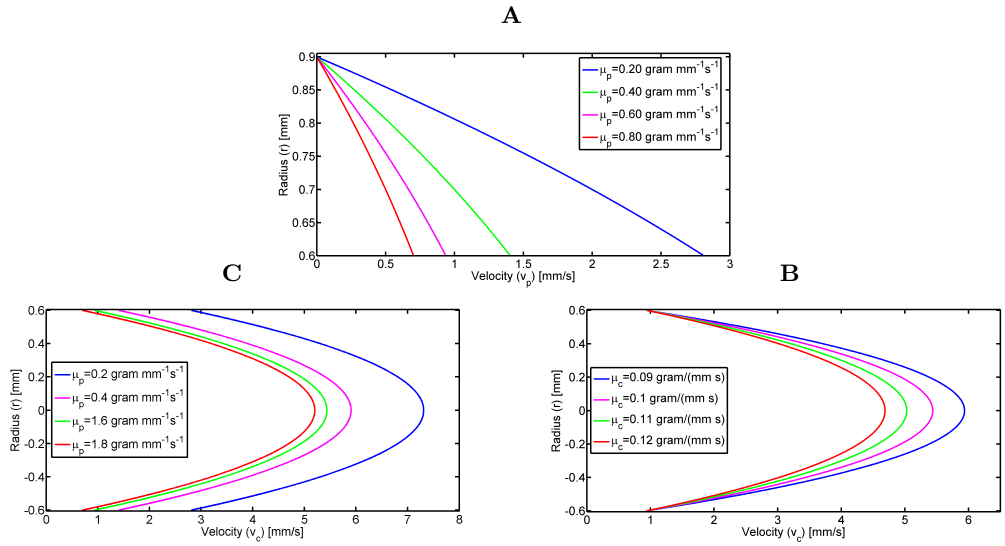


FIGURE 3. **A:** Variation of peripheral layer velocity (v_p) for different value of peripheral layer viscosity μ_p. **B:** Variation of core layer velocity (v_c) for different value of core layer viscosity μ_c. **C:** Variation of core layer velocity (v_c) for different value of peripheral layer viscosity μ_c.

is more than the effect of peripheral viscosity (μ_p). This can be seen clearly when we compare Fig. 3B and Fig. 3C. Compared with B, it has taken fewer viscosity values for different values of core velocity. This clearly indicates that core velocity (v_c) has been affected more due to core viscosity (μ_c). From these results, we conclude that the velocity of the fluid reduces rapidly in both layers of flow with increasing value of viscosity. The consequence of peripheral viscosity on varying velocity distribution has less contribution as compared to core viscosity.

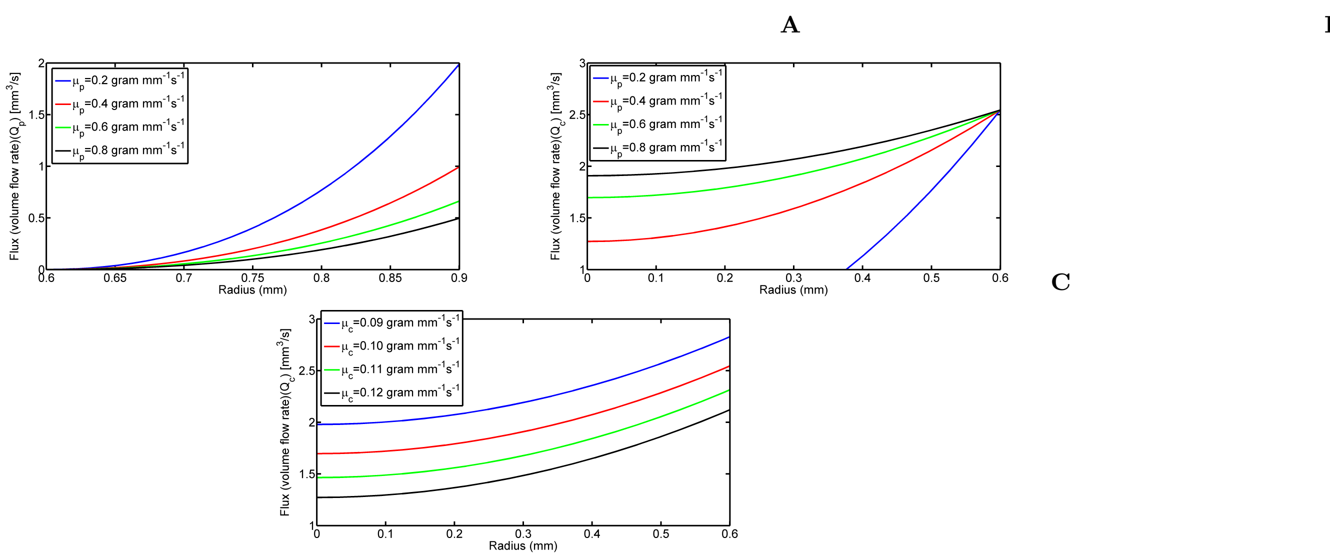


FIGURE 4. **A:** Volumetric flow rate (Q_p) for different values of peripheral layer viscosity (μ_p). **B:** Volumetric flow rate (Q_c) for different value of peripheral layer viscosity (μ_p). **C:** Volumetric flow rate (Q_c) for different value of core layer viscosity (μ_c).

4.2. **Two-layered Volumetric Flow Rate through a Stenotic Artery.** Figure 4A shows the volumetric flow rate of blood in an artery across stenosis with various values of radius (0.6-0.9) mm. Flow rate is least at value $\mu_p = 0.8$ gm mm^{-1} s^{-1} and maximum at value $\mu_p = 0.2$ gm mm^{-1} s^{-1} with radial value $R_s^P = 0.9$ mm. This figure shows that the volumetric flow rate decreases with the increased viscosity coefficient. A larger value of μ_p, the less steep the curve becomes. As stenosis height is elevated, the effect of μ_p tends to vanish due to the narrow artery and the curve becoming closer and closer. Figure 4 B depicts flux Q_c in the core layer for different values of viscosity coefficient μ_p. When μ_p changes from $(0.2 - 0.8)$ mm, flux rises up to 2.5 mm^3/s approximately. It concludes that the less viscous the fluid becomes, the more speedy it flows. Q_c attains maximum value when $R_s^p = 0.6$ mm. Flow increases rapidly when the viscosity coefficient μ_p is 0.2 gm mm^{-1} s^{-1} and the increment is least when the viscosity coefficient μ_p is 0.8 gm mm^{-1} s^{-1}. Figure 4C displays the relation between volumetric flow rate in the core layer with a radius for different values of core viscosity μ_c. When the viscosity μ_c is 0.09 gm mm^{-1} s^{-1} the volumetric flow rate increases from 2.0 mm^3/s to 2.8 mm^3/s. Similarly it increases from 1.70 mm^3/s to 2.60 mm^3/s for the core viscosity 0.10 gm mm^{-1} s^{-1}. Finally the volumetric flow rate increases from 1.30 mm^3/s to 2.20 mm^3/s for the core viscosity 0.12 gm mm^{-1} s^{-1}. From this figure, it is noted that the effect of core viscosity is less effective compared with the peripheral layer viscosity.

4.2.1. *Total Volumetric Flow Rate of Blood through Artery in presence of Stenosis.* In Fig. 5**A**, the relation between flow rate and radius of the artery is explained for different values of peripheral viscosity. For Fig. 5**A** core viscosity is kept constant and peripheral viscosity increases gradually from 0.2 gm mm^{-1}s^{-1} to 0.8 gm mm^{-1}s^{-1}. The volumetric flow rate in the core is less affected by the increment of the peripheral viscosity. So the line in the graph is increasing minutely in the core or say up to 0.6 mm. In the case of the peripheral layer, the volumetric flow rate is high for less viscosity which decreases gradually as the viscosity increases. The volumetric flow rate decreases gradually from 7.5 mm^3 s^{-1} to less than 4 mm^3 s^{-1} as the viscosity increases from 0.2 to 0.8 gm mm^{-1}s^{-1}. In comparison with the peripheral layer, the volumetric flow rate which is affected negligibly in the core nearly 1 mm^3 s^{-1} when the viscosity is 0.2 gm mm^{-1}s^{-1} and less for higher values of viscosity. In Fig. 5**B**, the relation between flow rate and radius of the artery is explained for different values of core viscosity. In this case, peripheral viscosity is kept constant and core viscosity increases gradually from 0.09 gm mm^{-1}s^{-1} to 0.12 gm mm^{-1}s^{-1}. Fig. 5 also depicts that the volumetric flow rate in the core is less effected by the change in viscosity in the peripheral layer. When the viscosity is 0.09 gm mm^{-1}s^{-1} the volumetric flow rate is about 2.4 mm^3 s^{-1} in the core layer but in the peripheral layer, it increases up to 4.5 mm^3 s^{-1}. The result is almost similar to other values of the viscosity coefficient also. In presence of progressive hemodynamic obstriction, it is observed that the volumetric rate of flow decreases to a considerable extent as peripheral layer viscosity increases and is more influenced by accumulating value of core layer viscosity.

4.3. **Two-layered Pressure Drop with and without Stenosis in Blood Flow.** Figure 6**A** depicts the effect of peripheral stenosis on the pressure drop. Both regions with and without stenosis of the artery are taken to study. To show the relation between these two variables other parameters (μ_p), volumetric flow

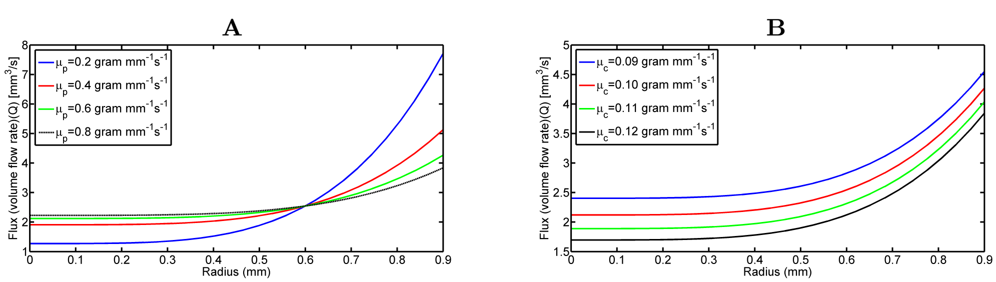


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FIGURE 5. Relation between volumetric flow rate and radius in **A:** peripheral viscosity. **B:** core viscosity.

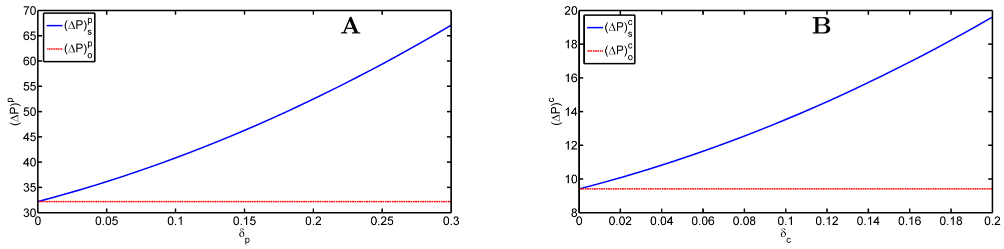

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FIGURE 6. **A:** Pressure drop in peripheral layer by variation of radius of stenosis. **B:** Pressure drop in core layerby variation of radius of stenosis.

rate Q_p, and z_0 are kept constant. If there is no stenosis, we see a uniform change in pressure drop and the line is almost straight. But in the region of stenosis, pressure drop changes immensely. In both cases, initial points are the same but as the height of the stenosis increases from $0 - 0.3$ mm, pressure drop changes from 32.5 pa to above 65 pa. This indicates that the stenosis in the peripheral layer plays a central role in the change in pressure drop.

Figure 6**B** describes the effect of core stenosis on pressure drop. As in case of Fig. 6**A** peripheral viscosity (μ_p), core viscosity (μ_c), core volumetric flux Q_c, and length of the stenosis z_0 are kept constant. In the part of the artery without stenosis, only a small change in pressure is seen which is denoted by a red line and is straight almost, but in the stenotic region, a huge change in pressure drop is seen. Value of the other parameters which are taken constantly are core viscosity ($\mu_c = 0.1$), peripheral viscosity ($\mu_p = 0.5$), length of stenosis $z_0 = 1$ mm, and a maximum height of the stenosis in core $(\delta_c) = 0.2$ mm. As the height of stenosis in the core increases from 0.0 mm to 0.2 mm, the pressure drop changes from 9.5 pa to 19 pa approximately. When we analyze these two figures effect of stenosis in the peripheral layer (δ_p) is more than stenosis in core layer (δ_c). These figures also show that the change in pressure drop is high in the stenotic region and there is a maximum alteration of blood flow noticed in the peripheral layer due to stenosis thickness.

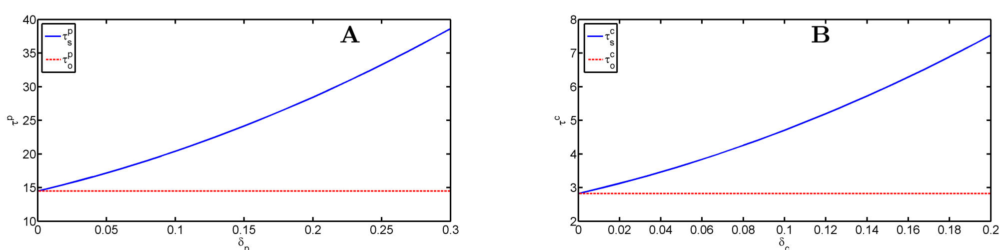

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FIGURE 7. **A:** Peripheral layer shear stress (τ^p) in an artery with and without stenosis by variation of radius of stenosis (δ_p). **B:** Core layer shear stress (τ^c) in an artery with and without stenosis by variation of radius of stenosis (δ_c).

4.4. **Peripheral and Core layer Shear Stress in Blood Flow.** Figure 7**A** depicts the effect of peripheral stenosis on shear stress in the peripheral wall in the regions with and without stenosis. The other parameters, peripheral viscosity and volumetric flux rate in the peripheral layer are kept constant. Maximum height of the stenosis in peripheral layer (δ_p) is kept 0.3 mm. We can see from the figure if there are no stenosis shear stress seems uniform. The shear stress in the peripheral layer increases from 15 pa to 38 pa approximately when the peripheral stenosis increases from 0 to 0.3 mm. In the region of the artery containing stenosis, shear stress increases with an increase in height of stenosis. Shear stress on the maximum height of peripheral stenosis is maximum and that is 38 pa.

Figure 7**B** describes the effect of core stenosis (δ_c) on core shear stress (τ_c). In the part of the artery if there is no stenosis only a small change in shear stress is seen and is denoted by a red line which is almost straight. . We have taken core stenosis height is (δ_c) is 0.2 mm. In this case also as the stenosis increases from 0.0 mm to 0.2 mm, the core shear stress increase from 2.8 pa to 7.5 pa. In the stenotic region, the shear stress in the core increases with an increase in core stenosis. Here core viscosity ($\mu_c = 0.1$) and peripheral viscosity ($\mu_p = 0.5$) are kept constant, length of the stenosis is taken 1 mm. As we see in the figures, shear stress is immensely affected by stenosis. The effect of stenosis size on shear stress is higher in the peripheral layer. This shows the presence of stenosis, shear stress brings significant alteration in the flow field and this effect is more in the peripheral layer as compared to the core layer.

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Data availability statement.: The plotting code are available from the first author upon request.

Authors' contributions: *Jeevan Kafle*: Main Idea of Research work, With a special contribution to the result and Discussion, and Main role in revision. *Samundra Timilsina Tripathee*: Substantial contribution to the extension of the Model, Designed the research plan, Development of the Model, Drafting the Final MS

5. Summary

In this article, the study focused on the analysis of steady, laminar blood flow through an artery in presence of the mild stenosis, consisting inner core layer of red blood cells and outer peripheral layer of plasma. The two-layered flow behavior of blood through a stenosed artery, velocity profile and volumetric flow rate, pressure gradient, and shear stress on the surface of stenosis with various values of viscosity coefficients have been analyzed. Significantly, analytical evaluation of the pressure drop and shear stress with and without stenosis are performed. There is a remarkable difference in the velocity variation in both layers. Maximum velocity attains at the center of the artery and decreases gradually towards the inner wall. It is observed that the velocity is increased with the increasing stenosis height and decreases for increasing coefficient of viscosity. The volumetric flow rate declines with the rise of stenotic thickness and less affected by the core layer viscosity as compared to peripheral layer viscosity. Further, volumetric flow rate rises with vessel radius. It is revealed that pressure drop is more in peripheral layer against thickness of stenosis with the radius of artery as compared to core layer and proportional relation to coefficient of viscosity. Higher viscous fluid accelerates a higher pressure gradient. Marginal change of pressure drop can be seen in the absence of stenosis in an artery. The result shows, the presence of stenosis, pressure drop brings significant alteration in the flow field and this effect is more in the peripheral layer as compare to core layer. Shear stress shows similar behavior, consequently, in the course of time, stenosis deviates the peripheral layer and reduce core layer. The results of the present analysis draw attention to the application and understanding of the blood flow mechanism in a two-layered model and provide decisive information in diagnosing and treating arterial stenosis.

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References

[1] F. Adam, What to know about cardiovascular disease, *Appl. Appl. Math.*: Vol. 4, pp 343-354, 2019.

[2] D. Liepsch, An introduction to mechanics, basic models and application, *J. Biomech.*: 35, pp 415-435, 2002.

[3] S. Chien, Haemorpheology in clinical medicine. *Adv.in Cardio Diseases* , Vol.(2), pp 21-26, 1981.

[4] H. Darcy, The Flow of Fluid Through Porous Media. Mc-GrawHill, New York, NY, USA, 1993.

[5] C. Diwakar, S. Kumar, Effects of axially symmetric stenosis on the blood flow in an artery having mild stenosis, *IJMTT*: 35(3), pp 163-167, 2016.

[6] R. Ellah, S.U. Rahman, The theoretical study of unsteady blood flow of Jeffery fluid through stenosed arteries wi'/th permable walls, *Zeitschr Naturforchung*; A68, pp 489-498, doi:10.5560/Zna, 2013-0032, 2013.

[7] J.N. Kapur, Mathematical Models in Biology and Medicine, *Affiliated East-West Press Pvt. Ltd.*, New Delhi, India, 1985.

[8] YC. Fung, Biomechanics: Mechanical properties of living tissues, *Spring-Verlag*, New York, 1993.

[9] K. Halder, Oscillatery flow of blood in a stenosed artery, *Bulletin of Mathematical Biology*: 49, pp, 279-287, 1987.

[10] K. Haldar, H.L. Andersson, Two-layered model of blood flow through stenosed arteries, *Acta Mechanica*: 117, pp 221-228, 1995.

[11] R.H. Haynes, Physical bases on dependence of blood viscosity on tube radius, *Am. J. Physiol.*: 198, pp 1193-1205, 1960.

[12] E.P. Hiatt, J.P. Mecchan, R. Galombos, Report on human acceleration, *Pub. 901, NAS-NRS*, Washington DC, USA, 1969.

[13] M. Jain, G.C. Sharma, R. Singh, Mathematical modeling of blood flow in a stenosed artery under MHD effec tthrough porous medium, *International Journal of Engineering, Transaction B*: 23(3-4), pp 243-251, 2010.

[14] S.S. Varghese, Numerical modeling of pulsatile turbulent flow in stenotic vessels, *Journal of Biomechanical Engineering*: 125(4), pp 445-460, 2003.

[15] S.S. Jasit, Numerical analysis of the effect of turbulance transition on the hemodynamics parameters in human coronary arteries, *Cardiovascular Diagnosis and Therapy*: 6(3), pp, 208-220, 2016.

[16] P. Joshi, A. Pathak, B.K. Joshi, The two layered model for blood flow through composite artery, *Applications and Applied Mathematics*: 4(2), 2009.

[17] J. Kafle, H.R. Gaire, P.K. Pokheral, P. Kattel, Analysis of blood flow through curved artery with mild stenosis, *Mathematical Modeling and Computing*: 1, pp 1-10, 2020.

[18] D.N. Ku, Blood flow in arteries, *Annu. Rev. Fluid Mech.*: 29, pp 399-434, 1997.

[19] G. Bulgliarello, J. Sevilla, Velocity distributed and other characteristic of steady and pulsatile blood flow in fine glass tube. *Biorheology*: Vol. 7, pp 85-107, 1970.

[20] J.S. Lee, Y.C. Fung, Flow in locally constricted tube at low Reynalds Number, *J. Mech. Trans.*: 37, pp 9-16, 1970.

[21] P.N. Gautam, C. Pokharel, G.R. Phaijoo, P. Kattel, and J. Kafle, Effect of Increasing Stenosis Over Time on Hemodynamics, *Biomath*, 12, 2310067, 2023. https://doi.org/10.55630/j.biomath.2023.10.067.

[22] M. Nakamura, T. Sawada, Numerical study on the flow of non-Newtonian fluid through an axisymmertric stenosis, *Journal of Biomechanical Engineering*: 110(1988), pp 137-143, 1988.

[23] V.P. Srivastava, R. Rastogi, R. Vishnoi, A two-layered suspension blood flow through an overlapping stenosis, *Comp. Maths. App.*: 60, pp 432-441, 2010.

[24] D. MacDonald, On steady flow through modelled vasculas stenosis, *J. Biomech*: 12, pp, 13-20, 1970.

[25] J.C. Misra, M.K. Patra, B.K. Sahu, Unsteady flow of blood through narrow blood vessels, A mathematical analysis, *Computers Math. Appli.*: 24(10), pp 19-31,1992.

[26] R. Ponalagusamy, Blood flow through an artery with mild stenosis,*Journal of Applied Science*: 7, pp 1071-1077, 2007.

[27] C. Pokharel, P.N. Gautam, S. Timilsina, C.R. Bhatta, J. Kafle, Analysis of flow parameters in blood flow through mild stenosis, *Nepalese Journal of Zoology*: Vol 6(2), pp 39-44, 2022.

[28] R. Ponalagusamy, R. Manchi, A study of an two-layered (K.L-Newtonian) model of blood flow in an artery with six types of mild stenoses, *Applied Mathematics and computation*: 367, 2020.

[29] J.B. Shukla, R.S. Parihar, S.P. Gupta, Effect of peripheral layer viscosity on blood flow through the artery with mild stenosis, *Bull. Math. Biol.*: 42, pp 797-805, 1980.

[30] P. Singh, B.K. Joshi, Analysis of blood flow Through an artery with mild stenosis, *Italian Journal of Pure and Applied Mathematics*: 33, 2014.

[31] S. Sreenadh, B. Samalatha, A.N.S. Srinivas, Study of Non-Newtonian blood flow of Jaffrey fluid in Elastic tube, *Journal of Applied Fluid Mechanics*: 13(6), pp 1807-1817, 2020.

[32] P. Chaturani, P.N. Kaloni, Two-layered poiseuille flow model for blood flow through arteries of small diameter and arterioles. *Biorheology*, Vol. 13(4), pp 243-250, 1976.

[33] V.P. Srivastava, M. Sexana, Two-layered model of cassion fluid flow through stenotic blood vessels application to cardiovascular system, *J. of Biomechanics*: 27(7), pp 921-928, 1994.

[34] V.P. Srivastava, A theoritical model for blood flow in small vessel, *Appl. Appl. Maths.* 2:51-65, 2007.

[35] G.B. Thurston, Plasma release cell-layering theory for blood flow, *Biorheology*: 26, pp 199-214, 1989.

[36] S. R. Shah, and R. Kumar, Mathematical modeling of blood flow with suspension of nanoparticle through a tapered artery with blood clot, *Front. Nanotechnol.* 2:596475 doi:10. 3389/fnano.2020(596475), 2020.