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ABSOLUTE SUMMABILITY FOR N-TUPLED TRIANGLE MATRICES ON SEQUENCE SPACE

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Abstract: In this paper, we determine that every n-tupled generalized Cesàro matrices $(C, \alpha_1, \alpha_2, ..., \alpha_n; \delta) \in B(A_k^n; \delta)$ for $k \geq 1$, $\delta \geq 0$ and $\alpha_1, \alpha_2, ..., \alpha_n > -1$, need not be absolute k^{th} power conservative since the Cesàro matrices of order α for $\alpha > -1$ are not conservative matrices, where for some given $k \geq 1$ and $\delta \geq 0$, if $T \in B(\mathcal{A}_k, \delta)$; i.e., if $\{s_0, s_1, ...s_n\}$ satisfying

(0.1)
$$\sum_{n=1}^{\infty} n^{\delta k+k-1} |s_n - s_{n-1}|^k < \infty,$$

implies
$$\sum_{n=1}^{\infty} n^{\delta k+k-1} |t_n - t_{n-1}|^k < \infty.$$

Then, T is said to be absolutely k^{th} power conservative.

Key Words: Absolute Summability, n-tupled sequence space, Bounded Operators, Absolute k^{th} power conservative.

AMS (MOS) Subject Classification. 40C05, 40F05, 40D20, 40G05.

1. Introduction

Let $\sum_{n=0}^{\infty} a_n$ be a given infinite series such that

(1.1)
$$s_k = a_0 + a_1 + a_2 + ... + a_k = \sum_{l=0}^{k} a_l,$$

where s_k denotes the k^{th} partial sum of the series $\sum_{n=0}^{\infty} a_n$ and $\{s_n\}$ define the sequence of partial sums. Then the n^{th} term of sequence-to-sequence transformation of $\{s_n\}$ is defined by

(1.2)
$$t_n = \sum_{k=0}^{\infty} t_{nk} s_k = \sum_{k=0}^{\infty} t_{n,n-k} s_{n-k}.$$

The sequence $\{t_n\}$ of the matrix means of the sequence $\{s_n\}$ is generated by the sequence of the coefficients $\{t_{nk}\}$. A sequence of the partial sums $\{s_n\} = \{s_0, s_1, ..., s_n\}$ is of bounded

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variation if the series $|s_1 - s_0| + |s_2 - s_1| + ... + |s_n - s_{n-1}|$ converges or

$$\sum_n |\Delta s_n| < \infty. \tag{1.3}$$

The infinite series $\sum_{n=0}^{\infty} a_n$ with the sequence of the partial sum $\{s_n\}$ is absolute summable by the method A (A-summable) to the limit s if it is A-summable to s, i.e. if $lim_{n\to\infty} t_n = s$ and if the sequence $\{t_n\}$ is of bounded variation:

$$\sum_n |\ t_n - t_{n-1}| < \infty. \tag{1.4}$$

Let n^{th} term of transform for the sequence $\{s_n\}$ with Cesàro matrix is t_n^{α}. The infinite series $\sum_{n=0}^{\infty} a_n$ is absolutely $|A|_k$-summable of degree $k \geq 1$, if $\sum_{n=1}^{\infty} n^{k-1}|t_n - t_{n-1}|^k$ converges. If this is the case, we can write $\sum_{n=0}^{\infty} a_n \in |A|_k$.

Das [2] defines the absolute conservation by transforming the sequence $\{s_n\}$ into $\{t_n\}$. Let T represents sequence-to-sequence transformation. If, whenever $\{s_n\}$ converges absolutely, $\{t_n\}$ converges absolutely, then T is called absolutely conservative. If the absolute convergence of $\{s_n\}$ implies absolute convergence of $\{t_n\}$ to the same limit, T is called absolutely regular. If $T \in B(\mathcal{A}_k^n)$ for some $k \geq 1$; i.e., if $\{s_0, s_1, ...s_n\}$ satisfying

$$\sum_{n=1}^{\infty} n^{k-1}|s_n - s_{n-1}|^k < \infty, \tag{1.5}$$

implies

$$\sum_{n=1}^{\infty} n^{k-1}|t_n - t_{n-1}|^k < \infty. \tag{1.6}$$

Then, T is called absolutely k^{th} power conservative. Note that when $k > 1$, (1.6) does not necessarily imply the convergence of $\{s_n\}$. There exists a sequence space $\mathcal{A}_k$ which is given by

$$\mathcal{A}_k = \left\{ \{s_n\} : \sum_{n=1}^{\infty} n^{k-1}|a_n|^k < \infty, a_n = s_n - s_{n-1} \right\}. \tag{1.7}$$

If $\alpha = 0$ in the inclusion statement involving (C, α) and (C, β), then we obtain the fact that $(C, \beta) \in B(\mathcal{A}_k)$ for each $\beta > 0$, where $B(\mathcal{A}_k)$ denotes the algebra of all matrices that map $\mathcal{A}_k$ to $\mathcal{A}_k$.

For some given $k \geq 1$ and $\delta \geq 0$, if $T \in B(\mathcal{A}_k, \delta)$; i.e., if $\{s_0, s_1, ...s_n\}$ satisfying

$$\sum_{n=1}^{\infty} n^{\delta k+k-1}|s_n - s_{n-1}|^k < \infty, \tag{1.8}$$

implies $\sum_{n=1}^{\infty} n^{\delta k+k-1}|t_n - t_{n-1}|^k < \infty.$

Then, T is said to be absolutely k^{th} power conservative for the sequence space $(\mathcal{A}_k, \delta)$ which is given by

$$(\mathcal{A}_k,\delta)=\left\{\{s_n\}:\sum_{n=1}^{\infty}n^{\delta k+k-1}|a_n|^k<\infty, a_n=s_n-s_{n-1}\right\} \tag{1.9}$$

Many research articles [13]-[20] devoted to the study of summability of infinite series due to its wide range of applications. Various investigations have been done to determine the most important results on absolute summability factor of infinite series by using different summability methods. The absolute summability (C,α), or $|C,\alpha|$ of a series was defined by Fekete [4], for the case where α is an integer, and in the general case by Kogbetliantz [6]. Whittaker [12] defined the absolute summability (A) or summability $|A|$ and was the first to investigate the summability $|A|$ of a Fourier series. In 1957, Flett [5] obtained an extension of summability $|C|$ and defined absolute summability. Mazhar [7] gave the necessary and sufficient conditions for the infinite series $\sum\limits_{n=0}^{\infty} a_n$ to be $|\bar{N},p_n|$ summable whenever it is $|C,\alpha|_k$ $(\alpha\geq 0, k\geq 1)$ summable. Dikshit [3] also has been given a general theorem on absolute summability factors for Cesàro summability of infinite series and rectified the deficiencies of the proof. Bor [1] gave a theorem dealing with $|\bar{N},p_n|$ summability factors taking an almost increasing sequence of the infinite series and provide the application of the almost increasing sequences of infinite series. After this Sulaiman [11] gave the applications and generalization of the result of Bor [1]. The absolute Cesàro summability, the absolute generalized Cesàro summability, the absolute Nölund summability, the absolute Riesz summability, the absolute Euler summability etc. have been become a topic of great interest since last two decades. In 2009, Savaş et al.[10] used the concept of absolute conservation for Cesàro means which is generalization of the Das [2]. Savaş et al. [10] have proved the theorems which give sufficient conditions of infinite series using the absolute summability factors. After reviewing several articles, we have dealt with absolute summability of an infinite series and obtained some general results included the minimal set of sufficient condition for n-tupled Triangle matrices $T\in B(\mathcal{A}_k^n)$.

2. Known results

In 2007, Savaş et al. proved that a Cesàro matrix of order $\alpha>-1$ is a bounded operator on $\mathcal{A}_k$ and in 2009, established a minimal set of sufficient conditions for a triangle $T\in B(\mathcal{A}_k)$ as follows:

Theorem 2.1. *$(C,\alpha)\in B(\mathcal{A}_k)$ for each $\alpha>-1$.*

3. Main results

Let T be the infinite matrix for the series $\sum\limits_{N_1=1}^{\infty}\sum\limits_{N_2=1}^{\infty}...\sum\limits_{N_n=1}^{\infty}a_{N_1,N_2,...,N_n}$ and

$\Delta_{11...n\,times}t^{i_1,i_2,...,i_n}_{N_1,N_2,...,N_n}$

$$
\begin{aligned}
&= t^{i_1,i_2,...,i_n}_{N_1-1,N_2-1,...,N_n-1} - \{t^{i_1,i_2,...,i_n}_{N_1,N_2-1,...,N_n-1} + ... + t^{i_1,i_2,...,i_n}_{N_1-1,N_2-1,...,N_n}\} \\
&+ \{t^{i_1,i_2,...,i_n}_{N_1,N_2,N_3-1,...,N_n-1} + t^{i_1,i_2,...,i_n}_{N_1,N_2-1,N_3,N_4-1,...,N_n-1} + ...\} \\
&- \{t^{i_1,i_2,...,i_n}_{N_1,N_2,N_3,N_4-1...,N_n-1} + t^{i_1,i_2,...,i_n}_{N_1-1,N_2,N_3,N_4,N_5-1,...,N_n-1} + ...\} \\
&+ ... + (-1)^n \; t^{i_1,i_2,...,i_n}_{N_1,N_2,...,N_n} \tag{3.1}
\end{aligned}
$$

There exists two infinite matrices $\bar{T}$ and $\hat{T}$ with T as follows:

$$\bar{t}^{i_1,i_2,...,i_n}_{N_1,N_2,...,N_n} = \sum_{\mu_1=i_1}^{N_1} \sum_{\mu_2=i_2}^{N_2} ... \sum_{\mu_n=i_n}^{N_n} t^{\mu_1,\mu_2,...,\mu_n}_{N_1,N_2,...,N_n} \tag{3.2}$$

and

$$\hat{t}^{i_1,i_2,...,i_n}_{N_1-1,N_2-1,...,N_n-1} = \triangle_{11...n\,times}\bar{t}^{i_1,i_2,...,i_n}_{N_1-1,N_2-1,...,N_n-1} \tag{3.3}$$

$$N_1, N_2, ..., N_n, i_1, i_2, ..., i_n = 0, 1, 2, ...$$

In the present paper, we generalize theorem 2.1 for n-tupled triangle matrices. Now, we shall prove the following:

Theorem 3.1. $(C, \alpha_1, \alpha_2, ..., \alpha_n; \delta) \in B(A^n_k; \delta)$ *for each* $\alpha_1, \alpha_2, ..., \alpha_n > -1$, $k \geq 1$ *and* $\delta \geq 0$.

4. Proof of the Theorem

Let $\sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2,...,N_n}$ denotes the $N_1N_2...N_n$ term of the $(C, \alpha_1, \alpha_2, ..., \alpha_n)$ transform for the order $(\alpha_1, \alpha_2, ..., \alpha_n)$ in the sequence $s_{N_1N_2...N_n}$; that is,

$$\sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2,...,N_n} = \frac{1}{E^{\alpha_1}_{N_1}E^{\alpha_2}_{N_2}...E^{\alpha_n}_{N_n}} \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} ... \sum_{i_n=1}^{N_n} E^{\alpha_1-1}_{N_1-i_1} E^{\alpha_2-1}_{N_2-i_2}...E^{\alpha_n-1}_{N_n-i_n} s_{i_1i_2...i_n}) \tag{4.1}$$

We shall show that $(\sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2,...,N_n}) \in (\mathcal{A}^n_k, \delta)$; i.e.,

$$\sum_{N_1=1}^{\infty} \sum_{N_2=1}^{\infty} ... \sum_{N_n=1}^{\infty} (N_1, N_2...N_n)^{\delta k+k-1} \left|\sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2,...,N_n} - \sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1-1,N_2-1,...,N_n-1}\right|^k < \infty, \tag{4.2}$$

Let $t^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2,...,N_n}$ denote the $N_1N_2...N_n$ term of the $(C, \alpha_1, \alpha_2, ..., \alpha_n)$ transform in term of $N_1N_2...N_na_{N_1N_2...N_n}$; that is,

$$
\begin{aligned}
t^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2,...,N_n} &= \frac{1}{E^{\alpha_1}_{N_1}E^{\alpha_2}_{N_2}...E^{\alpha_n}_{N_n}} \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} ... \sum_{i_n=1}^{N_n} E^{\alpha_1-1}_{N_1-i_1} E^{\alpha_2-1}_{N_2-i_2}...E^{\alpha_n-1}_{N_n-i_n} \times \\
&\times (i_1i_2...i_na_{i_1i_2...i_n}) \tag{4.3}
\end{aligned}
$$

For $\alpha_1, \alpha_2, ...\alpha_n > -1$,
Since

$$
\begin{aligned}
t^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2,...,N_n} = N_1N_2...N_n\Big[&\sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2,...,N_n} - (\sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1-1,N_2,...,N_n} + \sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2-1,...,N_n} \\
&+ ... + \sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2,...,N_n-1}) \\
&+ (\sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1-1,N_2-1,N_3,...,N_n} + \sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1-1,N_2,N_3-1,N_4,...,N_n} \\
&+ \sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1-1,N_2,N_3,...,N_n-1}) - ... + (-1)^n\sigma^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1-1,N_2-1,...,N_n-1}\Big], \tag{4.4}
\end{aligned}
$$

then condition (4.2) can also be written as,

$$\sum_{N_1=1}^{\infty}\sum_{N_2=1}^{\infty}...\sum_{N_n=1}^{\infty}(N_1,N_2...N_n)^{\delta k-1}\left|t^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2,...,N_n}\right|^k<\infty. \tag{4.5}$$

Using Holder's inequality, we have

$$\sum_{N_1=1}^{\infty}\sum_{N_2=1}^{\infty}...\sum_{N_n=1}^{\infty}(N_1,N_2...N_n)^{\delta k-1}\left|t^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2,...,N_n}\right|^k$$

$$=\sum_{N_1=1}^{\infty}\sum_{N_2=1}^{\infty}...\sum_{N_n=1}^{\infty}(N_1,N_2...N_n)^{\delta k-1}\left|\frac{1}{E^{\alpha_1}_{N_1}E^{\alpha_2}_{N_2}...E^{\alpha_n}_{N_n}}\times\right.$$

$$\left.\times\sum_{i_1=1}^{N_1}\sum_{i_2=1}^{N_2}...\sum_{i_n=1}^{N_n}E^{\alpha_1-1}_{N_1-i_1}E^{\alpha_2-1}_{N_2-i_2}...E^{\alpha_n-1}_{N_n-i_n}(i_1i_2...i_na_{i_1i_2...i_n})\right|^k$$

$$\leq\sum_{N_1=1}^{\infty}\sum_{N_2=1}^{\infty}...\sum_{N_n=1}^{\infty}\frac{(N_1,N_2...N_n)^{\delta k-1}}{E^{\alpha_1}_{N_1}E^{\alpha_2}_{N_2}...E^{\alpha_n}_{N_n}}\times$$

$$\times\sum_{i_1=1}^{N_1}\sum_{i_2=1}^{N_2}...\sum_{i_n=1}^{N_n}E^{\alpha_1-1}_{N_1-i_1}E^{\alpha_2-1}_{N_2-i_2}...E^{\alpha_n-1}_{N_n-i_n}(i_1i_2...i_n)^k|a_{i_1i_2...i_n}|^k\times$$

$$\times\left\{\frac{1}{E^{\alpha_1}_{N_1}E^{\alpha_2}_{N_2}...E^{\alpha_n}_{N_n}}\sum_{i_1=1}^{N_1}\sum_{i_2=1}^{N_2}...\sum_{i_n=1}^{N_n}E^{\alpha_1-1}_{N_1-i_1}E^{\alpha_2-1}_{N_2-i_2}...E^{\alpha_n-1}_{N_n-i_n}\right\}^{k-1} \tag{4.6}$$

By using,

$$\frac{1}{E^{\alpha_1}_{N_1}E^{\alpha_2}_{N_2}...E^{\alpha_n}_{N_n}}\sum_{i_1=1}^{N_1}\sum_{i_2=1}^{N_2}...\sum_{i_n=1}^{N_n}E^{\alpha_1-1}_{N_1-i_1}E^{\alpha_2-1}_{N_2-i_2}...E^{\alpha_n-1}_{N_n-i_n}=1,$$

we have

$$\sum_{N_1=1}^{\infty}\sum_{N_2=1}^{\infty}...\sum_{N_n=1}^{\infty}(N_1,N_2...N_n)^{\delta k-1}|t^{\alpha_1,\alpha_2,...,\alpha_n}_{N_1,N_2,...,N_n}|^k$$

$$\leq\sum_{N_1=1}^{\infty}\sum_{N_2=1}^{\infty}...\sum_{N_n=1}^{\infty}\frac{(N_1,N_2...N_n)^{\delta k-1}}{E^{\alpha_1}_{N_1}E^{\alpha_2}_{N_2}...E^{\alpha_n}_{N_n}}\times$$

$$\times\sum_{i_1=1}^{N_1}\sum_{i_2=1}^{N_2}...\sum_{i_n=1}^{N_n}E^{\alpha_1-1}_{N_1-i_1}E^{\alpha_2-1}_{N_2-i_2}...E^{\alpha_n-1}_{N_n-i_n}(i_1i_2...i_n)^k|a_{i_1i_2...i_n}|^k$$

$$\leq\sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty}...\sum_{i_n=1}^{\infty}(i_1i_2...i_n)^k|a_{i_1i_2...i_n}|^k\times$$

$$\times\sum_{N_1=i_1}^{\infty}\sum_{N_2=i_2}^{\infty}...\sum_{N_1=i_n}^{\infty}\frac{E^{\alpha_1-1}_{N_1-i_1}E^{\alpha_2-1}_{N_2-i_2}...E^{\alpha_n-1}_{N_n-i_n}}{(N_1,N_2...N_n)^{1-\delta k}E^{\alpha_1}_{N_1}E^{\alpha_2}_{N_2}...E^{\alpha_n}_{N_n}}$$

$$=O(1)\sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty}...\sum_{i_n=1}^{\infty}(i_1i_2...i_n)^{\delta k+k}|a_{i_1i_2...i_n}|^k\times$$

$$\times\sum_{N_1=i_1}^{\infty}\sum_{N_2=i_2}^{\infty}...\sum_{N_1=i_n}^{\infty}\frac{E^{\alpha_1-1}_{N_1-i_1}E^{\alpha_2-1}_{N_2-i_2}...E^{\alpha_n-1}_{N_n-i_n}}{N_1N_2...N_nE^{\alpha_1}_{N_1}E^{\alpha_2}_{N_2}...E^{\alpha_n}_{N_n}} \tag{4.7}$$

For $\alpha_1, \alpha_2, ..., \alpha_n > -1$ and $N_1, N_2, ..., N_n \geq 1$,

$$\sum_{i_1=1}^{N_1}\sum_{i_2=1}^{N_2}...\sum_{i_n=1}^{N_n}\frac{E_{N_1-i_1}^{\alpha_1-1}E_{N_2-i_2}^{\alpha_2-1}...E_{N_n-i_n}^{\alpha_n-1}}{N_1,N_2...N_nE_{N_1}^{\alpha_1}E_{N_2}^{\alpha_2}...E_{N_n}^{\alpha_n}}$$

$$= \sum_{N_1=i_1}^{\infty}\frac{E_{N_1-i_1}^{\alpha_1-1}}{N_1E_{N_1}^{\alpha_1}}\sum_{N_2=i_2}^{\infty}\frac{E_{N_2-i_2}^{\alpha_2-1}}{N_2E_{N_2}^{\alpha_2}}...\sum_{N_n=i_n}^{\infty}\frac{E_{N_n-i_n}^{\alpha_n-1}}{N_nE_{N_n}^{\alpha_n}} \tag{4.8}$$

We obtain

$$\sum_{N_n=i_n}^{\infty}\frac{E_{N_n-i_n}^{\alpha_n-1}}{N_nE_{N_n}^{\alpha_n}} = \sum_{r_n=0}^{\infty}\frac{E_{r_n}^{\alpha_n-1}}{(i_n+r_n)E_{i_n+r_n}^{\alpha_n}} = \sum_{r_n=0}^{\infty}E_{r_n}^{\alpha_n-1}B(i_n+r_n,\alpha_n+1)$$

$$= \sum_{r_n=0}^{\infty}E_{r_n}^{\alpha_n-1}\int_0^1(1-x)^{\alpha}x^{i_n+r_n-1}dx = \int_0^1(1-x)^{\alpha}x^{i_n-1}\left(E_{r_n}^{\alpha_n-1}x^{r_n}\right)dx$$

$$= \int_0^1 x^{i_n-1}dx = \frac{1}{i_n} \tag{4.9}$$

$$\sum_{i_1=1}^{N_1}\sum_{i_2=1}^{N_2}...\sum_{i_n=1}^{N_n}\frac{E_{N_1-i_1}^{\alpha_1-1}E_{N_2-i_2}^{\alpha_2-1}...E_{N_n-i_n}^{\alpha_n-1}}{N_1,N_2...N_nE_{N_1}^{\alpha_1}E_{N_2}^{\alpha_2}...E_{N_n}^{\alpha_n}}$$

$$= \frac{1}{i_n}\sum_{N_1=i_1}^{\infty}\frac{E_{N_1-i_1}^{\alpha_1-1}}{N_1E_{N_1}^{\alpha_1}}\sum_{N_2=i_2}^{\infty}\frac{E_{N_2-i_2}^{\alpha_2-1}}{N_2E_{N_2}^{\alpha_2}}...\sum_{N_n-1=i_{n-1}}^{\infty}\frac{E_{N_{n-1}-i_{n-1}}^{\alpha_{n-1}-1}}{N_{n-1}E_{N_{n-1}}^{\alpha_{n-1}}}$$

$$= \frac{1}{i_1i_2...i_n} = (i_1i_2...i_n)^{-1} \tag{4.10}$$

Thus,

$$\sum_{N_1=1}^{\infty}\sum_{N_2=1}^{\infty}...\sum_{N_n=1}^{\infty}(N_1,N_2...N_n)^{\delta k-1}|t_{N_1,N_2,...,N_n}^{\alpha_1,\alpha_2,...,\alpha_n}|^k$$

$$= O(1)\sum_{i_1=1}^{N_1}\sum_{i_2=1}^{N_2}...\sum_{i_n=1}^{N_n}(i_1i_2...i_n)^{\delta k+k}|a_{i_1i_2...i_n}|^k\frac{1}{i_1i_2...i_n}$$

(4.11)

$$= O(1)\sum_{i_1=1}^{N_1}\sum_{i_2=1}^{N_2}...\sum_{i_n=1}^{N_n}(i_1i_2...i_n)^{\delta k+k-1}|a_{i_1i_2...i_n}|^k = O(1),$$

since $s_n \in (\mathcal{A}_k^n, \delta)$. Hence proof of the theorem is complete.

5. Corollaries

Corollary 5.1. $(C, 1, 1, \ldots n\, times; \delta) \in B(\mathcal{A}_k^n; \delta)$, *with the condition*

$$\begin{aligned} t_{N_1,N_2,\ldots,N_n} &= (N_1N_2 \ldots N_n)^{\delta k-1} \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} \ldots \sum_{i_n=1}^{N_n} s_{i_1,i_2,\ldots,i_n} \\ &= (C, 1, 1, \ldots, n\, times; \delta)(s_{N_1N_2\ldots N_n}). \end{aligned}$$

Corollary 5.2. $(C, 1, 1, \ldots n\, times) \in B(\mathcal{A}_k^n)$, *with the condition*

$$\begin{aligned} t_{N_1,N_2,\ldots,N_n} &= \frac{1}{N_1N_2 \ldots N_n} \sum_{i_1=1}^{N_1} \sum_{i_2=1}^{N_2} \ldots \sum_{i_n=1}^{N_n} s_{i_1,i_2,\ldots,i_n} \\ &= (C, 1, 1, \ldots, n\, times)(s_{N_1N_2\ldots N_n}). \end{aligned}$$

Corollary 5.3. $(C, \alpha, 1, \ldots (n-1)\, times) \in B(\mathcal{A}_k^n)$ *with the condition*

$$\sum_{N_1=1}^{\infty} \sum_{N_2=1}^{\infty} \ldots \sum_{N_n=1}^{\infty} (N_1)^{k-\alpha-1} (N_2N_3 \ldots N_n)^{k-1} \left|a_{N_1N_2\ldots N_n}\right|^k = O(1).$$

Corollary 5.4. [9]. $(C, \alpha) \in B(\mathcal{A}_k)$ *with the condition*

$$\sum_{n=1}^{\infty} n^{k-1} |a_n|^k = O(1).$$

6. Conclusion

The goal of this research is a theorem on Cesàro matrix of order $\alpha_1, \alpha_2, \ldots, \alpha_n > -1$. Based on the derivation, it can be concluded that our result is a generalized which can be reduced for several well known summabilities. Our theorem is validated by corollary 5.4, which is a result of Savaş and Şevli [9].

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