



Bicomplex I -Convergent Sequence Spaces Defined by Orlicz Functions

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Abstract: In this paper, we extend the theory of I -convergent sequence spaces defined by Orlicz functions to the bicomplex framework. By employing the idempotent representation of bicomplex numbers, we introduce new classes of bicomplex-valued sequence spaces associated with ideals. Algebraic and topological properties of these spaces are investigated, including linearity, completeness, solidity, sequence algebra and inclusion relations. The obtained results extend several known outcomes from real and complex sequence space theory to the bicomplex setting.

Keywords: Bicomplex sequences, I -convergence, Orlicz functions, sequence spaces.

1 Introduction

The concept of I -convergence extends the classical notion of convergence and has been widely investigated in the context of real and complex sequence spaces. As an extension of complex analysis, bicomplex analysis has developed into an active area of research with growing applications in functional analysis and operator theory. Motivated by these developments, we study bicomplex-valued I -convergent sequence spaces generated by Orlicz functions.

The idea was first introduced by the Italian mathematician Corrado Segre in 1892 [12]. The most detailed exploration of bicomplex numbers was later conducted by Price [8]. Additionally, Alpay et al [1] developed a broad framework of functional analysis based on bicomplex scalars.

Several other researchers, including Bera and Tripathy [2, 3], Degirmen and Sagir [4, 5], Parajuli et al [7], Rochon and Shapiro [10], Sager and Sagir [11], and Wagh [13] have studied the algebraic, geometric, and topological aspects of bicomplex sequence spaces.

A bicomplex number z is given by

$$z = (a_1 + ib_1) + j(c_1 + id_1) = z_1 + jz_2,$$

where

$$z_1 = a_1 + ib_1, \quad z_2 = c_1 + id_1, \quad a_1, b_1, c_1, d_1 \in \mathbb{R}, \quad z_1, z_2 \in \mathbb{C}.$$

The imaginary units i and j are independent satisfying the relations

$$i^2 = j^2 = -1, \quad ij = ji = k,$$

where k is a hyperbolic unit with $k^2 = 1$. The set of all bicomplex numbers is

$$\mathbb{BC} = \{z_1 + jz_2 : z_1, z_2 \in \mathbb{C}\}.$$

In \mathbb{BC} , there exist two non-trivial idempotent elements e_1 and e_2 defined by

$$e_1 = \frac{1+ij}{2}, \quad e_2 = \frac{1-ij}{2}.$$

They satisfy the relations

$$e_1 + e_2 = 1, \quad e_1 e_2 = e_2 e_1 = 0, \quad e_1^2 = e_1, \quad e_2^2 = e_2.$$

Every bicomplex number $z = z_1 + jz_2$ ($z_1, z_2 \in \mathbb{C}$) admits a unique idempotent representation as

$$z = \mu_1 e_1 + \mu_2 e_2,$$

where the components are given by

$$\mu_1 = z_1 - iz_2, \quad \mu_2 = z_1 + iz_2.$$

The Euclidean norm on \mathbb{BC} is defined by

$$\|z\|_{\mathbb{BC}} = \sqrt{a_1^2 + b_1^2 + c_1^2 + d_1^2} = \sqrt{|z_1|^2 + |z_2|^2} = \sqrt{|\mu_1|^2 + |\mu_2|^2}.$$

Let I be an ideal on \mathbb{N} and let (x_k) be a sequence in \mathbb{BC} . Then (x_k) is said to be I -convergent to $L \in \mathbb{BC}$ if for every $\varepsilon > 0$,

$$\{k \in \mathbb{N} : \|x_k - L\|_{\mathbb{BC}} \geq \varepsilon\} \in I.$$

An Orlicz function [6] is a mapping $M : [0, \infty) \rightarrow [0, \infty)$ which is continuous, non-decreasing, convex, satisfies $M(0) = 0$, $M(t) > 0$ for $t > 0$ and $M(t) \rightarrow \infty$ as $t \rightarrow \infty$.

Let $w(\mathbb{BC})$ denote the space of all bicomplex-valued sequences. Throughout this paper, M denotes an Orlicz function and I denotes a non-trivial admissible ideal of \mathbb{N} .

A bicomplex-valued sequence $x = (x_k) \in w(\mathbb{BC})$ is said to be generalized I -convergent with respect to (M, u) if for every $\varepsilon > 0$, there exist $L \in \mathbb{BC}$ and $\rho > 0$ such that

$$\left\{k \in \mathbb{N} : M\left(\frac{u_k \|x_k - L\|_{\mathbb{BC}}}{\rho}\right) \geq \varepsilon\right\} \in I,$$

where $u = (u_k)$ be a sequence of positive real numbers.

In this case, we write

$$I\text{-}\lim M\left(\frac{u_k \|x_k - L\|_{\mathbb{BC}}}{\rho}\right) = 0.$$

2 Main Results

We now define the following classes of sequence spaces by extending the results of B.C. Tripathy et al [9] in the bicomplex framework. Let $u = (u_k)$ be a sequence of positive real numbers and $x = (x_k) \in w(\mathbb{BC})$.

1. The space of generalized bicomplex I -null sequences is defined by

$$c_{I,0}^{\mathbb{BC}}(M, u) = \left\{x \in w(\mathbb{BC}) : \left\{k \in \mathbb{N} : M\left(\frac{u_k \|x_k\|_{\mathbb{BC}}}{\rho}\right) \geq \varepsilon\right\} \in I\right\}.$$

2. The space of generalized bicomplex I -convergent sequences is defined by

$$c_I^{\mathbb{BC}}(M, u) = \left\{x \in w(\mathbb{BC}) : \left\{k \in \mathbb{N} : M\left(\frac{u_k \|x_k - L\|_{\mathbb{BC}}}{\rho}\right) \geq \varepsilon\right\} \in I\right\}.$$

3. The generalized bounded bicomplex I -convergent sequence space is defined by

$$m_I^{\mathbb{BC}}(M, u) = c_I^{\mathbb{BC}}(M, u) \cap \ell_{\infty}^{\mathbb{BC}}(u),$$

where

$$\ell_{\infty}^{\mathbb{BC}}(u) = \left\{x \in w(\mathbb{BC}) : \sup_k u_k \|x_k\|_{\mathbb{BC}} < \infty\right\}.$$

4. The generalized bounded bicomplex I -null sequence space is defined by

$$m_{I,0}^{\mathbb{B}\mathbb{C}}(M, u) = c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u) \cap \ell_{\infty}^{\mathbb{B}\mathbb{C}}(u).$$

Theorem 2.1. *The spaces $c_I^{\mathbb{B}\mathbb{C}}(M, u)$, $c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$, $m_I^{\mathbb{B}\mathbb{C}}(M, u)$, and $m_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$ are linear spaces over the bicomplex field $\mathbb{B}\mathbb{C}$.*

Proof. Let $x = (x_k)$ and $y = (y_k)$ be in $c_I^{\mathbb{B}\mathbb{C}}(M, u)$ and let $\alpha, \beta \in \mathbb{B}\mathbb{C}$. Then there exist $L_1, L_2 \in \mathbb{B}\mathbb{C}$ and positive real numbers ρ_1, ρ_2 such that, for every $\varepsilon > 0$,

$$A_1(\varepsilon) = \left\{ k \in \mathbb{N} : M\left(\frac{u_k \|x_k - L_1\|_{\mathbb{B}\mathbb{C}}}{\rho_1}\right) \geq \frac{\varepsilon}{2} \right\} \in I,$$

and

$$A_2(\varepsilon) = \left\{ k \in \mathbb{N} : M\left(\frac{u_k \|y_k - L_2\|_{\mathbb{B}\mathbb{C}}}{\rho_2}\right) \geq \frac{\varepsilon}{2} \right\} \in I.$$

Let $\rho = \max\{2|\alpha|\rho_1, 2|\beta|\rho_2\}$. Using the convexity and monotonicity of the Orlicz function M ,

$$\begin{aligned} M\left(\frac{u_k \|(\alpha x_k + \beta y_k) - (\alpha L_1 + \beta L_2)\|_{\mathbb{B}\mathbb{C}}}{\rho}\right) &\leq M\left(\frac{|\alpha|u_k \|x_k - L_1\|_{\mathbb{B}\mathbb{C}}}{\rho} + \frac{|\beta|u_k \|y_k - L_2\|_{\mathbb{B}\mathbb{C}}}{\rho}\right) \\ &\leq M\left(\frac{u_k \|x_k - L_1\|_{\mathbb{B}\mathbb{C}}}{\rho_1}\right) + M\left(\frac{u_k \|y_k - L_2\|_{\mathbb{B}\mathbb{C}}}{\rho_2}\right). \end{aligned}$$

Consequently,

$$\left\{ k \in \mathbb{N} : M\left(\frac{u_k \|(\alpha x_k + \beta y_k) - (\alpha L_1 + \beta L_2)\|_{\mathbb{B}\mathbb{C}}}{\rho}\right) \geq \varepsilon \right\} \subset A_1(\varepsilon) \cup A_2(\varepsilon).$$

Since I is an ideal, $A_1(\varepsilon) \cup A_2(\varepsilon) \in I$, and hence $\alpha x + \beta y \in c_I^{\mathbb{B}\mathbb{C}}(M, u)$.

Therefore $c_I^{\mathbb{B}\mathbb{C}}(M, u)$ is a linear space over $\mathbb{B}\mathbb{C}$. The proofs for other spaces can be done using analogous arguments. \square

Theorem 2.2. *The spaces $m_I^{\mathbb{B}\mathbb{C}}(M, u)$ and $m_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$ are Banach spaces when endowed with the norm*

$$\|x\| = \inf \left\{ \rho > 0 : \sup_{k \in \mathbb{N}} M\left(\frac{u_k \|x_k\|_{\mathbb{B}\mathbb{C}}}{\rho}\right) \leq 1 \right\}.$$

Proof. We prove $m_I^{\mathbb{B}\mathbb{C}}(M, u)$ is complete.

Consider a Cauchy sequence $\{x^{(n)}\}$ in $m_I^{\mathbb{B}\mathbb{C}}(M, u)$. Then for every $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$\|x^{(n)} - x^{(m)}\|_{\mathbb{B}\mathbb{C}} < \varepsilon \quad \text{for all } n, m \geq n_0.$$

By the definition of the norm,

$$\sup_{k \in \mathbb{N}} M\left(\frac{u_k \|x_k^{(n)} - x_k^{(m)}\|_{\mathbb{B}\mathbb{C}}}{\varepsilon}\right) \leq 1, \quad \text{for all } n, m \geq n_0.$$

Fix $k \in \mathbb{N}$. Then $\{x_k^{(n)}\}$ is a Cauchy sequence in $\mathbb{B}\mathbb{C}$. Since $\mathbb{B}\mathbb{C}$ is complete,

$$\lim_{n \rightarrow \infty} x_k^{(n)} = x_k,$$

where $x_k \in \mathbb{B}\mathbb{C}$. Let us define $x = (x_k)$. We now show that $x \in m_I^{\mathbb{B}\mathbb{C}}(M, u)$.

Taking the limit as $m \rightarrow \infty$ and using the continuity of M ,

$$\sup_{k \in \mathbb{N}} M\left(\frac{u_k \|x_k^{(n)} - x_k\|_{\mathbb{B}\mathbb{C}}}{\varepsilon}\right) \leq 1 \quad \text{for all } n \geq n_0.$$

Hence $\|x^{(n)} - x\| \leq \varepsilon$ for all $n \geq n_0$, which shows that $x^{(n)} \rightarrow x$ in the norm of $m_I^{\mathbb{B}\mathbb{C}}(M, u)$.

Finally, since $m_I^{\mathbb{B}\mathbb{C}}(M, u)$ is a subspace of $\ell_{\infty}^{\mathbb{B}\mathbb{C}}(u)$ and is closed under this norm, we conclude that $x \in m_I^{\mathbb{B}\mathbb{C}}(M, u)$.

Therefore, $m_I^{\mathbb{B}\mathbb{C}}(M, u)$ is complete and hence a Banach space. The proof for $m_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$ will follow similarly. \square

Theorem 2.3. *The generalized bicomplex sequence spaces admit the following idempotent decompositions:*

$$\begin{aligned} c_I^{\mathbb{B}\mathbb{C}}(M, u) &= c_I(M, u)e_1 \oplus c_I(M, u)e_2, \\ c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u) &= c_{I,0}(M, u)e_1 \oplus c_{I,0}(M, u)e_2, \\ m_I^{\mathbb{B}\mathbb{C}}(M, u) &= m_I(M, u)e_1 \oplus m_I(M, u)e_2, \\ m_{I,0}^{\mathbb{B}\mathbb{C}}(M, u) &= m_{I,0}(M, u)e_1 \oplus m_{I,0}(M, u)e_2. \end{aligned}$$

Proof. We prove only the first result.

Let $x = (x_k) \in c_I^{\mathbb{B}\mathbb{C}}(M, u)$. Using the idempotent representation of bicomplex numbers, each term x_k can be uniquely written as

$$x_k = x_{k,1}e_1 + x_{k,2}e_2, \quad x_{k,1}, x_{k,2} \in \mathbb{C}.$$

By the definition of $c_I^{\mathbb{B}\mathbb{C}}(M, u)$, there exist $L \in \mathbb{B}\mathbb{C}$ and $\rho > 0$ such that

$$\left\{ k \in \mathbb{N} : M\left(\frac{u_k \|x_k - L\|_{\mathbb{B}\mathbb{C}}}{\rho}\right) \geq \varepsilon \right\} \in I$$

for every $\varepsilon > 0$. Let $L = L_1e_1 + L_2e_2$.

By definition of the norm, we have

$$\|x_k - L\|_{\mathbb{B}\mathbb{C}} = \sqrt{|x_{k,1} - L_1|^2 + |x_{k,2} - L_2|^2}$$

Hence,

$$M\left(\frac{u_k |x_{k,i} - L_i|}{\rho}\right) \leq M\left(\frac{u_k \|x_k - L\|_{\mathbb{B}\mathbb{C}}}{\rho}\right), \quad i = 1, 2.$$

Therefore, for each $i = 1, 2$,

$$\left\{ k \in \mathbb{N} : M\left(\frac{u_k |x_{k,i} - L_i|}{\rho}\right) \geq \varepsilon \right\} \in I,$$

which shows that $(x_{k,i}) \in c_I(M, u)$. Consequently,

$$x \in c_I(M, u)e_1 \oplus c_I(M, u)e_2.$$

Conversely, let

$$x = ye_1 + ze_2, \quad y = (y_k), z = (z_k),$$

with $y, z \in c_I(M, u)$. Then there exist $L_1, L_2 \in \mathbb{C}$ and $\rho_1, \rho_2 > 0$ such that

$$I\text{-}\lim M\left(\frac{u_k |y_k - L_1|}{\rho_1}\right) = 0, \quad I\text{-}\lim M\left(\frac{u_k |z_k - L_2|}{\rho_2}\right) = 0.$$

Let $L = L_1e_1 + L_2e_2$ and $\rho = \max\{\rho_1, \rho_2\}$. Using the monotonicity of M ,

$$M\left(\frac{u_k \|x_k - L\|_{\mathbb{B}\mathbb{C}}}{\rho}\right) \leq M\left(\frac{u_k |y_k - L_1|}{\rho_1}\right) + M\left(\frac{u_k |z_k - L_2|}{\rho_2}\right).$$

Hence,

$$\left\{ k \in \mathbb{N} : M\left(\frac{u_k \|x_k - L\|_{\mathbb{B}\mathbb{C}}}{\rho}\right) \geq \varepsilon \right\} \subset A_1(\varepsilon) \cup A_2(\varepsilon),$$

where $A_1(\varepsilon), A_2(\varepsilon) \in I$. Since I is an ideal, their union belongs to I , and thus $x \in c_I^{\mathbb{B}\mathbb{C}}(M, u)$. □

Theorem 2.4. *Let M_1 and M_2 be Orlicz functions satisfying the Δ_2 -condition. Then the following inclusion relations hold:*

1. $c_{I,0}^{\mathbb{B}\mathbb{C}}(M_2, u) \subset c_{I,0}^{\mathbb{B}\mathbb{C}}(M_1 \circ M_2, u)$,
2. $c_{I,0}^{\mathbb{B}\mathbb{C}}(M_1, u) \cap c_{I,0}^{\mathbb{B}\mathbb{C}}(M_2, u) \subset c_{I,0}^{\mathbb{B}\mathbb{C}}(M_1 + M_2, u)$.

The above inclusions also hold for the spaces $c_I^{\mathbb{B}\mathbb{C}}(M, u)$, $m_I^{\mathbb{B}\mathbb{C}}(M, u)$, and $m_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$.

Proof. (i) Let $x = (x_k) \in c_{I,0}^{\mathbb{B}\mathbb{C}}(M_2, u)$. Then there exists $\rho > 0$ such that for every $\varepsilon > 0$,

$$\left\{ k \in \mathbb{N} : M_2 \left(\frac{u_k \|x_k\|_{\mathbb{B}\mathbb{C}}}{\rho} \right) \geq \varepsilon \right\} \in I. \quad (1)$$

Since M_1 is continuous at 0 and $M_1(0) = 0$, for a given $\varepsilon > 0$ there exists $\delta > 0$ such that $M_1(t) < \varepsilon$ for $0 \leq t \leq \delta$. Let

$$y_k = M_2 \left(\frac{u_k \|x_k\|_{\mathbb{B}\mathbb{C}}}{\rho} \right).$$

For $y_k \leq \delta$, we have $M_1(y_k) < \varepsilon$. For $y_k > \delta$, using the Δ_2 -condition on M_1 , there exists a constant $K > 0$ such that

$$M_1(y_k) \leq K \frac{y_k}{\delta} M_1(\delta).$$

Hence,

$$\left\{ k \in \mathbb{N} : M_1 \left(M_2 \left(\frac{u_k \|x_k\|_{\mathbb{B}\mathbb{C}}}{\rho} \right) \right) \geq \varepsilon \right\} \subset \left\{ k \in \mathbb{N} : M_2 \left(\frac{u_k \|x_k\|_{\mathbb{B}\mathbb{C}}}{\rho} \right) \geq \delta \right\}.$$

By (1), the right-hand side belongs to I , and hence $x \in c_{I,0}^{\mathbb{B}\mathbb{C}}(M_1 \circ M_2, u)$.

(ii) Let $x = (x_k) \in c_{I,0}^{\mathbb{B}\mathbb{C}}(M_1, u) \cap c_{I,0}^{\mathbb{B}\mathbb{C}}(M_2, u)$. Then there exists $\rho > 0$ such that

$$I\text{-}\lim M_1 \left(\frac{u_k \|x_k\|_{\mathbb{B}\mathbb{C}}}{\rho} \right) = 0 \quad \text{and} \quad I\text{-}\lim M_2 \left(\frac{u_k \|x_k\|_{\mathbb{B}\mathbb{C}}}{\rho} \right) = 0.$$

Using the linearity of limits over ideals, we have

$$I\text{-}\lim (M_1 + M_2) \left(\frac{u_k \|x_k\|_{\mathbb{B}\mathbb{C}}}{\rho} \right) = 0.$$

Therefore, $x \in c_{I,0}^{\mathbb{B}\mathbb{C}}(M_1 + M_2, u)$. □

Theorem 2.5. *The sequence spaces $c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$ and $m_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$ are solid.*

Proof. Let $x = (x_k) \in c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$. Then there exists $\rho > 0$ such that

$$I\text{-}\lim M \left(\frac{u_k \|x_k\|_{\mathbb{B}\mathbb{C}}}{\rho} \right) = 0. \quad (2)$$

Let $\alpha = (\alpha_k)$ be any sequence of scalars in $\mathbb{B}\mathbb{C}$ such that $|\alpha_k| \leq 1$ for all $k \in \mathbb{N}$. Consider the sequence $(\alpha_k x_k)$.

Using the definition of the norm and the monotonicity of M ,

$$M \left(\frac{u_k \|\alpha_k x_k\|_{\mathbb{B}\mathbb{C}}}{\rho} \right) \leq M \left(\frac{u_k \|x_k\|_{\mathbb{B}\mathbb{C}}}{\rho} \right), \quad \text{for all } k \in \mathbb{N}.$$

Therefore, from (2),

$$I\text{-}\lim M \left(\frac{u_k \|\alpha_k x_k\|_{\mathbb{B}\mathbb{C}}}{\rho} \right) = 0,$$

which implies that $(\alpha_k x_k) \in c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$. Hence $c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$ is solid.

Here

$$m_{I,0}^{\mathbb{B}\mathbb{C}}(M, u) = c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u) \cap \ell_\infty^{\mathbb{B}\mathbb{C}}(u).$$

As the intersection of two solid spaces is solid, it follows that $m_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$ is also solid. We can prove the results for $m_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$ using analogous arguments. □

Theorem 2.6. *The generalized bicomplex I -null sequence space*

$$c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$$

is a sequence algebra.

Proof. Let $x = (x_k)$ and $y = (y_k)$ be in $c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$. By the idempotent representation of bicomplex numbers, we can write

$$x_k = e_1 x_{k,1} + e_2 x_{k,2}, \quad y_k = e_1 y_{k,1} + e_2 y_{k,2},$$

where $x_{k,i}, y_{k,i} \in \mathbb{C}$ for $i = 1, 2$.

Since $x \in c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$, there exists $\rho_1 > 0$ such that

$$I\text{-}\lim_{k \rightarrow \infty} M\left(\frac{u_k |x_{k,i}|}{\rho_1}\right) = 0, \quad i = 1, 2.$$

Similarly, since $y \in c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$, there exists $\rho_2 > 0$ satisfying

$$I\text{-}\lim_{k \rightarrow \infty} M\left(\frac{u_k |y_{k,i}|}{\rho_2}\right) = 0, \quad i = 1, 2.$$

Using the multiplicative property of idempotents, we can write

$$x_k y_k = e_1(x_{k,1} y_{k,1}) + e_2(x_{k,2} y_{k,2}).$$

Choose $\rho = \rho_1 \rho_2$. By the monotonicity and convexity of the Orlicz function M ,

$$M\left(\frac{u_k |x_{k,i} y_{k,i}|}{\rho}\right) \leq M\left(\frac{u_k |x_{k,i}|}{\rho_1}\right) + M\left(\frac{u_k |y_{k,i}|}{\rho_2}\right), \quad i = 1, 2.$$

Since the sum of two I -null sequences is again I -null, it follows that

$$I\text{-}\lim_{k \rightarrow \infty} M\left(\frac{u_k |x_{k,i} y_{k,i}|}{\rho}\right) = 0, \quad i = 1, 2.$$

Consequently,

$$I\text{-}\lim_{k \rightarrow \infty} M\left(\frac{u_k \|x_k y_k\|_{\mathbb{B}\mathbb{C}}}{\rho}\right) = 0,$$

which shows that $(x_k y_k) \in c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$.

Hence, the space $c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$ is a sequence algebra. □

Now we give one example to illustrate that bicomplex I -convergence is determined by the combined behaviour of its idempotent components.

Example 2.7. Let I be the density ideal given by $I_d = \{A \subset \mathbb{N} : d(A) = 0\}$, where $d(A)$ denotes the natural density. It is an admissible ideal. Let $M(t) = t^2$ for $t \geq 0$, and let $u_k = 1$ for all $k \in \mathbb{N}$. Let us define a bicomplex-valued sequence $x = (x_k)$ by

$$x_k = e_1 \frac{1}{k} + e_2 (-1)^k, \quad k \in \mathbb{N}.$$

Using the idempotent decomposition, the component sequences are

$$x_{k,1} = \frac{1}{k}, \quad x_{k,2} = (-1)^k.$$

Clearly,

$$I\text{-}\lim_{k \rightarrow \infty} M(|x_{k,1}|) = I\text{-}\lim_{k \rightarrow \infty} \frac{1}{k^2} = 0,$$

which shows that $(x_{k,1})$ is I -null.

On the other hand, the sequence $(x_{k,2})$ is bounded but does not converge in the usual sense, since it oscillates between 1 and -1 . Moreover,

$$\{k \in \mathbb{N} : |x_{k,2}| \geq \varepsilon\} = \mathbb{N} \notin I_d \quad \text{for any } 0 < \varepsilon < 1,$$

and hence $(x_{k,2})$ is not I -null.

Here the first component is I -null. Due to the divergence of the second component, the bicomplex sequence (x_k) does not belong to $c_{I,0}^{\mathbb{B}\mathbb{C}}(M, u)$. This shows that bicomplex I -convergence depends on the joint behaviour of both components. However, since both component sequences are bounded, we have

$$x \in \ell_{\infty}^{\mathbb{B}\mathbb{C}}(u).$$

3 Conclusions

In this article, we have studied generalized bicomplex I -convergent sequence spaces defined by Orlicz functions and weight sequences using the idempotent decomposition of bicomplex numbers. This approach allowed us to extend several classical results, including linearity, completeness, solidity, inclusion relations, and the sequence algebra property, to the bicomplex framework. The example illustrates that bicomplex convergence depends on both components. These results provide a useful framework for further investigations on operators and summability methods in bicomplex functional analysis.

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References

- [1] Alpay, D., Luna-Elizarrarás, M.E., Shapiro, M., & Struppa, D.C. (2014). *Basics of functional analysis with bicomplex scalars and bicomplex Schur analysis*, Springer.
- [2] Bera, S. & Tripathy, B.C. (2023). Statistical convergence in bicomplex valued metric space. *Ural Mathematical Journal*, **9(1)**: 49–63.
- [3] Bera, S. & Tripathy, B.C. (2023). Cesàro convergence of sequences of bi-complex numbers using BC-Orlicz function. *Filomat*, **37(28)**: 9769–9775.
- [4] Degirmen, N. & Sagir, B. (2014). Some new notes on the bicomplex sequence space $\ell_p(\mathbb{C}_2)$. *Journal of Fractional Calculus and Applications*, **13(2)**: 66–76.
- [5] Degirmen, N. & Sagir, B. (2022). Some geometric properties of bicomplex sequence spaces. *Konuralp Journal of Mathematics*, **10(1)**: 44–49.
- [6] Krasnoselskii, M.A., & Rutickii, Y.B. (1961) Convex functions and Orlicz spaces, *P. Noordhoff ltd, Netherland*.
- [7] Parajuli, P., Pahari, N.P., Ghimire, J.L., & Jaisawal, M.P. (2025). On some sequence spaces of bi-complex numbers. *Nepal Journal of Mathematical Sciences*, **6(1)**: 35–44.
- [8] Price, G.B. (1991). *An introduction to multicomplex spaces and functions*. Marcel Dekker.
- [9] Tripathy, B.C., & Hazarika, B. (2011). Some I -Convergent Sequence Spaces Defined by Orlicz Functions, *Acta Mathematicae Applicatae Sinica*, English Series, **27(1)**: 149–154.
- [10] Rochon, D. & Shapiro, M. (2004). On algebraic properties of bicomplex and hyperbolic numbers. *Analele Universității din Oradea, Fasc. Matematica*, **11**: 1–28.

- [11] Sager, N. & Sagir, B. (2020). On completeness of some bicomplex sequence spaces. *Palestine Journal of Mathematics*, **9(2)**: 891–902.
- [12] Segre, C. (1892). Le rappresentazioni reali delle forme complesse e gli enti iperalgebrici. *Mathematische Annalen*, **40(3)**: 413–467.
- [13] Wagh, M.A. (2014). On certain spaces of bicomplex sequences. *International Journal of Physics, Chemistry and Mathematical Fundamentals*, **7(1)**: 1–6.

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