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The Actuarial Conditions for the Valuation of Pension Liability to Become Zero Under Minimum Funding Standard Architecture

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Abstract: *Pension valuation exercises for a defined benefit scheme requires an appraisal of both the schemes assets and its liabilities in different circumstances. The valuations are required to comply with regulatory standards, most notably the minimum funding standard. The objectives of this study are: (i) to compute the estimate of minimum funding standard of pension liability (ii) to establish the actuarial condition under which minimum funding standard liability will be zero. This study used minimum funding standard models for the computations of accruing liabilities for the current and past service liability of employees. Data in respects of different categories of employees were collected over 36 employees from a going concern located in Jos South local government of Plateau State in Nigeria. The data includes the employee's annual salary and their respective demographic data which includes sex, date of birth, date of employment. This was used to determine the number of years of pensionable service completed and the future years of services to be completed before retirement at the age of 65 years. The study also used life annuity table to compute the service liability of each member of the scheme. From the model used, the result shows that the total service liability of the plan will be vanishingly zero when the newly defined annuity component approaches zero hence this represents the condition for liability of the plan to the members to be zero.*

Keywords: *Annuity, Liability, Minimum funding standard, Valuation*

1. Introduction

Pension liability defines the size of fund that a scheme sponsor has to account for to make future pension payments. In [5] (Brown, 2015) and [10] (Gang & David,2017), we observe that the pension liability shows the level of variation between the total value due to retirees and the actual value of funds the sponsor has to disburse payments. When the plan sponsor has more funds than it requires to settle future pension, then we simply have pension surplus. In a defined benefit scheme, a pension liability occurs where the sponsor cannot pay all pension benefits to the retired. The expected future pension payments for every member of the scheme is computed by applying the member's data and scheme provisions. These future benefit payments address every member's benefits and service profile, the expected time of death, expected disability or the actual retirement date. In [2] (AAA, 2004), the norm is to compute the present value of future benefits($PVFB$) by discounting the future payment from the date of payment at the moment under correct actuarial assumptions. The $PVFB$ describes the present value of all benefits expected to be paid by the sponsor to current scheme members. Under correct actuarial assumptions, the sponsor could theoretically improvise the value of fund in the scheme presently such that it would cover payments on the scheme and such that this value may take care of both future service that the scheme member is expected to earn and future incremental payments.

However, pension plan sponsor does not seem to recognize the cost of unearned future service. This is seen to be equivalent to recognizing the cost of benefits prior to being paid. In order to distinguish the cost methods, three core areas have been developed. Following [2] (AAA,2004), Actuarial liability AL is the fraction of the $PVFB$ which is assigned to past services benefits. Again in [12] (Moshe,2006), this represents the current value of benefits which has been deferred in previous years and described as the projected benefit obligation (PBO). A scheme's obligation to members prior to retirement can be measured by both accumulated benefit obligation (ABO) and project benefit obligation (PBO). The (ABO) describes the present value of benefits which members of the scheme have earned with their current service based on the current age and salaries for the interval of time they would be collecting pension benefits after they reach the normal retirement age. The (PBO) defines the present value of benefits which members of the scheme are likely to earn with their current service and the projected future salaries at the time of retirement.

In [1](Adeyele & Ogungbenle, 2019); [10](Gang & David, 2017) ; [12] (Moshe, 2006);and [15] (Winklevoss, 1993), we observe that the AL may show the expected future payment increments since various pension schemes are developed to ascertain the retirement benefit depend on the payment at retirement. It is interesting that the plan sponsor should recognize the cost of the plan progressively over the member's working life time, consequently the actuarial assumption treats the fraction of the future benefit due to past service to cover expected future payment increments.

In [2] (AAA,2004), the percentage of the $PVFB$ which solely recognizes benefits accrued to date without observing future payment increases defines the present value of accumulated benefits $PVFB$ and confirms current services together with current salary profiles. In pension funding, this is the current liability.

The fraction of the $PVFB$ which is assigned to the current year of service represents the Normal Cost NC and defines the present value of benefits which is being deferred this today. This may be equivalent to service cost (SC) for the purpose of pensions accounting. Despite other different cost methods are applied to compute the NC, it should elicit the current year of services and could also bear the expected future payments increment.

Following [2](AAA, 2004), the percentage of the $PVFB$ which is assigned to the future years of service is the Present value of future Normal Cost $PVFNC$ and it is expected to include benefits which have not been earned.

In pension valuation, the projected unit credit cost method (PUC), the traditional unit credit (UC) cost methods, Entry age normal(EAN) are commonly employed. The PUC cost method addresses the expected future payment increments in the actuarial computations of liability and normal cost by distributing benefits to member's service period. It also seems to generate progressively increasing normal costs while the UC cost method does not adopt expected future payment increments but the liability. Following [10] (Gang and David, 2017), the cost prorated technique EAN which distributes costs over the members service period seems to have high normal cost at the initial years of members service. The unit credit addresses the expected growth in payment in the normal cost for a year.

Following [3] (Blake & Orszag,1997); [5] (Brown, 2015); [6](Brynjar, 2019) and [7] (Collinson, 2001); there is a particular distinction between the actuarial measures of pensions funding status and the appropriate measure of its market value. Following [3](Blake, 1999); [10](Gang &David, 2017);and [15] (Winklevoss, 1993), it is argued that in pensions fund valuation, the long term investment institution has liabilities of the most extended duration and although these liabilities are equivalent, there tends to be computational differences. Life insurance schemes make provisions for such futures as policy loan and early surrender options in such a manner that the pension fund would not consider. It is also apparent that defined benefits schemes have integrated options structures on the invested asset in such a way that life policies do not have such structures. The valuation of pension liabilities is necessary in the event that sponsors are unable to meet the pension overheads further.

The Purpose of an Actuarial Valuation

An actuarial valuation represents an appraisal of a pension scheme financial profile serving as regular check to ensure that the trustees funding program is not adversely altered. In [4] (Brien, 2020), [8] (Dufresne, 1988a); [9] Dufresne, 1988b); and [10] (Gang & David, 2017) valuation represents the actuaries' estimate of the scheme's solvency where the liabilities are computed as the cost of purchasing annuities from a life office. We observe that the valuation exercise is carried out at a specified date (valuation date) and it is conducted on behalf of the trustees by the scheme's actuary. It is a legal requirement that actuarial valuation be conducted on all defined benefit pension schemes at least once in three years. A defined benefits scheme pay a set of retirement income benefits to members depending on their promotional salaries and years of membership. This pension promise extends to many years into the future and payments continue irrespective of how long members would survive and when a member dies. Following [13] (Owadally & Haberman; 1999) ; [14](Steven & Igbal, 2001); and [15] (Winklevoss, 1993) defined benefits schemes are usually administered by the board of trustees who are legally responsible to ensure that scheme members receive their pensions at the appropriate time.

In carrying out an actuarial valuation, assumptions are made about a range of future events such as how long scheme members will survive, probable investment return, salary and price inflation.

The Main Elements of Defined Pension Model

s :planning horizon or date of the end of the pension plan, with $0 < s < \infty$

$f(s)$: value of the fund asset at time s

$P(s)$: Benefit advanced to the member at time s

$C(s)$: Contribution rate made by the sponsor at time s

$l(s)$: Actuarial liabilities at time s that is, total liabilities of the sponsor

$n(s)$: Normal cost at time s of the fund assets

$ul(s)$: Unfunded actuarial liability at time s equal to $l(s) - f(s)$

$S(s)$: Supplementary cost at time *t*, equal to $C(s) - n(s)$

$P(x)$ Percentage of the value of the future benefit accumulated until age $x \in [R, m]$ where R defines retirement age and $m > R$

$\delta(s)$: variable rate of valuation of the liabilities which is defined by the regulatory authorities.

$r(s)$: variable risk-free market interest rate

Actuarial liabilities which are estimated every year for each pension fund can be categorized into three. (i) total accrued liability, (ii) total future liability and future liability and total liability. Total accrued liability is the liability at hand when the estimation takes place and it is dependent on the accrued benefits of members. The total future liability represents the estimated future liability provided that all current members continuously pay premiums into the fund and receive benefits in accordance with these premium payments. Total liability is the addition of both total accrued liability and total future liability.

In actuarial valuation, we define the process of comparing pension assets with liabilities so as to confirm if variations occur between the two components. The actuarial valuation exercise is conducted on both the accrued position of the fund and with the future premiums and benefits. The actuarial valuation on the accrued position considers the current asset value, $A(s)$ and the accrued liabilities $l(s)$ due to benefits that members have currently received. The total actuarial valuation considers the present value of the current member's future premiums and the liabilities that are due as a result of these future premiums. It is often assumed that all current members will continuously pay premiums into the fund up to retirement and that no new members will enter the fund in the future. Again, the consideration of death is not accounted for so that the members are all assumed to survive to retirement. The accrued position would be defined as follows.

$$AP_s = \frac{A(s)}{l(s)} - 1 \tag{1}$$

where AP_s represent the accrued actuarial position of the fund

The total actuarial position would then be

$$\overline{AP}_s = \frac{\frac{A(s)}{l(s)} + \frac{PV(\Pi)}{l(s)} - 1 - \frac{\bar{l}}{l(s)}}{1 + \frac{\bar{l}}{l(s)}} = \frac{A(s) + PV(\Pi) - l(s) - \bar{l}}{l(s) + \bar{l}} \tag{2a}$$

$$\overline{AP}_s = \frac{\frac{A(s)}{l(s)} + \frac{PV(\Pi)}{l(s)} - 1 - \frac{\bar{l}}{l(s)}}{1 + \frac{\bar{l}}{l(s)}} = \frac{(AP_s + 1) + \frac{PV(\Pi)}{l(s)} - 1 - \frac{\bar{l}}{l(s)}}{1 + \frac{\bar{l}}{l(s)}} \tag{2b}$$

where AP_s is the accrued actuarial position of the fund, $PV(\Pi)$ defines the present value of all future premium payments from the current membership and $\bar{l}$ is the liability that arises from the future premium payments Π. $A(s)$ is the current asset value and $l(s)$ is the accrued liability of the pension fund.

The present value of an immediate yearly annuity of 1 per annum to (x) is denoted by a_x and can be considered as the sum of series of pure endowment of 1 1, i.e.

$$a_x = A_{x:\overline{1}|} + A_{x:\overline{2}|} + A_{x:\overline{3}|} + \ldots + A_{x:\overline{\omega - x - 1}|} = \frac{D_{x+1}}{D_x} + \frac{D_{x+2}}{D_x} + \frac{D_{x+3}}{D_x} + \ldots + \frac{D_{\omega-1}}{D_x} \tag{3}$$

$$a_x = \frac{D_{x+1}}{D_x} + \frac{D_{x+2}}{D_x} + \frac{D_{x+3}}{D_x} + \ldots + \frac{D_{\omega-1}}{D_x} \tag{4}$$

$$a_x = \frac{1}{D_x} \sum_{t=1}^{\omega-x-1} D_{x+1} \tag{5}$$

$$\ddot{a}_x = 1 + a_x \tag{6}$$

Material and Methods:

The Minimum Funding Standard Valuation Model

In [11] (Mc Nally & Connor, 2018), the Minimum funding standard valuation is required to ascertain whether the fund satisfies the standard set by the regulatory authority. The minimum funding standard valuation is essentially designed to confirm if the scheme holds adequate assets to meet the benefits which have accrued to members at the date of the valuation. Following [11] (Mc Nally & Connor, 2018), the MFS is designed to spell out the minimum assets that a defined benefits scheme must hold and what course of action should be taken where the asset of the scheme falls below this minimum. The funding standard is satisfied if in the actuary's sense of judgment, the schemes assets on the date of valuation exceed the addition of the transfer values at that date to which the members would be entitled to and the estimated expenses of winding up the scheme. The transfer values is computed by projecting the benefit payments to which the members will earn based on their date of employment, a correct margin for mortality improvement and assuming a prescribed investment rate of return as a discount factor

The precise models used to calculate the pension fund liability under minimum funding standard valuation are setout in the models below:

MODEL 1

$$L_{MFS} = \left(\frac{S_c}{60}\right) \times \left[SAL \times (1+EI)^x\right] \times \left(\frac{1}{(1+r_{pre})^x}\right) \times \left(\frac{\left(1-\left(\frac{1}{1+AF}\right)\right)^N}{AF}\right) \times MVA \tag{7}$$

$$L_{MFS} = f(\alpha, AF) \tag{8}$$

where $\alpha = a_x$ and $\ddot{a}_x = a_x + 1$

$$f(\alpha, AF) = \begin{cases} g(\alpha) \neq 0 \; for \quad \alpha = a_x \\ g(AF) = 0 \; for \quad \alpha = AF \end{cases} \tag{9}$$

MODEL 2

$$1 + AR = 1 + \frac{d_r - p_g}{1 + p_g} \tag{10}$$

$$1 + AR = \frac{1 + p_g + d_r - p_g}{1 + p_g} \tag{11}$$

$$1 + AR = \frac{1 + d_r}{1 + p_g} \tag{12}$$

$$(1 + AR)(1 + p_g) = 1 + d_r \tag{13}$$

$$d_r = (1 + AR)(1 + p_g) - 1 \tag{14}$$

$$\frac{1}{1 + AR} = \left(\frac{1 + p_g}{1 + d_r}\right) \tag{15}$$

$$AF = \left[1 - \left(\frac{1 + p_g}{1 + d_r}\right)\right]^{\frac{N}{AR}} \tag{16}$$

MODEL 3

$$AR = \left(\frac{(1 + d_r)}{1 + P_g}\right) - 1 \tag{17}$$

$$AR = \frac{1+d_r}{1+p_g} - 1 \quad (18)$$

$$AR = \frac{1+d_r - 1 - p_g}{1+p_g} \quad (19)$$

$$AR = \frac{d_r - p_g}{1+p_g} \quad (20)$$

Theorem 1

Under the condition that

(i). $AR = \dfrac{d_r - p_g}{1+p_g}$ (21)

(ii). $AF = \left[1 - \left(\dfrac{1+p_g}{1+d_r}\right)\right]^{\frac{N}{AR}}$ (22)

where N is expectation of life at retirement . Then $L_{MFS} \to 0$

To prove this, it is sufficient to prove that

$$AF = \left[1 - \left(\frac{1+p_g}{1+d_r}\right)\right]^{\frac{N}{AR}} = 0 \quad (23)$$

Proof

Note that $\lim_{n \to \infty} a^n = 0 \; if \quad |a| < 1$ (24)

We assume $\dfrac{1+p_g}{1+d_r} < 1$ (25)

Again since $N \in \mathbf{Z}^+$, $AR \in \mathbf{R}$ and specifically $AR < 1$, then $\dfrac{N}{AR} \to \infty$

Consequently $1 - \left(\dfrac{1+p_g}{1+d_r}\right) < 1$ and

$$1 - \left(\frac{1+p_g}{1+d_r}\right)^{\frac{N}{AR}} \to 0 \text{ hence } L_{MFS} = 0 \quad (26)$$

Q.E.D

Variable Description

Variable	**Description**
L_{MFS}	Total service liability
S_c	Numbers of years of pensionable service completed to date.
SAL	Current salary.
EI	Expected rate of inflation.
X	Numbers of years to retirement
r_{pre}	Pre-retirement discount rate

d_r	Post-retirement discount rate
N	Expected lifespan post-retirement
AF	Annuity factor
AR	Annuity rate
MVA	market value adjustment
P_g	Pension Increase.

Based on the current events in our local financial setting, we assume the following values of the parameter are shown below for the purpose of this work

N Life expectancy after retirement 10 years

P_g Pension increase from 1% to 4.5%

EI Expected rate of inflation 17.93%

d_r Pre-retirement discount rate 7.75%

d_r Post retirement discount rate 4.5%

MVA Market value adjustment 1.42

The market value adjustment (MVA) in equation (7) is an instrument which life offices and annuity issuer's use to manage risk. A market value adjustment could be favourable when life annuity is surrendered at the time when interest rates are low. Before an annuity is surrendered, it is necessary to ensure the terms of the contract is well understood or if an MVA or other penalties fees will come in. The interest rate trajectories should be well monitored since the timing matters for market value adjustments. The MVA of fixed annuities are more common and preferred by consumers since they offer bigger rates than the book value fixed annuities and many fixed annuity consumers buy with the aim that they would not surrender their contracts. In a continuously increasing interest rate regime, the MVA provides interest rate shield to the life office by charging the insured supplementary fees to surrender which reimburses them for having to sell assets at a loss. The supplementary charge would mitigate any potential benefit of higher rates a scheme holder would find somewhere else. Consequently, the protection against downside interest rates which the MVA fixed annuity offers permits the life office to experience higher rates because they are protected against an influx of liquidation requests in the event interest rates rise.

Method of Data Presentation and Analysis

The analysis evaluates the salary data of 36 employees from the human resource department of a manufacturing firm in Jos south local government area of plateau state. The salary data was then validated against error for each staff. Along the salary data, the demographic data of each employee were also collected to enable us carry out full computation of the service liability of the employees. The data was analyzed using minimum funding standard model in R–language. The findings were depicted in graphs and tabular form for easy access of the numerical values.

Table 1: Table of Liability L_{mfs}

PCN	A	B	C	D	E
001	0.183333	19609024.45	0.143599	0.032231	23626.91
002	0.183333	39267584.02	0.123685	0.031828	40243.50
003	0.183333	10980071.93	0.193562	0.033127	18328.80
004	0.183333	19285178.70	0.154728	0.032444	25202.83
005	0.183333	21432308.33	0.143599	0.032231	25823.79
006	0.066667	71765467.33	0.079034	0.030795	16534.91

007	0.15	32387973.38	0.114789	0.031638	25053.64
008	0.133333	23284038.12	0.13327	0.032026	18815.77
009	0.1	84630239.97	0.073349	0.030647	27014.22
010	0.15	49957759.07	0.091759	0.031111	30376.62
011	0.15	102043586.50	0.063177	0.03037	41703.17
012	0.1	60246037.80	0.079034	0.030795	20821.21
013	0.083333	124680871.40	0.058633	0.030241	26160.51
014	0.166667	31659412.17	0.073349	0.030647	16842.96
015	0.133333	71433030.14	0.068073	0.030505	28085.12
016	0.15	19401225.70	0.123685	0.031828	16268.24
017	0.15	11829224.45	0.154728	0.032444	12648.29
018	0.033333	14582864.86	0.123685	0.031828	2717.33
019	0.05	10899703.09	0.154728	0.032444	3884.80
020	0.15	71262730.08	0.063177	0.03037	29123.65
021	0.133333	7477340.13	0.166719	0.032663	7709.42
022	0.1	138802877.00	0.058633	0.030241	34948.30
023	0.15	9917670.86	0.193562	0.033127	13545.29
024	0.166667	7409496.71	0.154728	0.035245	9562.89
025	0.55	3808157.15	0.550379	0.036958	60497.28
026	0.466667	17228885.76	0.260909	0.034139	101692.97
027	0.466667	6404885.94	0.408312	0.035819	62074.72
028	0.483333	2932108.68	0.474053	0.036395	34720.13
029	0.333333	14220726.93	0.260909	0.034139	59955.29
030	0.483333	5384018.51	0.408312	0.035819	54044.30
031	0.283333	21077457.34	0.224727	0.03362	64069.31
032	0.283333	17842067.97	0.224727	0.03362	54234.67
033	0.25	47994445.74	0.143599	0.032231	78857.06
034	0.133333	398557513.20	0.050502	0.030001	114330.49
035	0.15	140564564.70	0.079034	0.030795	72869.30
036	0.1	60867941.71	0.114789	0.031638	31389.50
Total	**7.216663**	**1791128490**	**5.987537**	**1.166**	**1303777.19**

Source: Author's computation

In table 1 above we have computed the liabilities of the plan for each member. The total service liability of the plan was computed using minimum funding standard model. In column E, the total liability of the plan is **1303777.19** based on the parameters we have set below.
where PCN is the permanent code number

$$A = \frac{S_c}{60} \tag{27}$$

$$B = SAL \times (1 + EI)^X \tag{28}$$

$$C = \frac{1}{(1 + r_{pre})^X} = (1 + r_{pre})^{-X} \tag{29}$$

$$D = \left[\frac{\left(1 - \left(\frac{1}{1 + AF}\right)\right)^N}{AF} \right] MVA \tag{30}$$

$$E=\frac{M}{60}\times\left(CS\times\left(1+EI\right)^{X}\right)\times\left[\frac{1}{\left(1+r_{pre}\right)^{X}}\right]\times\left[\frac{\left(1-\left(\frac{1}{1+AF}\right)\right)^{N}}{AF}\right]MVA \qquad (31)$$

Table 2 : Table of Service Years Completed.

PCN	F	G	X	I
001	269292.5	11	39	26
002	387751.8	11	37	28
003	291655.8	11	43	22
004	312331.8	11	40	25
005	294331.8	11	39	26
006	263444.6	4	31	34
007	271193.4	9	36	29
008	271145.4	8	38	27
009	263436	6	30	35
010	255050	9	33	32
011	228395	9	28	37
012	221157.8	6	31	34
013	236633.6	5	27	38
014	98549.04	10	30	35
015	188549	8	29	36
016	191579.4	9	37	28
017	191579.4	9	40	25
018	144000	2	37	28
019	176525.4	3	40	25
020	159501	9	28	37
021	142811.8	8	41	24
022	263436	6	27	38
023	263436	9	43	22
024	120000	10	40	25
025	1017936	33	57	8
026	885156	28	47	18
027	885156	28	53	12
028	563556	29	55	10
029	730608	20	47	18
030	744072	29	53	12
031	778632	17	45	20
032	659112	17	45	20
033	659112	15	39	26
034	543900	8	25	40
035	516000	9	31	34
036	509664	6	36	29
Total	**13998690.54**			

Source: Author's computation

Table 2 above shows the computations of years of service completed the current age and the number of future service years. The annual salary of each member was also computed and the total annual salary of the plan is **13998690.54** based on the parameter set below.
Where; PCN = permanent code number.
F = (monthly salary)× 12
$G = S_C$ = current age –entry age
x = current year – year of birth
I = retirement age – current age

Table 3 : Table of Ordinary Annuity

PCN	J	K	L
001	0.01	0.034653	0
002	0.011	0.03363	0
003	0.012	0.032609	0
004	0.013	0.031589	0
005	0.014	0.030572	0
006	0.015	0.029557	0
007	0.016	0.028543	0
008	0.017	0.027532	0
009	0.018	0.026523	0
010	0.019	0.025515	0
011	0.02	0.02451	0
012	0.021	0.023506	0
013	0.022	0.022505	0
014	0.023	0.021505	0
015	0.024	0.020508	0
016	0.025	0.019512	0
017	0.026	0.018519	0
018	0.027	0.017527	0
019	0.028	0.016537	0
020	0.029	0.015549	0
021	0.03	0.014563	0
022	0.031	0.013579	0
023	0.032	0.012597	0
024	0.033	0.011617	0
025	0.034	0.010638	0
026	0.035	0.009662	0
027	0.036	0.008687	0
028	0.037	0.007715	0
029	0.038	0.006744	0
030	0.039	0.005775	0
031	0.04	0.004808	0
032	0.041	0.003842	0
033	0.042	0.002879	0
034	0.043	0.001918	0
035	0.044	0.000958	0
036	0.045	0	0
Total	**0.99**	**0.616883**	**0**

Source: Author’s computation

In table 3 a sensitivity analysis was carried out between annuity rate and annuity factor, it is apparent that annuity factor is zero. This was proved in theorem 1 above and the total value of annuity rate of the plan is **0.616883** based on the parameter set below.

PCN is the permanent code number

$$J = \frac{P_g}{100} \tag{32}$$

$$K \Rightarrow AR = \frac{d_r - p_g}{1 + p_g} \tag{33}$$

$$L \Rightarrow AF = \left[1 - \left(\frac{1 + p_g}{1 + d_r}\right)\right]^{\frac{N}{AR}} \tag{34}$$

Table 4 : Table of Life Annuity.

PCN	X	M	N
001	39	19.144	18.144
002	37	19.621	18.621
003	43	18.1	17.1
004	40	18.894	17.894
005	39	19.144	18.144
006	31	20.877	19.877
007	36	19.849	18.849
008	38	19.386	18.386
009	30	21.061	20.061
010	33	20.488	19.488
011	28	21.408	20.408
012	31	20.877	19.877
013	27	21.571	20.571
014	30	21.061	20.061
015	29	21.238	20.238
016	37	19.621	18.621
017	40	18.894	17.894
018	37	19.621	18.621
019	40	18.894	17.894
020	28	21.408	20.408
021	41	18.637	17.637
022	27	21.571	20.571
023	43	18.1	17.1
024	40	15.678	14.678
025	57	13.626	12.626
026	47	16.941	15.941
027	53	15.012	14.012
028	55	14.327	13.327
029	47	16.941	15.941
030	53	15.012	14.012
031	45	17.534	16.534
032	45	17.534	16.534
033	39	19.144	18.144
034	25	21.877	20.877
035	31	20.877	19.877
036	36	19.849	18.849

Source: Neill A. (1977)

Table 4 above shows the computation of life annuity due and annuity immediate and for each member of the scheme based on their respective age according to the parameters below.
where;
PCN is the Permanent code number.
x is the current year – year of birth

$$M = a_x + 1 = \ddot{a}_x \tag{35}$$

$$N = a_x = \ddot{a}_x - 1 \tag{36}$$

GRAPHS

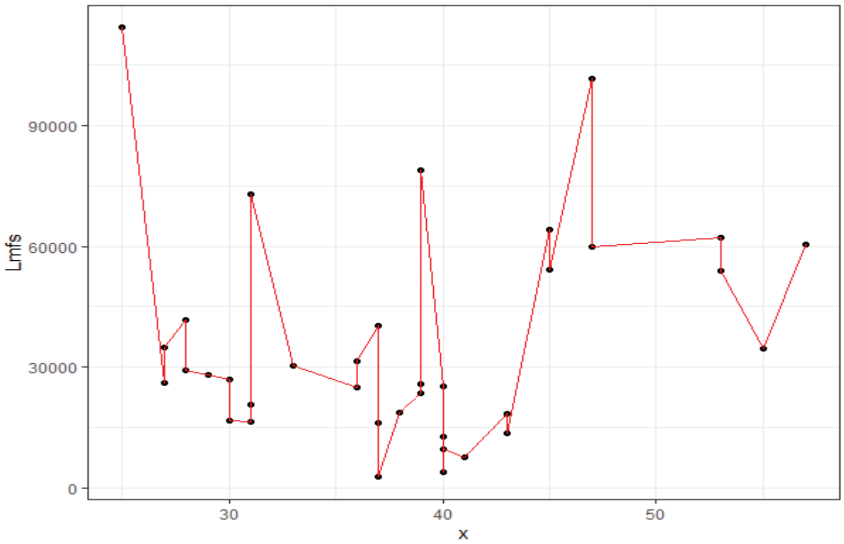

Figure 1: Graph of L_{mfs} against Age

Figure 1 above shows the graph of total service liability (L_{mfs}) against their respective ages (x) for each of the member in the scheme.

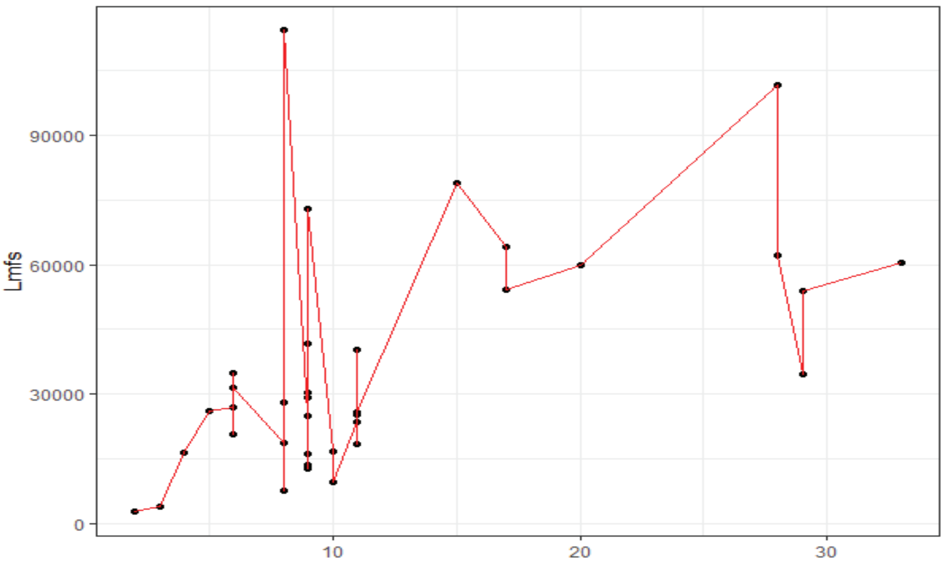

Figure 2: Graph of L_{mfs} against S_c

Figure 2 above shows the graph of total service liability L_{mfs} against the number of pensionable service completed (S_c) to date by each member of the scheme.

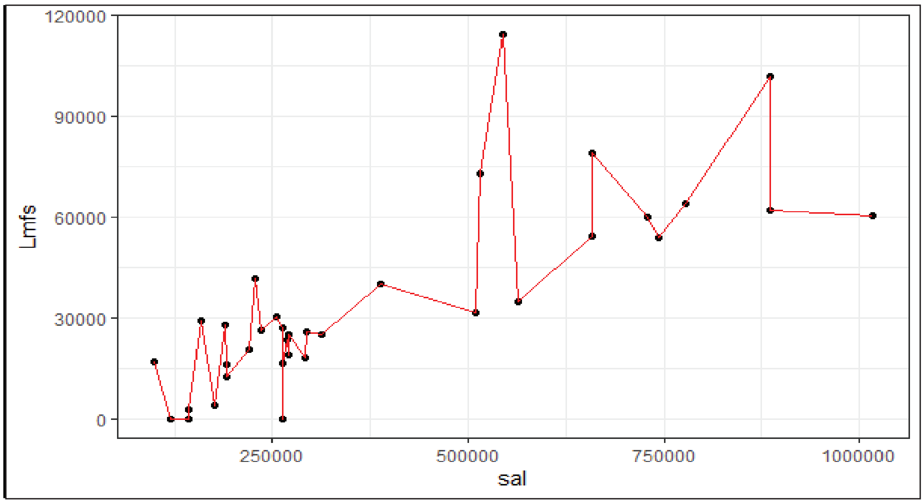

Figure 3 : Graph of L_{mfs} against Salary

Figure 3 above shows the graph of total service liability (L_{mfs}) against their respective salary (sal) of each member of the scheme.

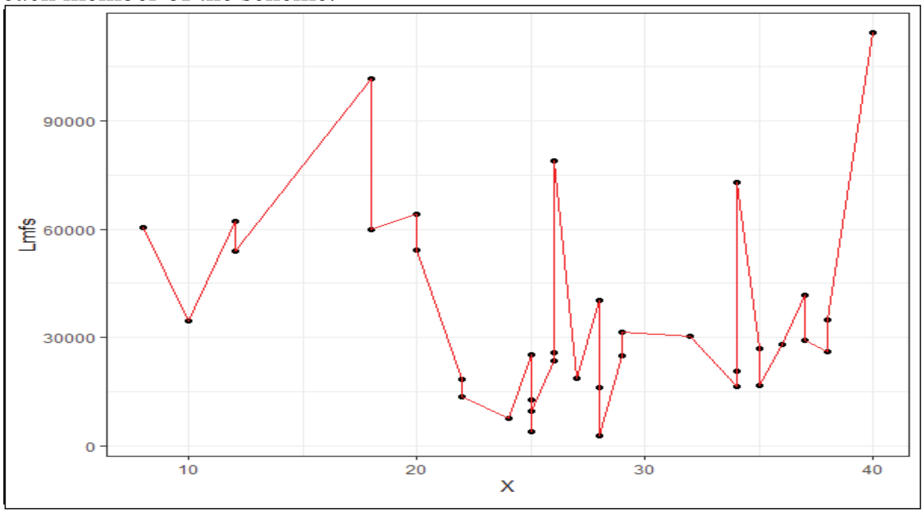

Figure 4 : Graph of L_{mfs} against Future Service

Figure 4 shows he graph of total service liability (L_{mfs}) against the future years of service X to be completed.

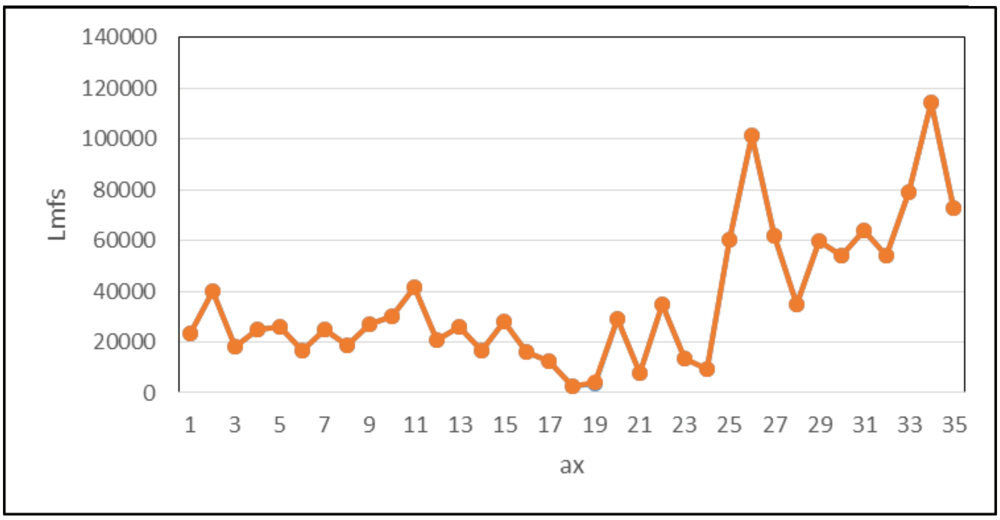

Figure 5: Graph of L_{mfs} against Annuity a_x

Figure 5 shows the graph of total service liability (L_{mfs}) against life annuity a_x

Discussion of Results

Theorem 2. Main Result

Given that

$$f(\alpha,\zeta)=\begin{cases} L_{MFS}(\alpha)\neq 0\ for \quad \alpha = a_x \\ L_{MFS}(\zeta)=0\ for \quad \zeta = AF \end{cases} \tag{37a}$$

$$L_{MFS}=\left(\frac{S_c}{60}\right)\times\left[SAL\times(1+EI)^x\right]\times\left(\frac{1}{(1+r_{pre})^x}\right)\times\left(\frac{\left(1-\left(\frac{1}{1+\alpha}\right)\right)^N}{\alpha}\right)\times MVA \tag{37}$$

$$L_{MFS}=\left(\frac{S_c}{60}\right)\times\left[SAL\times(1+EI)^x\right]\times\left(\frac{1}{(1+r_{pre})^x}\right)\times\left(\frac{\left(1-\left(\frac{1}{1+\zeta}\right)\right)^N}{\zeta}\right)\times MVA \tag{38}$$

In particular, if we define

$$\zeta = AF = \left[1-\left(\frac{1+p_g}{1+d_r}\right)\right]^{\frac{N}{AR}} \tag{39}$$

Then

AF = 0 and $L_{MFS} = 0$

Proof:

Under the condition that

(i) $AR=\dfrac{d_r-p_g}{1+p_g}$ (40)

(ii) $AF=\left[1-\left(\dfrac{1+p_g}{1+d_r}\right)\right]^{\frac{N}{AR}}$ (41)

N is expectation of life at retirement then $L_{MFS}\to 0$

To prove this, it is sufficient to prove that

$$AF=\left[1-\left(\frac{1+p_g}{1+d_r}\right)\right]^{\frac{N}{AR}}=0 \tag{42}$$

Note that $\lim_{n \to \infty} a^n = 0 \; if \quad |a| < 1$ (43)

We assume $\dfrac{1+p_g}{1+d_r} < 1$ (44)

Again since $N \in Z^+$, $AR \in R$ and specifically $AR < 1$, then $\dfrac{N}{AR} \to \infty$ (45)

Consequently $1 - \left(\dfrac{1+p_g}{1+d_r}\right) < 1$ and (46)

$$1 - \left(\frac{1+p_g}{1+d_r}\right)^{\frac{N}{AR}} \to 0 \tag{47}$$

and $\left(\dfrac{\left(1-\left(\dfrac{1}{1+AF}\right)\right)^N}{AF}\right) = 0$ (48)

hence $L_{MFS} = 0$ (49)

If we substitute $AF = 0$ into L_{MFS} then

$$L_{MFS} = \left(\frac{S_c}{60}\right) \times MVA \times \left[SAL \times (1+EI)^x\right] \times \left(\frac{1}{(1+r_{pre})^x}\right) \times \left(\frac{\left(1-\left(\frac{1}{1+AF}\right)\right)^N}{AF}\right) = 0 \tag{50}$$

If $\alpha = AF$ and given the size of the fund, we obtain

$$ul(s) = l(s) - f(s) \tag{51}$$

$$ul(s) = -f(s) \Rightarrow ul(s) + f(s) = 0 \tag{52}$$

If $\alpha = a_x$ (53)

$$L_{MFS} = \left(\frac{S_c}{60}\right) \times MVA \times \left[SAL \times (1+EI)^x\right] \times \left(\frac{1}{(1+r_{pre})^x}\right) \times \left(\frac{\left(1-\left(\frac{1}{1+a_x}\right)\right)^N}{a_x}\right) \tag{54}$$

$$L_{MFS} = \left(\frac{S_c}{60}\right) \times MVA \times \left[SAL \times (1+EI)^x\right] \times \left(\frac{1}{(1+r_{pre})^x}\right) \times \left(\frac{\left(1-\frac{1}{\ddot{a}_x}\right)^N}{a_x}\right) \tag{55}$$

$$L_{MFS} = \left(\frac{S_c}{60}\right) \times MVA \times \left[SAL \times (1+EI)^x\right] \times \left(\frac{1}{(1+r_{pre})^x}\right) \times \left(\frac{\left(\frac{\ddot{a}_x - 1}{\ddot{a}_x}\right)^N}{a_x}\right) \tag{56}$$

$$L_{MFS} = \left(\frac{S_c}{60}\right) \times MVA \times \left[SAL \times (1+EI)^x\right] \times \left(\frac{1}{(1+r_{pre})^x}\right) \times \left(\frac{\left(\frac{a_x}{\ddot{a}_x}\right)^N}{a_x}\right) \tag{57}$$

$$L_{MFS} = \left(\frac{S_c}{60}\right) \times MVA \times \left[SAL \times (1+EI)^x\right] \times \left(\frac{1}{(1+r_{pre})^x}\right) \times \frac{(a_x)^{N-1}}{\left(\ddot{a}_x\right)^N} \neq 0 \tag{58}$$

This is confirmed in column E of table 1 and consequently we obtain

$$ul(s) + f(s) = \left(\frac{S_c}{60}\right) \times MVA \times \left[SAL \times (1+EI)^x\right] \times \left(\frac{1}{(1+r_{pre})^x}\right) \times$$

$$\left(\frac{\left(1 - \left(\frac{1}{1+AF}\right)\right)^N}{AF}\right) = \mathbf{1303777.19} \tag{59}$$

Q.E.D

In table 1, column A shows the pension accrual of each member of the scheme which is the number of pensionable years of service completed divided by 60. The total pension accrual of the plan is $\mathbf{7.216663}$ .column B shows the projection of the annual salary figure of each member as compared to the expected rate of inflation in the country. This also depends on the number of years to retirement of each member of the scheme, the total value of the salary projection is $\mathbf{1,791,128,490}$. Column C shows the pre-retirement discount rate of each member of the scheme and the total value is $\mathbf{5.987537}$. Column D shows the computation of annuity factor based on the respective age of each member based on the number of years that each member is expected to survive. The number of years that each member is expected to survive after retirement is assumed to be 10 years. This also includes the market value adjustment (MVA). The total value is $\mathbf{1.166}$. Column E shows the total service liability of each member of the scheme. The computation was based on minimum funding standard model and the total value is $\mathbf{1303777.19}$

Therefore, the projected annual salary of $\mathbf{1791128490}$ is by far higher than the total service liability of $\mathbf{1303777.19}$. The pension accrual of $\mathbf{7.216663}$ is also higher than the discount rate pre-retirement.

In table 2 column F shows the annual salary of each member of the scheme, the total annual salary is **13998690.54**. Column G shows the number of pensionable service completed to date by each member of the scheme. The minimum is 2 years while the maximum is 33 years. Column X shows the current age of the members of the scheme, the younger member has the age of 25 years while the older member has the age of 57 years. Column I shows the number of years that each member still have to serve. The lowest age a member still has to serve is 8 years while the highest age is 40 years.

In table 3 column J shows the pension growth rate from 1.0% to 4.5% with the total pension growth rate of 0.99. Column K shows the computation of annuity rate which is based on post retirement discount rate d_r and the pension growth rate (P_g). The total value is 0.616883. Column L shows the computation of the annuity factor and the total is 0. The Column L confirms our theorem 2 forming the basis of this paper.

In table 4 column X shows the respective age of the members. Column M shows the life annuity due of each member according to their respective age. Column N shows the life annuity immediate of each member according to their respective age.

The distribution of liabilities in the figures $1-5$ are not evenly distributed. In figure 1, the total service liability of the plan was plotted against current age of the members. The graph shows that majority of the plan are clustered around ages 30 and 45 years while those that are above the age of 45 are dispersed. In figure 2, the total service liability of the plan was plotted against the number of pensionable years completed to date. The graph shows that high number of pensionable years of service completed are clustered between 2 years and 11 years while those that have completed more 11 years of service are dispersed. In figure 3, the total service liability of the plan was plotted against salary of each member of the scheme. The graph shows that the annual salary of the scheme members are clustered between 120000 to 257000 and those above 257000 are dispersed. In figure 4, the total service liability was plotted against future years of service. The graph shows that the years of future service of the members are dispersed below 20 years while those above 25 and 35 years of future service are clustered. In figure 5, the total service liability was plotted against life annuity. The graph shows that the liability of each member is unevenly distributed over the life annuity from 1 to 35 units.

Conclusion

Retirement plan funding anticipates that over long term, both contribution rate of administrative expenses and investment earnings less investment fees will be needed to cover benefit payments. Retirement plan asset changes as a result of the net impact of these income and expenses component. It is necessary to lower level and predictable plan cost from year to year. For this reason, the valuation method considered market value adjustment. The objective of this study is to determine the total service liability of employees in the plan using MFS valuation model. The MFS valuation computes the lump sum required to satisfy the future pension benefits of the scheme's member based on the completed years of pensionable service to date and their current salary. Future pension benefits is computed by taking the expected annual pension benefits in the year of retirement and multiplying it by an annuity factor to reflect the expected lifespan of the member after retirement. The lump sum computed is then discounted back to the present by considering the pre and post retirement investment growth rates

In this study, we have computed the total service liability of the plan based on the following key parameters; pension accrual, annual salary, expected rate of inflation, number of years to retirement, discount rate before retirement and the market value adjustment. In order that we establish the condition that $L_{MFS} \to 0$, the function AF must be defined as

$$AF = \left[1 - \left(\frac{1+p_g}{1+d_r}\right)\right]^{\frac{N}{AR}} \tag{60}$$

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