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New Modified Newton Type Iterative Methods

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Abstract: *In this work, we present two Newton type iterative methods for finding the solution of nonlinear equations of single variable. One is obtained as variant of McDougall and Wotherspoon method, and another is obtained by amalgamation of Potra and Pta'k method and our newly introduced method. The order of convergence of these methods are* $1+\sqrt{2}$ *and* 3.5615*. Some numerical examples are given to compare the performance of these methods with some similar existing methods.*

Keywords: *Newton's method, Nonlinear equations, Convergence order, Iterative methods.*

1. Introduction

Finding the solution of a single variable nonlinear equations is one of the most important tasks in numerical analysis. Nonlinear equations are frequently encountered in all fields of science and engineering. Most of the cases, it is impossible to solve such equations analytically. In those conditions, when an analytic solution cannot be obtained or the process of finding it is tedious, numerical methods are employed to get the approximate solutions. Main goal of numerical methods is to find the solution of given problem in allowed tolerance level. The construction of iterative method has been attracted the attention of mathematicians for more than four centuries. Because of the advent of different verities of computers and simulation software, the demand for numerical methods are increased rapidly in the applications to engineering and scientific fields. So during last two decades, large numbers of mathematicians are devoted to develop new numerical methods for solving nonlinear equations. One of the widely used and best known iterative methods for solving nonlinear equations is the Newton method. The iterative formula for Newton method to solve nonlinear equations $f(x)=0$ is given by [1]

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)} \tag{1}$$

This method converges quadratically to simple zero and need to evaluate two functions per iteration. Now days tremendous variant of this method have appeared, some of them are found in the papers [2] -[13] . In [9], McDougall and Wotherspoon proposed a new variant of Newton method using a different technique. Their method's iterative scheme is as follows:

If x_0 is the initial approximation, then

$$x_0^* = x_0 \tag{2a}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'\left(\frac{x_0+x_0^*}{2}\right)} = x_0 - \frac{f(x_0)}{f'(x_0)}. \tag{2b}$$

Subsequently for $n \geq 1$, the iteration can be obtained as

$$x_n^* = x_n - \frac{f(x_n)}{f'\left(\frac{x_{n-1}+x_{n-1}^*}{2}\right)} \tag{2c}$$

$$x_n = x_n - \frac{f(x_n)}{f'\left(\frac{x_n+x_n^*}{2}\right)}. \tag{2d}$$

This method is a predictor–corrector type method with convergence order $1+\sqrt{2}$. In this method, we have to evaluate two functions per iteration as in Newton's method but we calculate the derivative at some convenient point instead of previously iterated point.

2. The Methods

Here, we suggest the following method as a variant of method (2a)-(2d) by using harmonic mean instead of arithmetic mean. If x_0 is the initial approximation, then

$$x_0^* = x_0 \tag{3a}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'\left(\frac{2x_0x_0^*}{x_0+x_0^*}\right)} = x_0 - \frac{f(x_0)}{f'(x_0)}. \tag{3b}$$

Subsequently for $n \geq 1$, the iteration can be obtained as

$$x_n^* = x_n - \frac{f(x_n)}{f'\left(\frac{2x_{n-1}\,x_{n-1}^*}{x_{n-1}+x_{n-1}^*}\right)} \tag{3c}$$

$$x_n = x_n - \frac{f(x_n)}{f'\left(\frac{2x_nx_n^*}{x_n+x_n^*}\right)}. \tag{3d}$$

The third order method obtained by Potra and Pta'k in [11] is given by

$$x_{n+1} = x_n - \frac{f\left(x_n - \frac{f(x_n)}{f'(x_n)}\right) + f(x_n)}{f'(x_n)}. \tag{4}$$

Here, we suggest variant of method (4) whose iterative scheme is as follows:
If x_0 is the initial approximation, then

$$x_0^* = x_0 \tag{5a}$$

$$x_1 = x_0 - \frac{f\left[x_0 - \frac{f(x_0)}{f'\left(\frac{2x_0x_0^*}{x_0+x_0^*}\right)}\right] + f(0)}{f'\left(\frac{2x_0x_0^*}{x_0+x_0^*}\right)} = x_0 - \frac{f\left[x_0 - \frac{f(x_0)}{f'(x_0)}\right] + f(x_0)}{f'(x_0)} \tag{5b}$$

Subsequently, for $n \geq 1$, the iterations can be obtained as below:

$$x_n^* = x_n - \frac{f\left[x_n - \frac{f(x_n)}{f'\left(\frac{2x_{n-1}x_{n-1}^*}{x_{n-1}+x_{n-1}^*}\right)}\right] + f(x_n)}{f'\left(\frac{2x_{n-1}x_{n-1}^*}{x_{n-1}+x_{n-1}^*}\right)} \tag{5c}$$

$$= x_n - \frac{f\left[x_n - \frac{f(x_n)}{f'\left(\frac{2x_nx_n^*}{x_n+x_n^*}\right)}\right] + f(x_n)}{f'\left(\frac{2x_nx_n^*}{x_n+x_n^*}\right)} \tag{5d}$$

3. Convergence Analysis

For the convergence of the method (3a)-(3d), we prove the following results.

Theorem 3.1 *Let* α *be a simple zero of a function* f *which has sufficient number of smooth derivatives in a neighborhood of* α. *Then for solving the nonlinear equation* $f(x) = 0$, *the method* (3a)-(3d) *is convergent with order of convergence* $1+\sqrt{2}$.

Proof:

Suppose e_n^* and e_n denote the errors in the iterates x_n and x_n^* respectively and

$c_k = \frac{f^k(\alpha)}{k!\,f'(\alpha)}$, $k = 2,3,4$ .... Then the error equation of the Newton's method (1) is given by [1]

$$e_{n+1} = c_2\, e_n^2 \tag{6}$$

where the terms involving higher power of e_n are ignored. Next, we proceed the analysis of convergence of method (3a)-(3d). Clearly, $e_0^* = e_0$ and from view (6), the error equation for (3b) is given by

$$e_1 = c_2 e_0^2 \tag{7}$$

Using this, the Taylor series expansion, and binomial expansion, the error in x_1^* is given by

$$e_1^* = e_1 - \frac{f(\alpha+e_1)}{f'(\alpha+e_0)} = e_1 - \frac{e_1+c_2e_1^2+c_3e_1^3+O(e_0^4)}{1+2c_2e_0+3c_3e_0^2+O(e_0^3)}$$

$$= e_1 - (e_1 + c_2e_1^2 + c_3e_1^3 + O(e_0^4))(1 - 2c_2e_0 - 3c_3e_0^2 - 4c_2^2e_0^2 + O(e_0^3))$$

$$= 2c_2e_0e_1$$

$$= 2c_2^2e_0^3 \tag{8}$$

where the terms involving higher power of e_0 are neglected.

Next,

$$\frac{2x_1x_1^*}{x_1+x_1^*} = \frac{2(\alpha+e_1)(\alpha+e_1^*)}{(\alpha+e_1)+(\alpha+e_1^*)} = \left(\alpha+e_1+e_1^*+\frac{e_1e_1^*}{\alpha}\right)\left(1+\frac{e_1+e_1^*}{2\alpha}\right)^{-1} = \alpha+\frac{e_1+e_1^*}{2},$$

where the terms involving higher powers of e_1 and e_1^* are neglected. Therefore, the error equation for (3d) with $n = 1$ and using (7) and (8) is given by

$$e_2 = e_1 - \frac{f(\alpha+e_1)}{f'(\alpha+\frac{e_1+e_1^*}{\alpha})} = e_1 - (e_1 + c_2e_1^2 + c_3e_1^3)\left[1+2c_2\left(\frac{e_1+e_1^*}{2}\right)+3c_3\left(\frac{e_1+e_1^*}{2}\right)^2\right]^{-1}$$

$$= c_2e_1e_1^* = 2c_2^4e_0^5 \tag{9}$$

In general, it can be shown that for $n \geq 2$, the errors in x_n^* and x_n can be obtained recursively by the relations

$$e_n^* = c_2e_ne_{n-1} \tag{10}$$

and

$$e_{n+1} = c_2\, e_n\, e_n^* \tag{11}$$

In order to find the order of convergence of the method, we need a relation of the form

$$e_{n+1} = Ke_n^p \tag{12}$$

where *K* is some constant. Thus,

$$e_n = Ke_{n-1}^p \quad \text{or} \quad e_{n-1} = K^{-\frac{1}{p}}e_n^{\frac{1}{p}} \tag{13}$$

From (10), (11), (12) and (13), we have

$$Ke_n^p = Kc_2\, e_ne_n^* = Kc_2e_n\, c_2e_ne_{n-1} = K^{1-\frac{1}{p}}c_2^2\, e_n^{2+\frac{1}{p}}$$

Equating the power of e_n,

$$p = 2+\frac{1}{p}$$

$$\therefore\ p = 1+\sqrt{2}$$

Hence the method (3a)-(3d) is convergent with order of convergence $1+\sqrt{2}$.

Theorem 3.2 ***Let* α *be a simple zero of a function* f *which has sufficient number of smooth derivatives in a neighborhood of* α *. Then for solving the nonlinear equation* $f(x) = 0$, *the method*** (5a)-(5d) ***is convergent with order of convergence*** 3.5615

Proof:

Let e_n and e_n^* denote, respectively, the errors in x_n and x_n^*. Also, we denote $c_k = \frac{f^k(\alpha)}{k!\,f'(\alpha)}$, $k =$ 2,3,4, ...,which are constants. Then from (5a), $x_0^* = x_0$ implies $e_0^* = e_0$. Next, we proceed to calculate the error e_1 in x_1. By using Taylor series expansion and binomial expansion, we get

$$x_0 - \frac{f(x_0)}{f'(x_0)} = \alpha + e_0 - \frac{f(\alpha+e_0)}{f'(\alpha+e_0)} = \alpha + e_0 - \frac{e_0+c_2e_0^2+c_3e_0^3+O(e_0^4)}{1+2c_2e_0+3c_3e_0^2+O(e_0^3)}$$

$$= \alpha + e_0 - [e_0 + c_2e_0^2 + c_3e_0^3 + O(e_0^4)]\,[1 + 2c_2e_0 + 3c_3e_0^2 + O(e_0^3)]^{-1}$$

$$= \alpha + c_2e_0^2 + (2c_3 - 2c_2^2)e_0^3 + O(e_0^4),$$

So that after some calculations, we get

$$f\left(x_0 - \frac{f(x_0)}{f'(x_0)}\right) = f'(\alpha)[c_2e_0^2 + (2c_3 - 2c_2^2)e_0^3 + c_2^3e_0^4 + O(e_0^5)],$$

$$f\left(x_0 - \frac{f(x_0)}{f'(x_0)}\right) + f(x_0) = f'(\alpha)[e_0 + 2c_2e_0^2 + 3c_3e_0^3 - 2c_2^2e_0^3 + c_2^3e_0^4 + O(e_0^5)]$$

and

$$\frac{f\left(x_0 - \frac{f(x_0)}{f'(x_0)}\right) + f(x_0)}{f'(x_0)} = [e_0 + 2c_2e_0^2 + 3c_3e_0^3 - 2c_2^2e_0^3 + c_2^3e_0^4 + O(e_0^5)]\,[1 + 2c_2e_0 + 3c_3e_0^2 + O(e_0^3)]^{-1}$$

$$= e_0 - 2c_2^2e_0^3 + O(e_0^4)$$

Thus from (5b),

$$\alpha + e_1 = \alpha + e_0 - e_0 + 2c_2^2e_0^3 + O(e_0^4)$$

$$\therefore\ e_1 = ae_{0,}^3 \tag{13}$$

where $a = 2c_2^2$ and the terms involving higher power of e_n are neglected. Again, from (5c)

$$x_1^* = x_1 - \frac{f\left[x_1 - \frac{f(x_1)}{f'(x_0)}\right] + f(x_1)}{f'(x_0)}$$

Here $f\left[x_1 - \frac{f(x_1)}{f'(x_0)}\right] = f\left[\alpha + e_1 - \frac{f(\alpha+e_1)}{f'(\alpha+e_0)}\right] = f\left[\alpha + e_1 - \frac{e_1+c_2e_1^2+c_3e_1^3+O(e_1^4)}{1+2c_2e_0+3c_3e_0^2+O(e_0^3)}\right]$

After some calculation, we get

$$= f'(\alpha)[1 + 2c_2e_0e_1 + 3c_3e_0^2e_1 - 4c_2^2e_0^2e_1 + O(e_0^6)]$$

Also,

$$f\left[x_1 - \frac{f(x_1)}{f'(x_0)}\right] + f(x_1) = f'(\alpha)[e_1 + 2c_2e_0e_1 + 3c_3e_0^2e_1 - 4c_2^2e_0^2e_1 + \cdots]$$

and

$$\frac{f\left[x_1 - \frac{f(x_1)}{f'(x_0)}\right] + f(x_1)}{f'(x_0)} = [e_1 + 2c_2e_0e_1 + 3c_3e_0^2e_1 - 4c_2^2e_0^2e_1 + \cdots][1 + 2c_2e_0 + 3c_3e_0^2 + \cdots]^{-1}$$

$$= e_1 - 4c_2^2e_0^2e_1 + O(e_0^4) \tag{14}$$

Thus the error e_1^* in x_1^* is given by

$$e_1^* = e_1 - [e_1 - 4c_2^2e_0^2e_1 + O(e_0^4)] = abe_0^5$$

where $b = 4c_2^2$ and the terms involving higher power of e_0 are neglected.

Next, we compute the error e_2 in x_2.

Now

$$\frac{f(x_1)}{f'\left(\frac{2x_1x_1^*}{x_1+x_1^*}\right)} = \frac{f'(\alpha)(e_1+c_2e_1^2+c_3e_1^3+O(e_1^4))}{f'(\alpha+\frac{e_1+e_1^*}{2})} = \frac{e_1+c_2e_1^2+c_3e_1^3+O(e_1^4)}{1+c_2\,e_1+c_2\,e_1^*+\frac{3}{4}c_3\,e_1^2+O(e_0^3)}$$

$$= e_1 + \frac{1}{4}\,c_3\,e_1^3 - c_2\,e_1\;e_1^* - c_2^2\,e_1^2\,e_1^* + \cdots.$$

$$\therefore\;\; x_1 - \frac{f(x_1)}{f'\left(\frac{2x_1x_1^*}{x_1+x_1^*}\right)} = \alpha - \frac{1}{4}\,c_3\,e_1^3 + c_2\,e_1\;e_1^* + c_2^2\,e_1^2\,e_1^* + \cdots$$

where the higher power terms are neglected. Thus

$$f\left(x_1 - \frac{f(x_1)}{f'\left(\frac{2x_1x_1^*}{x_1+x_1^*}\right)}\right) = f'(\alpha)\left(c_2\,e_1\;e_1^* + c_2^2\,e_1^2\,e_1^* - \frac{1}{4}\,c_3\,e_1^3 \ldots.\right)$$

and

$$f\left(x_1 - \frac{f(x_1)}{f'\left(\frac{2x_1x_1^*}{x_1+x_1^*}\right)}\right) + f(x_1) = e_1 f'(\alpha)\;(1 + c_2\,e_1\; + c_3\,e_1^2 + c_2\;e_1^* + c_2^2\,e_1\,e_1^* - \frac{1}{4}\,c_3\,e_1^3 + \cdots).$$

$$\frac{f\left(x_1 - \frac{f(x_1)}{f'\left(\frac{2x_1x_1^*}{x_1+x_1^*}\right)}\right) + f(x_1)}{f'\left(\frac{2x_1x_1^*}{x_1+x_1^*}\right)} = e_1\; - \frac{3}{2}c_3e_1^2\,e_1^* \ldots$$

From (5d),

$$x_2 = x_1 - \frac{f\left(x_1 - \frac{f(x_1)}{f'\left(\frac{2x_1x_1^*}{x_1+x_1^*}\right)}\right) + f(x_1)}{f'\left(\frac{2x_1x_1^*}{x_1+x_1^*}\right)}$$

So, after substituting the values, we get

$$\alpha + e_2 = \alpha + e_1\; - (e_1\; - \frac{3}{2}c_3e_1^2\,e_1^*\; + \cdots)$$

$$\therefore e_2 = \frac{3}{2}c_3e_1^2\,e_1^*\; = a^3bce_0^{11},$$

Where $c = \frac{3}{2}\; c_3$. In fact it can be worked out for $n \geq 1$, that the following relation holds:

$$e_{n+1}\; = c\,e_n^2\,e_n^* \tag{15}$$

To compute e_{n+1} explicitly, we need e_n^*. We already obtained the value of e_1^* and next we compute e_2^*. From (5d)

$$x_2^*\; = \; x_2 - \frac{f\left[x_2 - \frac{f(x_2)}{f'\left(\frac{2x_1x_1^*}{x_1+x_1^*}\right)}\right] + f(x_n)}{f'\left(\frac{2x_1x_1^*}{x_1x_1^*}\right)}$$

Proceed stepwise as above, the error e_2^* in x_2^* is given by the relation

$$e_2^* = de_1^2\,e_2,$$

Where $d = c_2^2$ and, it can be checked that, in general, for $n \geq 2$, the following relation holds:

$$e_n^*\; = d\,e_{n-1}^2\,e_n \tag{16}$$

From (15) and (16), we conclude that the errors e_n^* in x_n^* and e_{n+1} in x_{n+1} for $n \geq 2$ in method (5a)-(5d) satisfy the recursion formula given below:

$$e_n^* = d\, e_{n-1}^2\, e_n \tag{17}$$

$$e_{n+1} = c\, e_n^2\, e_n^* \tag{18}$$

To obtained convergence order of the method, we need a relation of the form

$$e_{n+1} = K e_n^p, \tag{19}$$

where K is some constant. Thus,

$$e_n = K e_{n-1}^p \quad \text{or} \quad e_{n-1} = K^{-\frac{1}{p}} e_n^{\frac{1}{p}} \tag{20}$$

From (17), (18), (19) and (20),

$$K e_n^p = c\, e_n^2\, e_n^* = c\, e_n^2 d\, e_{n-1}^2\, e_n = cd\, K^{-\frac{2}{p}}\, e_n^{3+\frac{2}{p}}$$

Equating the power of e_n,

$$p = 3 + \frac{2}{p} \qquad \therefore\ p = \frac{3 \pm \sqrt{17}}{2}$$

Hence the method (5a)-(5d) is convergent with convergence order $\frac{3+\sqrt{17}}{2} \cong 3.5615$.

4. Numerical Experiments

In order to check the performance of the introduced methods (3a)-(3d) and (5a)-(5d), we exhibit the numerical results on some nonlinear equations of single variable. We also compare the results of these methods with Newton method (NM), McDougall and Wotherspoon (MW) method, and Potra and Pta'k method. Numerical computations have been performed using the Matlab software. We use the stopping criteria $|x_{n+1} - x_n| < (10)^{-12}$ or $|x_{n+1}| < (10)^{-14}$ for the iterative process of our results.

For the numerical examples, we use following text functions and their roots α

(i) $f_1 = (x-2)^{23} - 1$, $\alpha = 3$

(ii) $f_2 = \cos x - x e^x + x^2$, $\alpha = 0.639154069332008$

(iii) $f_3 = x^6 - x - 1$, $\alpha = 1.134724138401519$

Table 1: $f_1 = (x-2)^{23} - 1$, and initial guess $x = 4$

No. of Iterations	Newton Method	M-W Method	Newly introduced Method (3a)-(3d)	Potra and Pta'k Method	Present Method (5a)-(5d)
1	3.913043488626895	3.913043488626895	3.913043488626895	3.881762281757005	3.881762281757005
2	3.829867712336497	3.813403617069735	3.813331074452638	3.770514691012868	3.751168249608405
3	3.750308319871752	3.719262465432603	3.719132009936569	3.665844113356311	3.629953250636192
4	3.674208153011303	3.629998067126574	3.629817244700443	3.567362235341004	3.517123737847595
5	3.601417012345112	3.545369541447942	3.545145511948340	3.474705145072179	3.412115738950753
6	3.531791563484456	3.465137701182709	3.464876921225920	3.387535965688113	3.314417477331388
7	3.465195593647396	3.389080467269836	3.388788790381042	3.305558823234900	3.223669633399193
8	3.401501177624357	3.317001067351732	3.316683886407281	3.228575744377471	3.140096774314826
9	3.340592238813068	3.248757430234993	3.248420046055467	3.156705026488715	3.066464605301569

10	3.282374444500791	3.184357717630727	3.184006488699039	3.091180490016622	3.014477358683509
11	3.226801820893740	3.124261554464694	3.123907609777858	3.036856948205285	3.000181100048933
12	3.173947045831000	3.070261667428139	3.069931829367179	3.005553045154960	3.000000000030649
13	3.124182323440681	3.027446469454418	3.027203785176629	3.000036207524484	3.000000000000000
14	3.078615102622522	3.004438034641327	3.004352969040154	3.000000000011477	
15	3.039945182614113	3.000074769586678	3.000071395956669	3.000000000000000	
16	3.013097347814351	3.000000003879636	3.000000003465490		
17	3.001704377684283	3.000000000000000	3.000000000000000		
18	3.000031522843926				
19	3.000000010927831				
20	3.000000000000001				
21	3.000000000000000				

Table 2: $f_2 = \cos x - xe^x + x^2$ and initial guess $x = 2$

No. of Iterations	Newton Method	M-W Method	Present Method (3a)-(3d)	Potra and Pta'k Method	Present Method (5a)-(5d)
1	1.413190095241312	1.413190095241312	1.413190095241312	1.221708204826725	1.221708204826725
2	0.960950362222547	0.890336187671187	0.884765708414379	0.744561133808421	0.698655987801723
3	0.708489670672238	0.664695575326538	0.662363993421890	0.640375324008681	0.639188120754996
4	0.642803431695474	0.639281518751900	0.639251779238558	0.639154098569363	0.639154096332008
5	0.639164526016085	0.639154096646505	0.639154096497285	0.639154096332007	
6	0.639154096417341	0.639154096332008	0.639154096332008		
7	0.639154096332008				

Table 3: $f_3 = x^6 - x - 1$ and initial guess $x = 2$

No. of Iterations	Newton Method	M-W Method	Present Method (3a)-(3d)	Potra and Pta'k Method	Present Method (5a)-(5d)
1	1.680628272251309	1.680628272251309	1.680628272251309	1.576686172124548	1.576686172124548
2	1.430738988239062	1.387614413767816	1.386095807015648	1.287827012394893	1.249344732145450
3	1.254970956109436	1.205999615828861	1.204636665057341	1.152665486084360	1.138755278835567
4	1.161538432773313	1.140433658814203	1.140138942841691	1.134784110035479	1.134724181083601
5	1.136353274170505	1.134740819036991	1.134738745696497	1.134724138404039	1.134724138401520
6	1.134730528343629	1.134724138412645	1.134724138409576	1.134724138401520	
7	1.134724138500221	1.134724138401519	1.134724138401519		
8	1.134724138401519				

5. Conclusion

In this work, we have presented two new Newton type iterative methods having convergence order $1+\sqrt{2}$ and 3.5615 for solving nonlinear equations of single variable. From the numerical as well as theoretical result, the newly introduced method (3a)-(3d) whose order of convergence and efficiency index are higher than Newton's methods and same with McDougall and Wotherspoon method (2a)-(2d). Also hybrid method (5a)-(5d) is converge to the root faster than Potra and Pta'k method but in this method we have to evaluate one more function after first iteration.

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