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Research Article

Air pollutant dispersion using advection-diffusion equation

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Abstract
The advection-diffusion model is one of the mathematical models that may be used to understand how air pollutants are spread in the environment. It uses the advection-diffusion system with time-dependent prediction of the spread of environmental contamination under various environmental conditions to determine whether the contaminants are more concentrated at ground level or close to the point of emissions. The times, diffusivity, emission sources, and wind profile are all considered by the model. In order to determine the approach's descriptive characteristics in terms of advection and dispersion coefficients, we evaluate the exact solution to a flow field problem as an initial value problem in space.

Keywords: Advection–diffusion equation; analytical solution; Gaussian plume model

Introduction
Fick's first law is used to generate the parabolic partial differential equation (PDE) known as the advection-diffusion equation (ADE) from the idea of mass conservation. The physical quantities resulting from advection and diffusion are two processes whereas advection is the movement of a substance or quantity in bulk and diffusion is the term used to describe the movement of substances caused by random motion (Andallah & Khatun, 2020). One of the most significant partial differential equations (PDEs) is the advection-diffusion equation, which is used in a variety of industrial and engineering applications, particularly in the study of transport processes (Singh & Tanaka, 2000). Mathematical models developed for various environmental pollution problems such as the dispersion of pollutants in rivers, estuaries, lakes, and air transportation (Fig. 1) have been mostly defined with advection-diffusion equation (Haluk & Sertac, 2019). It has been used to describe a variety of phenomena like mass, heat, energy, and velocity, including temperature change in a draining film (Isenberg & Gutfinger, 1972), water movement in soil (Parlarge, 1980), pollution lengthy transmission in the atmosphere, solute spread in a liquid flowing via a tube, and pollutant dispersion in lakes (Zlatev et al., 1984), solubility salt's distribution in groundwater (Guvanasen & Volker, 1983), a pollutant in groundwater, and spread of pollutants in the land.


Figure 1 An illustration of the primary sources of air pollution, with a focus on the variables that the wind most frequently affects as it transports pollutants through the atmosphere

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The concept of diffusion modeling was introduced, revealing the Gaussian nature of the average cross-wind concentration (Goyal & Kumar, 2011). The methodology behind the Gaussian diffusion theory was elaborated (Daly & Zannetti, 2007). Two key factors explaining the turbulent diffusion process that results in the Gaussian model were identified (Hanna et al.,1982; Khaled et al., 2014). The development of the gradient transport or eddy diffusion model was led by applying a mass balance to a differential volume, considering source strength, average wind speed, and dispersion coefficients. Similarly, pollution dispersion under low wind convective conditions was investigated (Subbiah, 2012). The Gaussian plume model (GPM) and two low wind models were used to predict ground-level concentrations (GLCs) of SO_2 from point sources in Delhi. A general analytical model for pollutant dispersion in the atmospheric boundary layer using crosswind-integrated concentrations was developed (Goyal & Kumar, 2011). Several numerical methods, including FTCS, FTBSCS, BTCS, BTBSCS, and Crank-Nicholson schemes, were explored (Abolfazl et al., 2011). The results showed that the Crank-Nicholson method closely matched the analytical solution, while the other methods showed varying degrees of deviation. Likewise, the Lax-Wendroff scheme was found to be the most effective for solving an ADE with initial and boundary conditions after applying three numerical methods (Appadu, 2013).

The concentration of pollutants depends upon time, diffusion rate, emission sources, and wind speed. This paper presents a variation of physical and geometrical parameters in the solutions of the advection-diffusion equation in a single spatial dimension as well as analyzes analytical solutions in different scenarios. In practically every environmental compartment, the numerical solution technique which is analyzed this paper can be used to solve a variety of problems.

Mathematical Model

One dimension Advection-Diffusion equation is (Andallah & Khatun, 2020; Abolfazl et al., 2011; Thongmoon & Mckibbin, 2006; Karahan, 2006; Kafle et al., 2024),

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2}, \quad 0 < x < L, 0 < t < T \qquad (1)$$

With initial condition (IC),
$c(x,0) = f_0(x);\ 0 \le x \le L$

And boundary condition (BC),
$c(0,t) = g_0(t);\ 0 < t \le T,\ c(L,t) = g_1(t);\ 0 < t \le T$

The instance where the plume traveled inside a flow field can be addressed using the equation (1) technique. The solution when the fluid moves with a velocity of u in the direction of x is (Andallah & Khatun, 2020; Sankaranarayanan et al., 1998):

$$c(x,t) = \frac{Q}{\sqrt{4\pi tD}}\ exp\left[\frac{-(x-ut)^2}{4tD}\right] \qquad (2)$$

where $f_0$is function of x and g_0, and g_1 are functions of time. c is the concentration of pollutants and u represents the wind speed in the x-direction. D stands for diffusion coefficient whereas the pollutant system's aggregate weight per unit area is indicated by the symbol Q.

Results and Discussion

This section describes the simulation of concentration pollution along horizontal distance by variation of time, diffusion coefficient, emission sources, and wind speed. Figure 2 explains the differences in pollutant concentrations at various times. Figure 3 describes the pollutant concentration by variation of diffusion rates. Similarly, Fig. 4 describes the concentration of pollutants by variation of emission sources. Finally, Fig. 5 shows the variation of wind speed for the pollutant concentration. Specifically, the relationship between the concentration of pollutants and wind speed, times, emission source, and diffusion rate is quite relevant to analyzing the pollutant's concentration.

Concentration Pollutant with Variation of Time

The different times in the concentration of pollutant dispersion are shown in Fig. 2. We use constant values for the emission sources, diffusion rate, and wind speed. There are many alternative ways to depict periods between one and seven seconds. According to our observations, pollutant dispersion concerns in time 1 second involving some height and low-range diffusion. Similar to this, we tested various time intervals of 2s, 3s, 4s, 5s, 6s, and 7s which resulted in a modest reduction in height and a slight expansion of the x-direction dispersion range.

The result, which was somewhat modified during the simulation, is shown in Fig. 2 plotted the concentration distribution at seven distinct times. Clearly, the shape of the function changes as the plume moves downstream. The time interval during which higher concentrations emerge widens, while the peak concentration decreases with increasing times. The pollution concentration and dispersion are low in the x −direction but reach their maximum height in this illustration of low time and extending the time in this scenario results in a minor decrease in the height concentration and an increase in the x −direction pollution dispersion.

Comparison between maximum concentration height and front position of concentration is shown in Table 1. The heights are 1.2614 m, 0.7042 m, 0.5772 m, 0.4969 m, 0.4081 m, and 0.3779 m at the various times of 1s, 2s, 3s, 4s, 5s, 6s, and 7s, respectively. We compared the height of other times in this table using the initial height (H1) of the time 1s. The various growing periods give decreasing heights and determine the concentration of pollutants. Additionally, their front position of concentration or pollutant dispersion in the x-direction is growing.

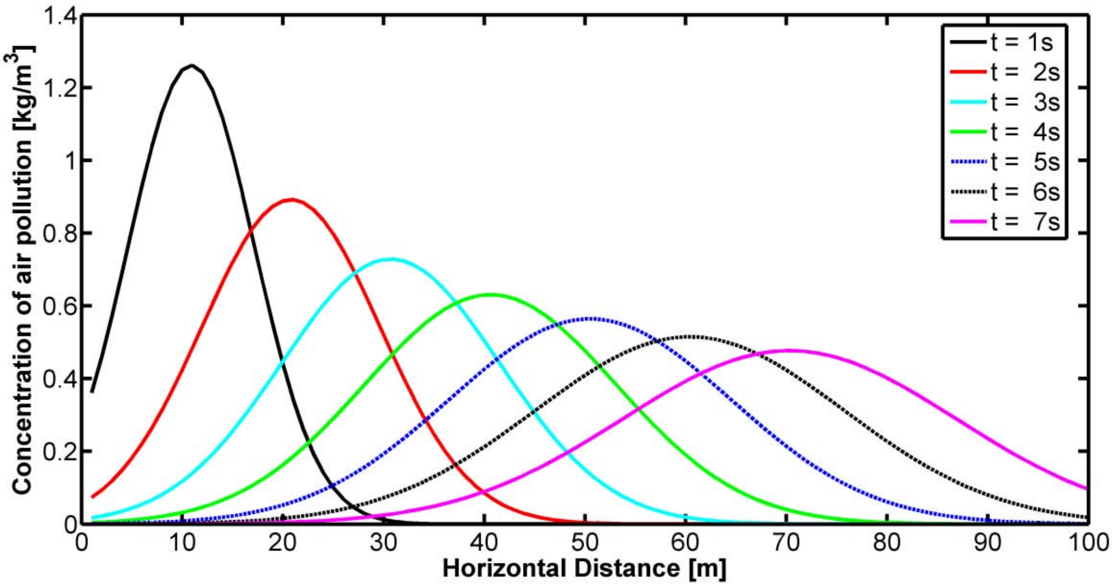

Figure 2. The transport equations are t = 1s, 2s, 3s, 4s, 5s, 6s, and 7s, respectively, for the concentration pollutant with variation of time instantaneous sources illustrated

Table 1. Comparison of maximum height of concentration (H_t) and front position of concentration (F_t) for different time

Time (t) $[s]$	Front position of concentration (F_t) $[m]$	Maximum height of concentration (H_t) $[m]$	$\frac{H_t}{H_1}$	$\frac{F_t}{F_1}$	$\frac{H_t}{F_t}$
1	30 (=F1)	1.2614 (=H1)	1	1	0.0420
2	50	0.8884	0.7042	1.67	0.0177
3	65	0.7281	0.5772	2.167	0.0112
4	80	0.6269	0.4969	2.67	0.0078
5	90	0.5638	0.4469	3	0.0062
6	100	0.5149	0.4081	3.33	0.0051
7	120	0.4767	0.3779	4	0.0039

We examined the ratio between beginning height and maximum height of concentration in Table 1. We noticed a height ratio that depicts how various growing periods are dropping over time. Additionally, we examined the relationship between the first front position of focus and the front position, both of which are growing over time and in the x −direction, respectively.

In the final column of Table 1, we additionally calculate the proportion of maximum concentration height to the front position. We concluded that although the ratio climbed from 1s to 7s, there was a declining relationship between the front location of attention and the maximum height.

Concentration Pollutant with Variation of Diffusion Coefficient

The various diffusion coefficients for pollution dispersion are displayed in Fig. 3. We use diffusion coefficients with varied values of 0.01 m^2/s, 0.05 m^2/s, 0.025 m^2/s, and 0.0625 m^2/s, respectively. The blue line represents the diffusion rate, which is 0.00625 m^2/s, and we can see that it disperses as it takes on more height and narrows in the x −direction. Similar to this, we raise the rate of diffusion, which is 0.025 m^2/s, 0.01 m^2/s, and 0.05 m^2/s accordingly. We observe that this marginally reduces the height and slightly lengthens the x −direction diffusion length.

Finally, we conclude that, as a result of the slow rate of diffusion, disperses significantly less and slightly increases height. If the diffusion coefficient is raised, the height is slightly reduced and the diffusion in the x-direction is increased.

Concentration Pollutant with Variation of Emission Sources

Figure 4 shows the variety of sources from which pollution is released. We planned to use a range of emission sources, like 3 kg/s, 2 kg/s, 1 kg/s, and 0.5 kg/s. Black dot line with 3 kg/s emission sources is displayed, and we could observe that while diffusion is quite low, it does require some height. The blue line represents the 2 kg/s of the emission source, and it can be seen that the x −direction dispersion is somewhat larger, and the height is slightly decreasing. The figure's red line, which represents a 1 kg/s emission source, showed that the height was slightly lower than expected and that the rate of x −axis dispersion was rising. The low value of the emission source in the final line is 0.5 kg/s. As a result, it was clear that the dispersion was more pronounced in the $x-$ direction and less likely to heighten.

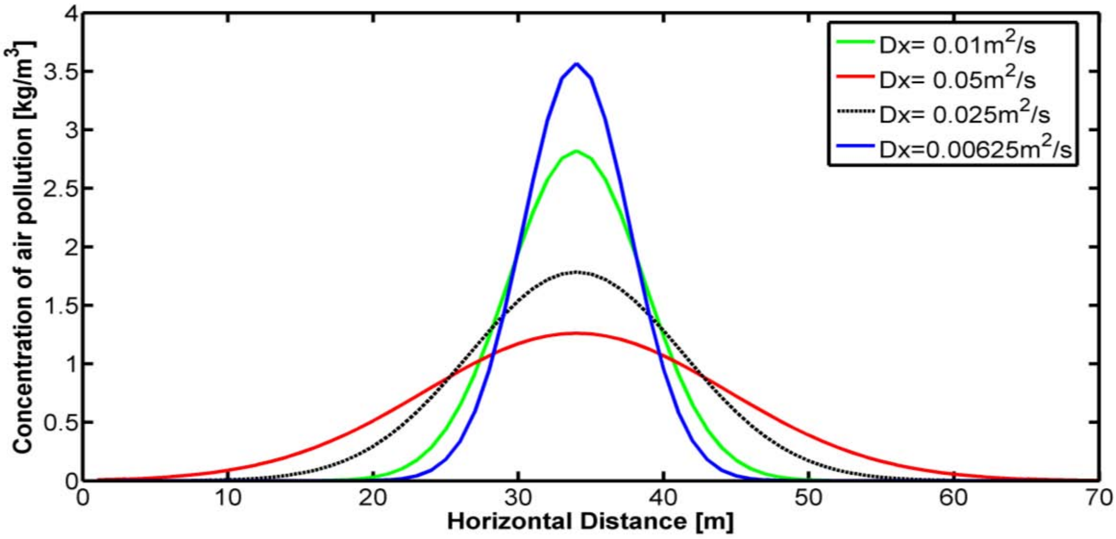

Figure 3. Transport solutions for various diffusion coefficients

Eventually, we conclude that a large number of emission sources led to low directional dispersion and somewhat increased height, and that if the emission source somewhat lowers, the height will also slightly decrease, and the dispersion length will increase in the $x-$direction. In this case, we asserted that the big industrial companies, lakes, reservoirs, canals, channels, the ocean, and groundwater might reduce the amount of pollutants in the air by using this method.

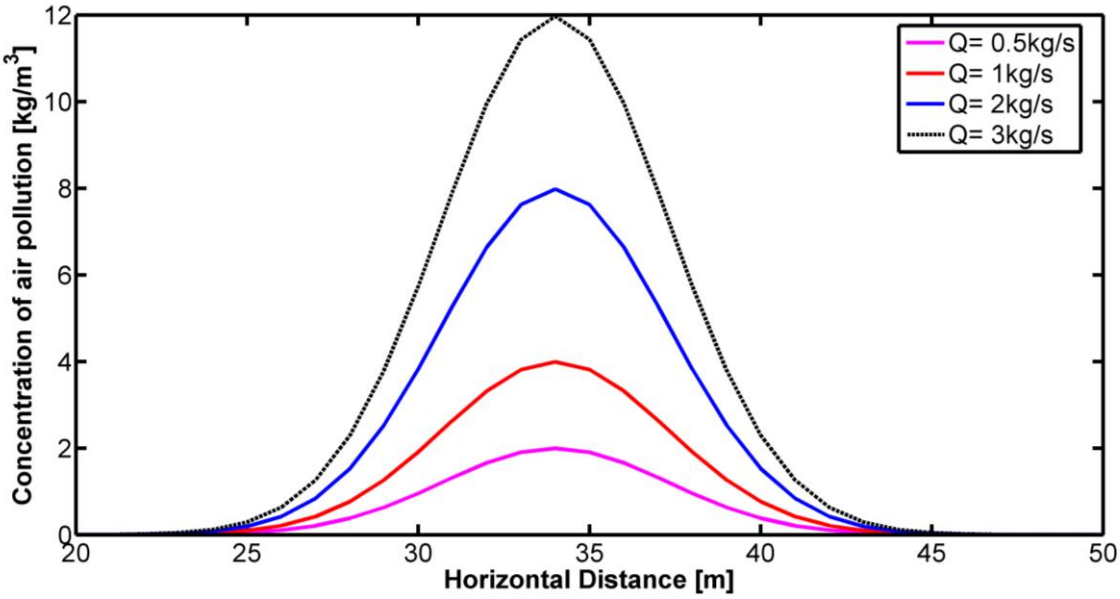

Figure 4. Transport solutions for various Emission Sources

Concentration Pollutant with Variation of Wind Speed

Figure 5 displays the various wind speeds for the dispersion of pollution. The red, black dot, blue, and magenta in the image represent various wind speeds in increasing order of 0.5 m/s, 0.8 m/s, 1 m/s, and 1.5 m/s. The height can be quickly raised or taken while traveling a short distance if the wind speed is 0.5 m/s, which causes rapid dispersion in the x-direction. Similarly, we increased the wind speed by 0.8 m/s, 1 m/s, and 1.5 m/s, which showed a slight change in the x-direction dispersion and a similar height of some other wind speed.

Finally, it reached the conclusion that a increase in wind speed will cause some changes in the $x-$direction, whereas a slight increase in wind speed will cause some changes in the x −direction. If we can operate in the industry more during times of low wind speed and less during times of high wind speed, we can lower the amount of pollutants in the environment. Many industrial companies employ this procedure to produce more emissions.

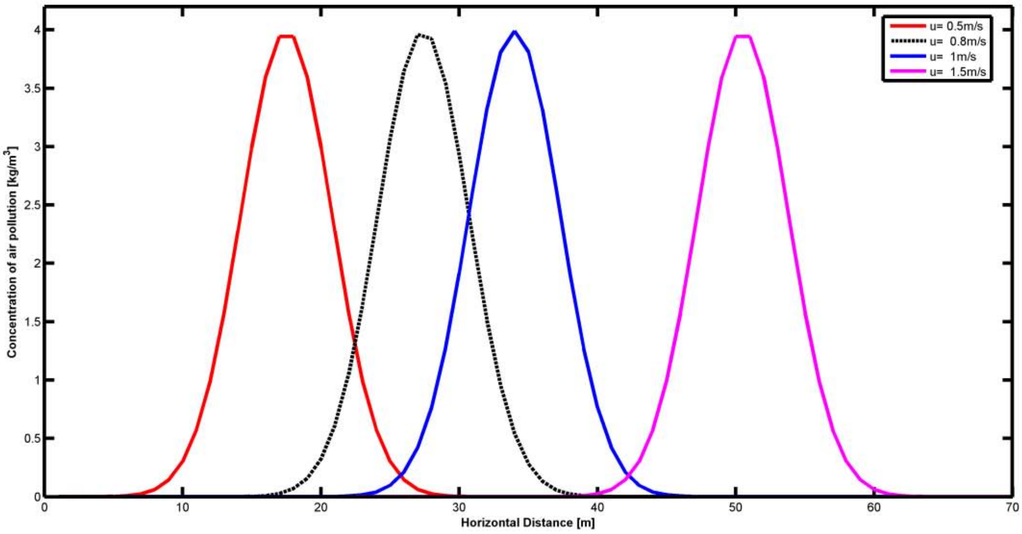

Figure 5. Transport solutions for various wind speeds.

Conclusions

In this study, these variables represent times, diffusivity, wind speed, and emission sources. Based on the results, numerous simulations were run to see how altering any one of the climatic elements would impact how environmental contamination is distributed. The shape of the function changes as the plume moves downwind at varied pollutant concentrations. The peak concentration gradually decreases with time, and the period during which elevated concentrations emerge widens. The state of various pollution diffusion coefficients is shown. Due to the low rate of diffusion, we have seen that the result disperses far less and climbs greatly. When the diffusion coefficient is raised, the height is slightly decreased, and the diffusion in the *x*-direction is boosted. The various emission sources that affect the spread of pollutants are displayed. There were many emission sources, which resulted in a small height rise and modest directional dispersion. Additionally, if the emission source is significantly lowered, the height will also slightly decrease, and the dispersion length will increase in the *x*-direction. For the dispersal of pollution at different wind speeds. While a minor increase in wind speed will produce some changes in the *x*-direction and some height gain, a low-rate rise in wind speed will cause the low-rate wind speed to scatter quickly in the x-direction and gain some height. After analyzing the concentration of pollutants and the rate of diffusion in the x-direction for each pollutant concentration variation over time, diffusion source, emission source, and wind speed, these models can be used to address source contamination issues in lakes, reservoirs, waterways, channels, the ocean, groundwater, and dirty water. Following this work, the two-dimensional advection diffusion equation, which is used in air pollution and other fields, will be studied. We will also discuss how to apply this topic to real-world problems and industrial settings.

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Conflicts of Interest: The author declares no conflicts of interest.

Data Availability Statement: The data that support the finding of this study are available from the corresponding author, upon reasonable request.

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