

Algebra Problems in Middle School Textbooks from Six Countries

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Abstract

This study compared the algebra content of middle school mathematics textbooks from Finland, the United States, Singapore, China, Taiwan, and Nepal. Content analysis was employed to investigate five aspects of the algebra problems: the total number of algebra problems, representational forms, contextualization, response types, and cognitive demand. The results revealed that the six mathematics textbook series differed in all five aspects, indicating uneven opportunities to learn middle school algebra across countries. The insights gained can be valuable in designing mathematics textbooks and curricular materials. We provide recommendations for stakeholders and curriculum developers based on the results.

Keywords: algebra, cross-national comparison, mathematics textbook, middle school

Introduction

Studies have documented that the content of mathematics textbooks affects students' learning opportunities, the quality of textbooks affects students' learning of mathematics, and textbooks differ in their effectiveness (Cai et al., 2011; Grouws et al., 2015; Tarr et al., 2008). Teachers around the world use mathematics textbooks significantly in their daily instructions. For example, Chávez-López (2003) reports the daily use percentages of mathematics textbooks in the classroom in the United States to be about 60%, and in Japan, it is approximately 70% (p. 4). Three-fourths of 8th graders in the United States use textbooks on a daily basis (Grouws & Smith, 2000), whereas, as recently as 2020, about 85% of 8th graders from Nepal are reported to use textbooks to learn mathematics (Education Review Office, 2022). Evaluating the impact of textbooks in different ways (Heon & Mills, 2023). Yet, instructional materials profoundly impact teachers' actions and students' learning (Tarr et al., 2008). The quality of textbooks positively affects students' performance in mathematics, and there are various ways to

evaluate the quality of textbooks. For example, the evaluation of features embedded in the problems, such as representations, cognitive demand, contexts, and textual/non-textual features of a mathematics problem, inform mathematical and pedagogical aspects of the textbook (Kim, 2009; Zhu & Fan, 2006). Furthermore, using these features as a tool, various cross-national comparative studies on mathematics textbooks have revealed the strengths and limitations in the content and design of textbooks across different countries (e.g., Choi & Park, 2013; Hong & Choi, 2014; Kar & Işık, 2015; Özer & Sezer, 2014; Son & Senk, 2010).

Recognizing the importance of cross-national analysis in the realm of mathematics textbooks, this study employed content analysis to assess the quality of middle school mathematics textbooks from six nations: China, Finland, Nepal, Singapore, Taiwan, and the United States. The primary reason for selecting Taiwan, Singapore, China, and Finland is that the students in these countries perform well in international mathematics assessments, such as the Programme for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS). The United States was included in our sample because of its leading role in mathematics education worldwide. Nepal was selected for a few reasons. First, textbooks from Nepal are rarely included in cross-national comparisons. Second, students from Nepal have never participated in large-scale international assessments, such as PISA and TIMSS. Third, one of the coauthors of this study, a native of Nepal, wished to gain insight into how Nepal's middle school mathematics textbooks from Nepal were included.

This investigation is carried out with the hope that the insights gained can serve as a basis for the revision and development of instructional and curriculum materials (Fan et al., 2013). To achieve the goal of assessing the quality of textbooks from the sample countries, we aimed to identify the differences in the total number of algebra problems in the representational forms, contextual features, response type, and cognitive demand for algebra problems. We aim to answer the following research questions:

- (1) Are there differences in the total number of algebra problems in mathematics textbooks among the six countries?
- (2) Are there differences in the representational forms of problems in mathematics textbooks among the six countries?
- (3) Are there differences in the contextual features of problems in mathematics textbooks among the six countries?

- (4) Are there differences in the response type of problems in mathematics textbooks among the six countries?
- (5) Are there differences in the cognitive demand for algebra problems in mathematics textbooks among the six countries?

Literature Review

Studies on Middle School Algebra

Algebra provides a tool to solve many arithmetic problems at a time. It works as a powerful tool to enable students to generalize and find answers to many problems in daily life (Grønmo, 2018; National Council of Teachers of Mathematics (NCTM), 2000). Algebra is a topic in middle school or high school curricula in most countries. In many countries, middle school students are expected to learn to use mathematical models to represent word problems and interpret quantitative relationships in mathematical models (NCTM 2000). Charles (2005) proposed that the ability to use variables, expressions, and equations is a key concept that must be taught in elementary and middle schools. Algebra is an essential topic in middle school and beyond because it provides a gateway to more abstract mathematics (Grønmo, 2018).

The importance given to teaching and learning algebra underscores its significance. However, learning algebra is challenging. Students worldwide report difficulties in learning algebra. For example, Jupri et al. (2014) discovered that Indonesian students struggle with translating word problems into symbolic forms and interpreting symbolic problems into words. Moreover, they reported that students had difficulty understanding variables, algebraic expressions, and varying meanings of equal signs. Studies show even students who are experts in arithmetic problem-solving find algebra difficult (Bush & Karp, 2013; Cai & Moyer, 2008; Kieran, 2004). As a result, algebra becomes a barrier for many students in middle and high school. Therefore, it is not surprising that algebra skills strongly indicate successful college graduation (Parker, 2005).

Kulm et al. (1999) noted that middle school mathematics programs are repetitive and non-challenging. Therefore, the middle school program is considered a fertile ground for incorporating educational reform. Project 2061 (Waldron, 2017), a long-term project that aims to make citizens science literate, initiated a benchmark-based textbook evaluation. In their first-ever study of textbooks, Project 2061 investigated middle school mathematics textbooks against their benchmark. They concluded that many middle school textbooks do not meet the standards that are highly regarded in many curriculum and research studies. One factor that affects learning is the teaching method, and the teaching of mathematics in many countries is heavily textbook-dependent (Tarr et al., 2013; Valverde et al., 2002). A probable link between students' inability to solve certain types of problems and the absence of such problems in their textbooks has been established in earlier studies (Törnroos, 2005; Wijaya et al., 2015a; Zhu & Fan, 2006). Therefore, by counting the number of algebra problems, analyzing their representational forms, contextualization, and non-contextualization attributes, as well as closed- and open-endedness, and assessing the level of cognitive demand, this study aims to identify similarities and differences in problems contained in the sample textbooks.

Algebra is recognized as an essential topic in the middle school mathematics curriculum. Few studies have comprehensively examined middle school mathematics textbooks, and the textbooks from the sample countries in this research have never been evaluated for their middle school algebra contents. This comparative study aims to fill that particular gap in the research.

Studies on Counting Certain Aspects of Textbooks

The most basic frequency counts in textbook research include counting the number of tasks, pages, topics, subtopics, and chapters. Charalambous et al. (2007) noted that this particular type of counting provides very little information about the pedagogical and didactical aspects of the textbook; however, it does portray a tentative sketch of the investigated textbook.

Yang et al. (2017) analyzed middle school textbooks and found that the number of geometry problems was 896 in Taiwan, 945 in the United States, 1679 in Singapore, and 2065 in Finland. The numbers indicate that students in Finland and Singapore have more opportunities to solve geometry problems compared to Taiwanese and U.S. students. In another study, the number of pages in elementary mathematics textbooks (grades 1-5) from Japan, Kuwait, and the United States was reported to be 761, 1129, and 3039 pages, respectively (Alajmi, 2012). Elementary mathematics textbooks from the United States are, on average, four times larger than their Japanese counterparts.

Studies on Representational Forms

The various forms of representation, such as symbolic, verbal, visual, and combined forms, and their role in teaching and learning have received considerable attention in studies (Yang & Sianturi, 2022; Goldin, 2008; Zhu & Fan, 2006). These

studies defined four representational forms: symbolic, verbal, visual, and combined. If a problem is written using mathematical symbols, then it is considered a "symbolic form." If a problem is written in words, then it is considered a "verbal form." If a problem involves visual contents: pictures, graphs, tables, etc., then it is considered a "visual form." If a problem consists of two or more of the symbolic, verbal, and visual forms, then it is considered a "combined" form.

The ability to move between modes of representation is a prerequisite for increasing proficiency in the mathematics (Cuoco & Curcio, 2001; Goldin, 2008). As a tool for communication, representational forms in mathematics help students develop an understanding of mathematical concepts (Zazkis & Liljedahl, 2004). Studies have demonstrated the effectiveness of multiple representational forms in solving algebra problems and understanding concepts (Kang & Liu, 2018; Soneira, 2021). The role of representation is underscored as an important pedagogical tool in many curriculum guidelines. The widely accepted document NCTM (2000) suggests that representation should be a process standard in the teaching and learning of mathematics, citing its crucial role as a pedagogical tool (Cuoco & Curcio, 2001; Goldin, 2008; Moreno-Armella et al., 2008).

Several studies have revealed that textbooks from Eastern and Western countries emphasize different representational forms (Yang & Sianturi, 2022; Zhu & Fan, 2006). For example, when comparing middle school textbooks from China and the United States, Zhu and Fan (2006) discovered that the textbooks from the United States contained more problems with visual and verbal forms than those in China. In contrast, the mathematics textbooks in China emphasized symbolic forms and contained fewer visual forms. Examining the differences in geometry problems among middle school textbooks in Finland, Singapore, Taiwan, and the United States, Yang et al. (2017) discovered that the Taiwanese and Singaporean textbooks contained more problems with combined forms, whereas the Finnish and US textbooks contained more problems with verbal and visual forms.

Studies on Contextualization and Noncontextualization

This study adopted the definition of contextual and noncontextual problems from earlier literature (Yang & Sianturi, 2022; Charalambous et al., 2010; Zhu & Fan, 2006). Contextual problems involve real-world situations. A problem without context is a noncontextual problem.

Graumann (2011) emphasized that mathematics should be taught as a tool that enables students to solve problems in various situations in daily life. To address the need for context-based problems in mathematics education, large-scale assessments, such as PISA consist of problems related to situations that students encounter in their everyday lives. Studies show lack of exposure to context-based problems may affect students' ability to solve such problems (Wijaya et al. 2015a, 2015b). The contextual problem can make the problem more familiar, easier to identify solution strategies, and render problems more natural and solvable through various approaches (Van den Heuvel-Panhuizen, 2005).

Many cross-national comparisons have investigated this topic and cited its crucial role of context in mathematics education (e.g., Charalambous et al., 2010; Zhu & Fan, 2006).

Studies on Closed-Ended and Open-Ended Problems

Wijaya et al. (2015a) categorized problems as open and closed-ended based on the required response. For example, if multiple solutions to a problem are possible, then it is considered an open-ended problem. A problem with only one correct answer is a closed-ended problem. The framework of this study adopted the definitions of open- and closed-ended problems from the study of Wijaya et al. (2015a).

Mathematical problems with multiple correct answers can broaden students' perspectives and help them think flexibly (NCTM, 2000; Sullivan et al., 2009; Zhu & Fan, 2006). Kwon et al. (2006) observed that open-ended problems in mathematics education promotes divergent thinking skills in students. A lack of opportunity to solve open-ended problems can result in difficulties for students when solving complex problems (Cai, 2000). Students' mathematical creativity can be stimulated through open-ended problems (Aziza, 2018), which can also help students develop a growth mindset by demonstrating that high achievement is possible with effort and perseverance (Boaler, 2013). Overall, the aforementioned studies suggest that open-ended problems are crucial in mathematics education.

Studies on the Cognitive Demand of a Problem

Cognitive skills are required to solve mathematics problems (Charalambous et al., 2010). Therefore, the cognitive demand of a problem is defined as the degree of mental effort that students experience while solving a problem. Studies have divided the

cognitive demand of mathematical tasks into four categories: memorization, procedures without connection, procedures with connections, and doing mathematics (Charalambous et al., 2010; Jones & Tarr, 2007; Stein & Smith, 1998). The definition of cognitive demand of mathematical problems used in the framework of this study is based on related studies (Charalambous et al., 2010; Jones & Tarr, 2007; Stein & Smith, 1998). According to their definitions, problems that require the memorization and recall of formulas and facts are considered "memorization." Those that require the use of algorithmic procedures without connection to underlying concepts are considered "procedures without connections." Problems require students to make connections between algorithms or procedures and underlying mathematical concepts are considered "procedures with connections." Problems requiring the use of nonalgorithmic and complex thinking are considered "doing mathematics." Memorization and procedures without connections belong to lower cognitive demand, and procedures with connections and doing mathematics belong to higher levels of cognitive demand (Charalambous et al., 2010; Jones & Tarr, 2007; Stein & Smith, 1998).

The use of problems with high cognitive demand in teaching helps students improve their thinking and problem-solving skills (Stein & Lane, 1996). Studies have demonstrated that optimal conditions for learning are created in classrooms where problems with high cognitive demand are regularly implemented (Boaler & Staples, 2008; Stein et al., 1996). The rise and wide acceptance of frameworks for analyzing problems, such as the Task Analysis Guide for Science, evidences the crucial role of cognitive demand in accomplishing tasks (Tekkumru-Kisa et al., 2015).

Research Methods

Sample Textbooks

We selected representative middle school mathematics textbooks from each sample country. Table 1 lists each country, the name and publisher of the sample textbooks, publication dates, and their market share within the respective country. In the selection process, we tried to choose sample textbooks with the highest market share. As textbook problems mostly stay the same over time, even if new curriculum documents specifically warrant change (Bakken and Andersson-Bakken, 2021), we have decided to use older editions from the US and China due to convenience.

Country	Textbook	Grades	Publisher	Year	Market share
China	Compulsory Education Curriculum Standard Experimental Mathematics	6, 7, 8	People's Publishing House	2006	50%
Finland	Laskutaito	7, 8, 9	WSOY	2009	70%
Nepal	My Mathematics	6, 7, 8	Curriculum Development Center	2018	76%
Singapore	New Syllabus Mathematics	7, 8, 9	Shing Lee Publishers Pte Ltd.	2011 2012 2011	80%
Taiwan	Kang Husan	7, 8, 9	KH Education Publishing Group	2019	39%
United States	Connected Mathematics	6, 7, 8	Prentice Hall	2006	Used in 2500+ school districts

Table 1

The Related Backgrounds of the Selected Textbooks

China's CMCST (Grades 7–9) is based on the national mathematics curriculum established by China's Ministry of Education. The CMCST series (2006) has the largest market share (approximately 50%) among middle school mathematics textbooks in China (Ding & Sun, 2011). Of the 94 chapters in the series, 33 cover algebra.

Finland's Laskutaito series prioritizes combining learning with real-life situations to make education practical and engage students. Covering approximately 70% market in Finland, Laskutaito series contains 41 chapters, 17 if which cover algebra.

The Nepali government publishes and distributes the MM middle school textbooks (Grades 6–8) (Ministry of Education, 2018). The MM series covers 76% of the market share (Basyal et al., 2023), and aims to make education more purposeful, practical, relevant, and job-oriented, in preparation for students to face the challenges of the 21st century. Out of 46 chapters in the series, 8 focus on algebra.

Singapore's NSM series (Grades 7–9) was developed following the national mathematics curriculum guidelines developed by the Ministry of Education in Singapore

(2001) and covers 80% of market share (Yang et al., 2017). NSM series emphasizes cognitive development, viewpoint construction, and the development of problem-solving strategies. Of the 41 chapters in the series, 14 involve topics related to algebra.

The Kang Husan (KH) textbooks combine real-life situations, problem-solving, communication, and computational skills. With 39% market share (Yang et al., 2017), the KH textbook series has 62 chapters, 24 covering algebra.

The CMP of the United States, a middle school reform mathematics curriculum, was analyzed because it has the highest utilization among middle school textbooks in the United States and is used in approximately 2,500 school districts (Cai et al., 2011; Rivette et al., 2003). Several studies have reported that the CMP positively affects students' performance (e.g. Tarr et al., 2008). Of the 24 chapters in the CMP series, 6 are related to algebra.

Analytical Framework

Earlier studies employed horizontal and vertical approaches to textbook analysis. In horizontal analysis, the physical characteristics of textbooks, such as page size, page numbers, topics, and sequencing of topics, are the subjects of investigation. On the other hand, vertical analysis delves into an in-depth examination of particular topics, such as the use of representation, contextual features, and cognitive demand of textbook problems. While horizontal analysis provides a very rough picture of the textbook, vertical analysis provides more in-depth details about the textbook and the textbook author's intention (Charalambous et al., 2010). To get the most out of this investigation, we employed both horizontal and vertical frameworks in this study. In horizontal analysis, we counted the total number of problems and the total number of algebra problems. For vertical investigation, we investigated the four features of the problems, namely: representation, context, response type demanded by the problem, and cognitive demand. The definition of each feature (discussed in the literature review section) was taken from earlier studies (Yang et al., 2017; Charalambous et al., 2010; Jones & Tarr, 2007; Larina, 2016; Zhu & Fan, 2006).

Coding and Reliability Measures

Table 2 demonstrates the coding method utilized in this study. In the second row of Table 2, the example is considered as three separate problems, all of which received the same codes: symbolic, closed-ended, non-contextual, and procedures without

connections. Another example in the third row is counted as two problems. The first problem is coded as verbal, closed-ended, contextual, and procedures with connections, while the second one is coded as verbal, open-ended, contextual, and procedures with connections.

Table 2

Problem	Codes received
Expand the following expression.	3 problems
Expand the following expression: $(2x+2x)^2$	Symbolic
a. $(2x+3y)^2$	Closed-ended
b. $(5x-3y)^2$	Non-contextual
c. $(3x+2y)(3x-2y)$	Procedures without
(NSM, Grade 8, p.84)	connections
Problem 3.2 Equations With Two Operations	2 problems
When Liz tells Theo about the idea of visiting Wild World	l, he
suggests she check to see whether the part offers special p	ricesVerbal
for large groups. She finds this information on the park's	Web
site:	Closed-ended (Problem 1)
Regular Admission: \$21.00 per person	
Special Group Price: \$50.00 plus \$10 per group member	Open-ended (Problem 2)
A. 1. Find the price of admission for a group of 20 pe	ople
a group of 35 people, and a group of 42 people.	Contextual
2. Describe in words how you can calculate the	
admission price for a group with any number of	Procedures with
people.	connections
r r ·	

Example of Coding Procedures

About 10% of algebraic problems for each textbook series were selected and independently coded by two researchers using the analytical framework of this study. Two researchers (the first author and his research assistant) independently coded the KH (Taiwan), CMP (USA), NSM (Singapore), CMCST (China), and Laskutaito series (Finland). Both researchers (the corresponding author and the first author) independently coded the Nepali MM series. We note that coding the tasks for cognitive demand of a problems was challenging, confusing, and time-consuming. It required multiple

discussions to reach a perfect agreement on tasks that were initially coded differently by the two coders. The agreement scores between two coders ranged from .88 to .93, with an average of .91, indicating a strong agreement. As stated earlier, any discrepancies in coding were discussed until a consensus was reached.

Results and Discussion Differences in the Total Number of Algebra Problems

Table 3 presents the cross-tabulation of the total chapter, algebra chapters, total numbers, and algebra problems for each country. The results of Table 3 show a big difference in the total problems and number of algebra problems. The proportion of algebra problems among the textbooks from the six countries is ranging from 45.6% to 27.4%. This shows that algebra problems play different roles in the six textbook series. A comparison of the total number of algebra problems for the six countries revealed that the textbooks in Finland had the highest proportion of algebra problems (45.6%) and that Nepal had the lowest proportion (27.4%). The textbooks from China contained the fewest algebra problems (n = 2,302). However, the proportion of algebra problems (34.8%) was higher than that of Nepal. The Nepali textbooks contained 1,005 algebra problems (27.4%), which is more than that in the Chinese textbooks. More than 40% of the problems in the middle school textbooks from Finland, the United States, and Singapore were algebra problems.

Table 3

Country	Total chapters	Algebra chapters	Total Problems	Algebra problems
China	94	33	2302	802 (34.8%)
Finland	41	17	9214	4202 (45.6%)
Nepal	46	8	3663	1005 (27.4%)
Singapore	41	14	8311	3429 (41.3%)
Taiwan	62	24	3033	1146 (37.8%)
United States	24	6	5159	2239 (43.4%)

Cross-tabulation of Number of Chapters Problems by Country

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Differences in Representational Forms

The textbooks differed in terms of the type of representational forms used in the algebra problems. Table 4 presents the cross-tabulation of the representational forms by country. The data in Table 3 showed that there is a difference in symbolic form, visual form, verbal form, and combined form among the six textbook series. The symbolic form was predominant in the Nepali textbooks (69%). More than half of the problems in the textbooks from Singapore and Finland were in the symbolic form, whereas most of the problems in the US textbooks were presented in the verbal form (42%). The textbooks from Finland contained more problems in the visual form, and the textbooks from China contained the most problems in the combined form among the six countries. The textbooks from Nepal contained the lowest proportion of problems in the verbal form. The textbooks from China, Taiwan, and the United States emphasized the combined form, whereas the textbooks from Singapore and Finland offered few problems with the combined form.

Table 4

	Symbolic	Verbal	Visual	Combined
Finland	51.6%	18.0%	19.2%	11.2%
United States	22.3%	42.0%	9.4%	26.3%
Singapore	58.4%	20.7%	7.6%	13.3%
China	38.8%	17.0%	1.2%	43.0%
Taiwan	29.2%	23.6%	12.6%	34.6%
Nepal	69.0%	6.6%	3.2%	21.2%

Cross-Tabulation of Representational Forms by Country

Differences in Problem Type

Table 5 presents the results of the problem type by country. The results indicated that there is a difference between contextual and non-contextual problems among the six textbook series. Noncontextual problems were prevalent across all the textbooks, and US and Chinese textbooks contained more contextual problems compared with the textbooks of the other nations. The US textbooks contained the largest proportion of contextual problems (37.5%) and the China textbooks had the second-highest proportion

of contextual problems (30.7%). Nepal's textbooks are dominated by whopping 92.1% noncontextual problems. The textbooks from Singapore, Taiwan, and Finland also contained a high proportion of noncontextual problems.

Table 5	
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	Contextual	Non-contextual
Finland	21.1%	78.9%
United States	37.5%	62.5%
Singapore	15.8%	84.2%
China	30.7%	69.3%
Taiwan	16.6%	83.4%
Nepal	7.9%	92.1%

Cross-Tabulation of Problem Type by Country

Differences in Response Type

Table 6 presents the results of the response type by country. As shown, there is a big difference in the availability of closed- and open-ended problems among the six textbook series. The proportions of open- and closed-ended problems differed heavily among the mathematics textbooks of the six countries. The US textbooks contained the highest proportion of open-ended problems (25.4%). More than one-fourth of the algebra problems in the US textbooks were open-ended. However, more than 96% of the algebra problems in the textbooks of the other five countries were closed-ended. In addition, more than 98% of the algebra problems in the textbooks from Finland, Singapore, Taiwan, and Nepal were closed-ended.

Table 6

<i>J</i> 1	Open-ended	Close-ended
	Open-ended	Close-elldeu
Finland	1.8%	98.2%
United States	25.4%	74.6%
Singapore	0.6%	99.4%
China	3.9%	96.1%
Taiwan	1.1%	98.9%
Nepal	0.1%	99.9%

Cross-Tabulation of Response Type by Country

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Differences in Cognitive Demand

Table 7 presents the results for the level of cognitive demand of the problem for each country. It demonstrates differences among memorization, procedure without connections, procedure with connections, and problem-solving in mathematics across the six textbook series. This indicates that the distribution of cognitive demand levels for algebraic problems varied among the textbooks from the six countries. The data reveals that all six countries emphasized problems requiring procedures both with and without connections. The China textbooks had the highest percentage of problems that required doing mathematics (3.0%). The textbooks from US contained the most problems requiring procedures with connections (35.0%). The Nepali textbooks contained the most problems requiring procedures without connections (89.8%). The Taiwanese textbooks contained the most memorization problems (38.0%).

The data revealed that the textbooks from Finland, China, and the United States contained a higher proportion of problems that require doing mathematics, whereas those of Taiwan and Singapore contained few such problems. Approximately 70% of the problems in all of the textbooks other than those from Taiwan and Nepal required procedures with or without connections. The textbooks of every country except Nepal contained problems requiring each level of cognitive demand, and more than 89% of the problems in the textbooks from Nepal required procedures without connections, with a few requiring memorization (6.9%), and none required doing mathematics.

	Memorization	Procedures without Connections	Procedures with Connections	Doing Mathematics
Finland	16.5%	53.0%	27.6%	2.9%
United States.	21.9%	40.2%	35.0%	2.9%
Singapore	23.2%	44.2%	30.5%	2.1%
China	29.0%	44.4%	23.6%	3.0%
Taiwan	38.0%	44.1%	16.8%	1.1%
Nepal	6.9%	89.8%	3.3%	0%

Table 7

Cross Tabulation of Level of Cognitive Demand by Country

Discussion

This study analyzed middle school mathematics textbooks from six countries and a framework was designed to compare five aspects (i.e., the total number of algebra problems, representational forms, problem type, response type, and cognitive demand) of the textbook problems. While textbooks from all six countries share some similarities, they vary across five key aspects, likely reflecting each nation's curriculum priorities. These differences may also stem from the textbook authors' motivations, experience, and interpretations of the curriculum (Basyal & Mainali, 2023). Nevertheless, the findings reveal significant variation in opportunities to learn middle school algebra across countries.

The results revealed differences in the percentage of algebra problems in mathematics textbooks. Over 40% of the problems in the textbooks from Finland, the United States, and Singapore were algebra problems. In contrast, about one-third of the problems in the Chinese and Taiwanese textbooks and roughly one-fourth of the problems in the Nepali textbooks were algebra problems. This suggests that textbooks from Finland, the United States, and Singapore place more emphasis on algebra in middle school compared to those from China, Taiwan, and Nepal. While this distribution highlights the emphasis on algebra in middle school, it doesn't reflect the priority given to algebra in earlier grades, as some nations take different approaches to developing algebraic thinking from an early stage. Research studies highlight how different countries' elementary textbooks prioritize teaching algebra concepts in the early grades (Cai et al., 2005; Cai & Moyer, 2008; Yang & Sianturi, 2022).

The distribution of representational forms significantly differed among the six textbook series. The textbooks in each country emphasize specific forms for the students to use while learning algebra. The results revealed that more than half of the problems in the textbooks from Nepal, Singapore, and Finland were presented in the symbolic form. This finding is consistent with that of another study, which reported that the elementary school mathematics textbooks of Finland and Singapore emphasize the symbolic form for algebra problems (Yang & Sianturi, 2022). This suggests that the textbooks on the topic of algebra in these three countries put more emphasis on symbolic form problems. That is, they highlight the proficiency of algebra problems by the use of symbolic form problems. Many problems in the Taiwanese and Chinese textbooks contained the highest proportions of combined-form problems among the textbooks of the six countries. This

may reflect the new mathematics standards of China and Taiwan which highly highlight the importance of the use of multiple representations in mathematics learning and the realistic situation should be integrated into mathematics classes. Therefore, the algebra problems in the China and Taiwan textbooks include more combined forms than in other countries. Studies have demonstrated that students understand mathematical concepts if they have more opportunities to engage with problems in the combined form (Christine, 2012; NCTM, 2000; Zhu & Fan, 2006). Given the lack of visual problems in Chinese and Nepali textbooks and studies highlighting their importance, incorporating more visual content could benefit student learning.

The textbooks varied significantly in terms of problem type. The US and Chinese textbooks contained more contextual problems (more than 30%), whereas approximately 15% of the problems in the textbooks from Finland, Taiwan, and Singapore were contextual. This finding aligns with Yang et al. (2017), who noted a similar distribution of problem types across textbooks. The main difference was that Taiwanese textbooks had significantly more contextual algebra problems (16.58%) than contextual geometry problems (3.3%). Approximately 8% of the algebra problems in the Nepali textbooks were contextual. The fact that the US and Chinese textbooks contained more contextual problems may reflect the standards of their mathematics curricula (Common Core State Standards for Mathematics, 2010; Ministry of Education of the People's Republic of China, 2011; NCTM, 2000), which prioritize connecting mathematics to students' daily life. Providing more opportunities to solve context-based problems in middle school mathematics can promote meaningful learning and encourage higher-level mathematical thinking (Gu et al., 2004; NCTM, 2000). Taiwan's new mathematics standards (Ministry of Education in Taiwan, 2019) highly emphasize the integration of context-based problems into mathematics textbooks. Finland, Singapore, and Nepal may consider including more context-based problems in their textbooks. Recent editions of Nepali textbooks appear to include little more context-based problems, but studies have raised concerns about their quality (Nepal et al. 2024; Basyal and Mainali, 2025).

Closed-ended problems dominated the textbooks from each country. Most of the textbooks in our sample contained only a few open-ended problems. However, in the US textbooks, more than one-fourth of the algebra problems were open-ended, which is the highest proportion among the six textbook series. Yang and Sianturi (2022) found only one open-ended algebra problem in elementary textbooks from Finland, Singapore, and Taiwan. However, the number of open-ended algebra problems increased significantly

in middle school textbooks, though they still accounted for less than 2% of all algebra problems.

Moreover, we observed only one open-ended algebra problem in the Nepali textbooks. The inability of most 8th graders from Nepal to solve open-ended problems in the National Assessment of Student Achievement (NASA) (Educational Review Office, 2015) may be connected to this discovery. Studies have revealed that students with limited experience with open-ended problems encounter difficulties in solving nonroutine problems (Cai, 2000; Wijaya et al., 2015a). Kwon et al. (2006) discovered that the opportunity to frequently solve open-ended problems strengthens students' divergent thinking skills (p. 51) and may promote their mathematical creativity. Considering the role that open-ended algebra problems play in mathematics learning, and the scarcity of open-ended problems in all textbook series, it is desirable to put more emphasis on open-ended algebraic problems in the six textbook series. However, the question remains: Is algebra suitable for open problems? Further studies are necessary to explore this.

In terms of the cognitive demand of the problems, our results show more than 30% of the problems in the textbooks from the United States, Finland, and Singapore involved high cognitive demand. However, the textbooks from China and Taiwan had slightly fewer problems with high cognitive demand. In contrast, Nepali textbooks had the lowest proportion of high-demand problems, focusing more on algorithmic procedures and calculations, cosistent with findings by Basyal et al. (2023). Studies have suggested that the use of problems with high cognitive demand in education can strengthen students' thinking, explanation, and problem-solving skills (Boaler & Staples, 2008; Stein & Lane, 1996; Tekkumru-Kisa et al., 2015). Nepal, China, and Taiwan could benefit from incorporating more high-demand problems into their math textbooks and teaching. High-demand tasks can engage students more than low-demand ones if implemented effectively, but learning depends on several other factors. Tarr et al. (2008) emphasized that student learning is complex, though cognitively challenging tasks have the potential to enhance it.

In summary, our findings showed that the algebra problems in the six textbook series varied in representational forms, contextual features, response types, and cognitive demand. This indicates that students' opportunities to learn middle school algebra is unevenly distributed among countries. Our findings can benefit textbook writers, policymakers, researchers, and stakeholders worldwide.

Implication and Significance

The results revealed that the six mathematics textbook series differed in all five aspects. This may reflect the differences in culture and educational philosophy for each country, as suggested by earlier studies (Fan et al., 2013; Ka & Leung, 2012). Studies have argued that the content of mathematics textbooks affects students' learning opportunities, which in turn influence their learning of mathematics and subsequently impact their achievement in the subject (Cai et al., 2011; Grouws et al., 2015; Stein et al., 2007; Tarr et al., 2008). We should be concerned about students' experience working with certain problem styles (e.g., this study focused on five specific aspects), as it can lead to difficulties when they attempt to solve nonroutine or challenging mathematical problems. Hence, textbooks should provide varying opportunities for students to engage with the five specified aspects in future textbook, countries not included in the sample can gain insights into the advantages and disadvantages of the textbook designs identified in this study. These insights may contribute to the creation of improved textbooks in the future.

Our findings and earlier studies show that the algebraic content in elementary to middle school textbooks in Taiwan emphasizes memorization and computation. These computational skills contributed to the strong performance of Taiwanese students in international comparison tests. However, students in Taiwan reported mathematics as their least favourite subject (Mullis et al., 2020). Therefore, including more contextual and cognitively challenging problems may enhance Taiwanese textbooks and students' attitudes toward mathematics. Policymakers and stakeholders in Nepal can draw upon the findings presented here. Nepal may benefit from incorporating more problems in its textbooks, encompassing various representational forms and real-life situations. Furthermore, Nepal should increase the proportion of contextual problems and those with high cognitive demand. Previous studies have reported a lack of contextual, cognitively challenging problems in Nepali mathematics textbooks (Jones & Basyal, 2019; Basyal et al., 2023; Nepal et al., 2024; Basyal & Mainali, 2025). Additionally, results from NASA (Education Review Office, 2013, 2015, 2019, 2022) have indicated that most students in Nepal struggle with solving contextual problems and problems requiring high cognitive demand. Our findings revealed that Nepali students needed to be afforded more opportunities to engage with such problems. Studies have demonstrated that students can only grasp concepts covered in their textbooks (Li et al., 2008) and that their proficiency in certain types of problems decreases if they are not exposed to them (Wijaya et al., 2015a). Consequently, Nepal's students' poor performance on the NASA assessment may be attributed to their textbooks lacking exposure to certain problem types.

Limitations

This study examined one textbook series from each sample country. Although the market shares of each series are high, they are not representative of the textbooks in each country. Therefore, the generalizations of the findings need to consider the context used in this study. In selecting samples from each country, we chose what was available to us. Therefore, China and the United Textbooks are from 2006 editions. Textbook problems mostly stay the same, even if new curriculum documents specifically warrant change (Bakken & Andersson-Bakken, 2021). However, researchers may want to investigate newer editions of the textbooks, including online interactive books, in conjunction with the newer curriculum documents from the respective countries. It's crucial to note that we should have examined how the textbooks were used in the classroom. Teachers may employ varying methods when using textbooks, which could result in different student learning outcomes. Subsequent studies may investigate classroom implementation of these textbooks on students' learning.

Furthermore, the mathematics education community is shifting from discussing properties embedded within problems to examining the interactions between textbooks and students. Researchers may wish to investigate such interaction in a cross-cultural setting. We deliberately selected these five aspects of the textbooks cited as essential features in earlier studies. Future cross-cultural studies may want to investigate the textbooks using newer approaches of textbook analysis suggested in studies (Petersson et al., 2021; Lee & Guajardo, 2023; Gracin, 2018).

Despite the abovementioned limitations, this study contributes to content analysis and cross-national comparisons of mathematics textbooks.

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