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Ways of Developing Creative Thinking and Reasoning of Students in Mathematics Learning

Sandip Dhungana and Ramesh Thapa

Kathmandu University School of Education, Lalitpur, Nepal

Corresponding Author Email: sandip@kusoed.edu.np

Abstract

Reasoning skill is one of the main components of mathematical competency (NCTM, 2014). Mathematics students use mainly two types of reasoning skills namely imitative reasoning and creative reasoning in solving mathematical tasks. The textbook tasks in our context are of routine types which are solvable using imitative reasoning. Most mathematics teachers use prescribed textbooks in lesson planning and teaching (Thomson & Fleming, 2004) which ultimately promotes imitative reasoning among the students as compared to creative reasoning. Moreover, many mathematics teachers still follow the traditional chalk-and-talk banking pedagogy which does not support the enhancement of creative thinking and reasoning skills. This literature-based research study has identified some of the ways to develop the creative thinking and reasoning skills of mathematics students through the intense study of the available research articles, thesis, and books in the field of mathematical thinking and creativity. This study has found the positive impact of productive struggle and adversity quotient on students' creative thinking and reasoning skills. Likewise, problem-posing and problem-solving skills have been found to be essential skills for enhancing creativity. This study has also found modern student-centric teaching approaches like the ADI (Argument-Driven Inquiry) approach and the RME (Realistic Mathematics Education) approach beneficial in improving reasoning skills. This study also found CMR (Creative Mathematically Founded Reasoning) tasks to be helpful in increasing the creative reasoning skills of mathematics students.

Keywords: Reasoning skills, Creative thinking, Creative reasoning, Imitative reasoning

Background of the Study

According to Kings et al., (2013), higher-order thinking skills demand higher cognitive abilities rather than remembering only. In the revised Bloom's taxonomy, the skills like analyzing, evaluating, and creating are kept in the higher-order thinking skills while the skills like remembering, understanding, and applying are kept in the lower-order thinking skills (Moore & Stanley, 2010). When students face unusual problems, uncertainties, questions, or dilemmas, their higher-order thinking skills are activated (Tanujayal et al., 2017). Whereas students engaged in usual procedures using imitative reasoning helped them to be guided and using their own procedures brought novelty, flexibility, and plausibility which could be seen as Creative Mathematically founded Reasoning (Boesen et al., 2010). This creates space for students' procedural knowledge.

Developing procedural knowledge through rigorous problem-solving practice and procedural knowledge is associated with the problem type (Hiebert & Lefevre, 1986). According to Rittle-Johnson and Schneider (2015), procedural knowledge is the knowledge of procedures and procedures can either be (i) a predetermined sequence of actions that yields correct answers when executed correctly or (ii) possible actions that must be tailored fittingly to solve the problem. According to Waluya (2018), straightforward simple geometrical problems do not cultivate geometrical reasoning in students. He claimed divergent problems are powerful drivers of geometric thinking. Divergent problems are non-routine tasks that may have multiple answers and can be solved by various strategies or algorithms (Hajesfandiari et al., 2014 as cited in Waluya, 2018). For familiar problems, many of us apply reasoning based on established experience (Lithner, 2002). The reasoning based on established experience (EE) is the reasoning that has already been used and tested and can be applied in similar routine tasks. According to Vinner (1997), pseudo–analytical reasoning is not analytical but might give the impression of analytical in the routine task. The pseudo-analytical reasoning even leads to the correct answer.

Spending much time in the classroom teaching textbook problems and allowing students to invest a large amount of time in practicing textbook or textbook-related tasks promotes algorithmic and rote learning as claimed by Sidenvall et al., (2015) and Wakefield (2006). Many mathematics teachers are using this type of pedagogy as per the literature available in the field. In a cross-national study of textbooks from twelve countries including Nepal (Jador, 2015), it has been found that 79% of tasks are of routine type. That means teachers have been promoting algorithmic reasoning to the students. Students cannot acquire mathematical competency by being proficient in solving routine tasks only. Mathematical competency is acquired when students are able to solve non-routine problems and translate classroom mathematics in a contextual setting. It can be possible only when students are trained to use creative reasoning.

If we analyze the results of students in mathematics in the context of Nepal, we can see a pathetic situation in mathematics (Dhungana, 2018). According to status results published by the Ministry of Education (MOE), 47.229% of students failed mathematics in 2012. In 2013, the failed percentage reached 52.90%. The failed percentage in mathematics increased to 65% in 2014. In 2015 the failed percentage again increased by 5% and reached 70% (MOE, 2012, 2013, 1014, 2015 & 2019). Since 2016, the letter grade system has been introduced and the grades secured by students in mathematics since then till this date show a similar status (Dhungana, 2023).

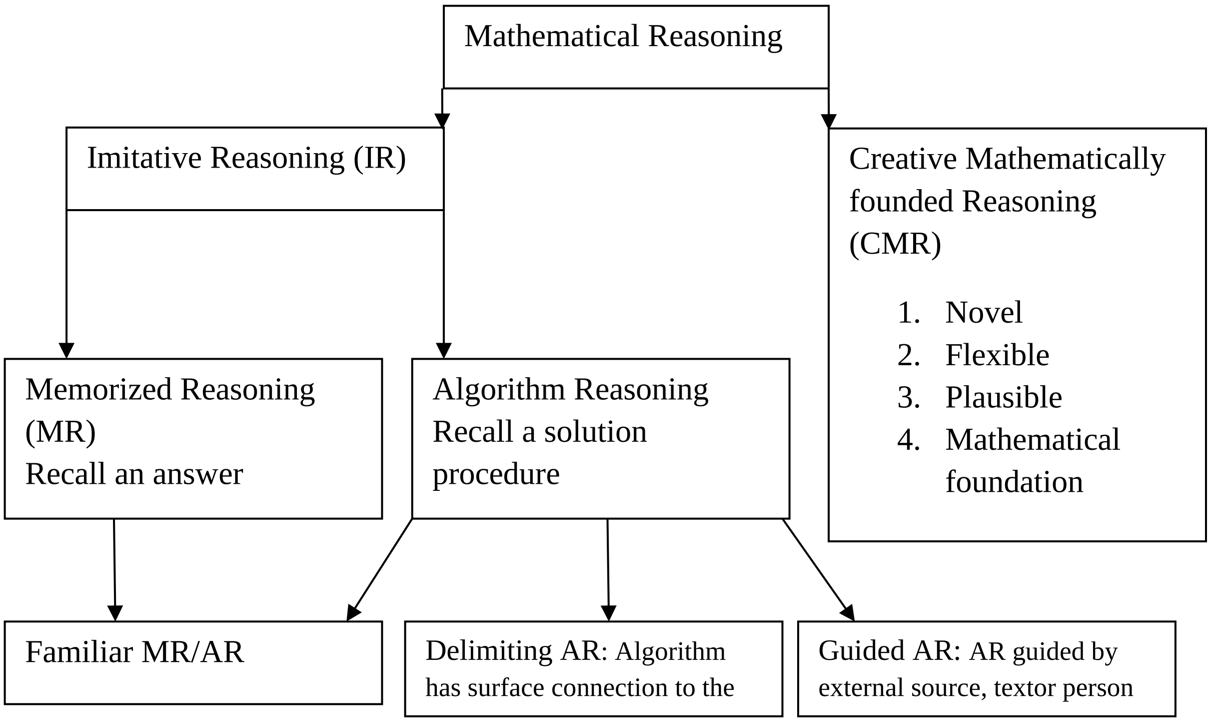
In Nepal, the National Assessment of Student Achievement (NASA) was started in 2011 by the Education Review Office (ERO) to assess students' achievement in selected grades in subjects like Nepali, Mathematics, and social studies. The assessments done by NASA in 2013 among eighth-grader students claimed that students are weak in reasoning, critical thinking, and making figures and shapes (Manandhar, 2018). From the NASA brief report of 2017, it has been observed that 46% of students in grade 8 are below level 4 out of 6 levels of performance according to their capacity. The report claims that the result in mathematics in 2017 is less as compared to the result in 2013. This indicates that a huge mass of students is underperforming level in mathematics.

Why huge mass of students is not performing well in mathematics at the school level? There could be various reasons like school and home environment, students' motivations and self-efficacy, teaching strategies, reasoning skills of students, and many more. According to Frosch and Simms (2015), research has shown a close relation between reasoning skills and mathematical achievement. Mathematics reasoning is the sense-making process. It is the process of understanding the ideas and concepts associated with the procedures (Bieda et al., 2013). Mathematical reasoning skills is the combination of five interconnected processes of mathematical thinking such as sense-making, conjecturing, convincing, reflecting, and generalization (Blujand, 2007). Sense-making is an endeavor to give meaning to something. In mathematical problem solving, sense-making is an attempt to dig out mathematical ideas intertwined with the question. Conjecturing is an assumption based on the information obtained from the problem. Convincing is making one or others believe the truth of something. Reflecting is the careful and thoughtful observation of own process. Generalization is deducing one general conclusion from the observations which can be implemented in other similar situations. Reasoning skill is an integral part of understanding mathematics (National Council of Teachers of Mathematics [NCTM], 2014). Lithner (2008) has divided reasoning skills mainly into two categories- imitative reasoning and creative reasoning. Both types of reasoning skills are important to foster mathematical competency. Imitative reasoning is used to solve mathematical problems with lower-order thinking skills and creative reasoning skills are used to tackle with higher-order thinking skills.

Imitative Reasoning (IR)

Imitative reasoning is the students' memory that is often suitable for the routine task (Mumu & Tanujaya, 2019) which can be further branched into two branches memorized reasoning and algorithmic reasoning. Memorized reasoning is based on simply recalling the formulae, answers, and strategies which students have already practiced in a similar situation and solely depends on the memory capacity. On the other hand, algorithmic reasoning is remembering the sequential steps that should be carried out to reach the answer.

Figure 1: *An Overview of Empirically Established Reasoning Types*



Source: Lithner (2008); A framework for logical reasoning

Creative Reasoning (CR)

Lithner (2008) has found creativity, plausibility, and mathematical foundation as three key elements to distinguish creative reasoning from imitative reasoning from the analysis of literature. A strategy designed by students to tackle a problem is said to be creative if it is not copied from previously existing templates. A plausible strategy is one that has arguments based on mathematical ground which can increase the certainty of the strategy. If we combine the idea of Haylock (1987) and Silver (1997), we get fluency, flexibility, and novelty as distinguishers of creative reasoning from imitative reasoning. Fluency is the smoothness and speed that is seen in solving the task. Flexibility is the capacity of the problem solver tale different approaches to adopt in a new situation. According to Lithner (2008), a task-solving situation is said to be Creative Mathematically founded Reasoning (CMR) if it fulfills the conditions of novelty, plausibility, flexibility, and mathematical foundation.

In every examination, a certain percentage of questions demand the high-order thinking skills of the students. A low number of students securing high grades and a high number of students securing low grades in examinations is an indication of using creative reasoning in problem-solving by very few students. For example, in the SLC result of 2016, 23,334 students had secured A^+ and 1, 46, 935 secured E in mathematics out of 495786 students. If we analyze the international assessment in mathematics, we see a similar result. The Program for

International Student Assessment (PISA) is a worldwide study that compares the competency of 15-year-old in mathematics, science, and reading. According to the result of PISA – 2018, China is at the top position with an average mean score of 591 out of 78 countries whereas the Dominican Republic is at the bottom with an average mean score of 336. Among 78 participating countries, only 21 countries scored more than 500. In terms of percentage, only 26.92 percent have secured more than 500 points. Why were only a few countries able to score above 500? There could be various reasons. One reason may be the lack of proper reasoning skills among mathematics students. Even the high scorers might not have used creative reasoning. By the use of imitative reasoning, only students might have secured good grades. Mumu and Tanujaya (2019) argued that most of the tasks exhibit a heavy focus on imitative reasoning and there is a lack of research works on measuring creative reasoning skills of mathematics students. This helped us inquire on the ways of developing creative thinking and reasoning of students in mathematics learning. Mathematics students use imitative reasoning and creative reasoning in task-solving. Most of the routine tasks can be solved by imitative reasoning while non-routine tasks demand creative thinking and reasoning. There must be ways of increasing creative thinking and reasoning skills. In this scenario, this study is guided by a research question; What are the ways of developing creative thinking and reasoning of students in mathematics learning?

Research Methodology

This is literature-based research. The data were collected from various research articles, thesis, journals, and books with intensive study. As this research issue was focused on finding ways of increasing creative reasoning, both authors started to research the appropriate literature. We reviewed literatures on productive struggle, problem posing and problem-solving, adversity quotient, argument-driven inquiry approach, realistic mathematics education etc. to articulate this study. Based on this literature we generated five themes for this research. Then we analyzed and interpreted the data using these five themes and reached a conclusion.

To enhance this article, we started with figuring out the types of reasoning used by mathematics students first we studied the literature articles like Lithner (2008), Permatasari, Darhim, & Jupri (2020) and Jonsson, Mossegård, Lithner, and Wirebring (2022). To enhance knowledge about the types of reasoning we studied another research article by Berqviest (2007) on titled, *"Types of reasoning required in university exam in mathematics."* The finding of his research article was that about 70% of university exam questions are solvable by imitative reasoning. To make this search intense we reviewed the books "How to solve it" by Polya (1954) and "How we think" by Dewey (1997). After going through these books, we acquired the knowledge on the solving steps of mathematical problems. Furthermore, we gained the concept on demonstrative reasoning and plausible reasoning (Polya, 1954). We found similarities between imitative reasoning and demonstrative reasoning. Likewise plausible reasoning and creative reasoning both have similarities. Both of these reasoning advocated for fluency, flexibility and novelty. We also went through some of the national and international review reports such as ERO, NASA, and PISA etc.

Data Analyses and Interpretation

After going through the research article "A research framework for creative and imitative reasoning" by Lithner (2008), we found the term "productive struggle" in the research article by Norqvist (2017) and Daily (2021), in which the importance of struggle of students was argued as an essential component for enhancing mathematical reasoning competency. We collected more literatures on productive struggle like "Productive struggle for deeper learning" by Bullmaster-Day (2015), "Strategies to support productive struggle" by Warshauer (2015), "Productive struggle in mathematics" by Pasquale (2016) etc. We also skimped through many literatures related to the struggle of the students in tackling with the mathematical problems and found that productive struggle induces the creative reasoning of the students. That is why we generated the first theme of this research article as "Productive struggle boosts the creative reasoning". We also went through many literatures (like Calabrese, Capraro, & Thompson, 2022; Polat, & Ozkaya, 2023; Bonotto, & Santo, 2015; Akben, 2020) on problem posing and problem solving to investigate their effect on the creativity of the students. A study by Silver (1997) showed the connection of problem posing and problem solving with fluency, flexibility and novelty which are the core components of creativity. Similarly, the study by Ayllon et al. (2016) revealed the existence of relationship between the development of mathematical thinking and creativity with the problem posing and problem solving. We found the similar results after reviewing more research work by other scholars such as Garcia (1998), Harpen and Sriraman (2013), Sinuger et al. (2011) etc. So, based on these available research works and literatures, we generated second theme as problem "problem posing and problem solving enhance the creative thinking". In the search of third theme for research article we acquired the concept of Adversity Quotient (AQ) through the book "Adversity quotient: Turning obstacles into opportunities" by Stoltz (1997). We found research works of many scholars like Hendriana (2017), Suryadi and Santosh (2017), Hendriana et al., (2018) etc. on the effect of adversity quotient on mathematical reasoning. Similarly, we also found more scholars like Hidayat et al. (2018), Rosidin et al. (2019), etc. who studied the effect of adversity quotient and argument-driven inquiry on mathematical creative reasoning skills. As we found the positive influence of adversity quotient and adversity quotient on creativity and mathematical reasoning, we generated the third theme as "Adversity quotient, argument-driven inquiry, and reasoning ability". The further study acquainted us with the Realistic Mathematics Education (RME) approach was developed in Netherlands in 1971. We studied the characteristics of RME approach. We searched the research work on RME and mathematical reasoning ability. We found the experiment research of Lestari and Surya (2017) which showed a better conceptual understanding of students after applying RME approach. Likewise, we studied the research work of Laurens et al. (2017) and found their research work showed an improvement on the cognitive level of students with RME approach as compared to the conventional approach. We also studied few more literatures on the RME approach and generated “Realistic Mathematics Education (RME) and creative thinking” as our fourth theme. In the search for our fifth theme, we began to investigate on the types of task that students are facing in mathematics class. Studies by many scholars like Newton and Newton (2007), Shield and Dole (2008), Norqvist (2018) etc. showed that more textbooks are solvable by using imitative reasoning and only few tasks need creative reasoning. In 2010 Boesen, Lithner, and Palm carried out a study to investigate the relation between the types of assessment tasks and the mathematical reasoning used by students by dividing task into two categories as high relatedness task and low relatedness task. The five themes are below that we generated after reviewing the available literatures.

Productive Struggle Boosts the Creative Reasoning

The struggle in any field which helps to develop creative ideas, strategies and procedure to reach to the final answer is taken in the positive sense as a productive struggle. Productive struggle in mathematics is an opportunity for delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions (NCTM, 2014). To acquire a deep understanding of the mathematical structure of the problem students must persevere. Perseverance is the essential element of the productive struggle (Pasquale, 2016) and it is the continuing forward to explore the different dimensions of the procedures of the problems irrespective of the difficulty. When persistent students reach the dead end of a challenging problem, they follow another strategy leaving the previous one. The application of various strategies one by one to solve a demanding problem broadens the analytical power of the students. Hiebert and Wearne (2003) stated that students need to struggle with challenging problems to learn mathematics deeply. Along the same line, Hiebert and Grouws (2007) claimed that struggle is a necessary component of learning mathematics with understanding. Warshsuer (2015) also mentioned that the productive struggle help in learning mathematics with deep understanding.

Students engage in productive struggle when they attempt to solve confusing problems or try to make sense of challenging ideas. Therefore, a good teacher involves students in a problem-solving situation where the students feel unfamiliarity, difficulty, and a little bit of frustration that could serve as a learning opportunity (Star, 2015). To prevent the productive struggle from becoming total frustration for the students, a teacher has to be very careful about his/her role. Teachers should acknowledge students’ anxiety; guide them with probing questions; encourage them to reflect on their work and should give them enough time to struggle by not entering into their work too soon or too much (Warshsuer, 2015). In the same way, Permatasari (2016) mentioned the function of the teacher. According to him, teachers provide support and guidance for needy students to move through uncertainties. They help with representing a mathematical relationship, justifying reasoning, or finding the solution strategy. In the productive struggle, students should have faith in their capabilities of doing well in mathematical tasks with their effort and perseverance in reasoning, sense-making, and problem-solving (Permatasari, 2016). When teachers and students believe that understanding and abilities can be developed through persistence, productive struggle gives a positive output.

From the analysis of the literature, it is seen that students have to struggle with challenging problems to enhance creative reasoning. In a class that accepts and supports the productive struggle of the students, the mistakes of the students are expected and accepted. The paths taken by the students are more valued than the answers believing that students learn the best from their own mistakes. Students are encouraged to create their own solution procedures investigating on “how” and “why” rather than just mimicking the readymade procedures. Hence, it can be concluded that the productive struggle boosts the creative thinking and reasoning of mathematics students.

Problem-posing and Problem-solving Enhance Creative Thinking

Problem-posing and problem-solving are two sides of the same coin. Posing and solving problems strengthen learning (Allyon et al., 2016). Both of these problems posing and problem-solving are necessary activities for mathematical knowledge building (Noda, 2000, as cited in Allyon et al., 2016).

Creativity

Scholars have defined creativity in various ways. Some scholars (like Vuichard, Botella, & Capron Puozzo, 2023; Forster, 2015) have defined creativity as a process and some scholars have defined it as a product (Haylock, 1987). Some of the definitions have connected creativity to thinking style. According to Nadjafikhah and Yaftian (2013), creative thinking is a dynamic mental process that includes convergent and divergent thinking. Convergent thinking brings all the ideas to a single path. Divergent thinking involves the creative generation of multiple answers or multiple solution strategies to a problem. Ervynck (1991) has connected mathematical creativity with advanced mathematical thinking, defining creativity as the ability to raise important mathematical questions and find inherent relationships. Likewise, Liljedahl and Sriraman (2006) have defined mathematical creativity as the ability to produce original work that significantly extends the body of knowledge or opens up avenues of new questions for other mathematicians.

Problem Posing

In the traditional transmission–reception model of teaching students solves textbook problems and problems posed by teachers. Students are given less chance to pose new problems. But the contemporary constructivist theories of teaching and learning have acknowledged the importance of problem-posing by the students (Silver, 1994). Mathematical problem posing is the generation of new problems and the reformulation of given problems (Silver, 1994). Students can generate problems before, during, and after the solution of a problem. The new problem is posed before the solution. During the solution process, students can pose various problems reformulating the given problems to investigate the different avenues of the problem and then focus on one avenue for better understanding the problem and solving it (Silver et al., 1996). According to Silver (1994) even after solving a problem, students can pose the problem by modifying the objectives, conditions, and questions in the problem. In mathematics class, students have to be very imaginative, thoughtful, and critical before posing a problem. While posing a new problem, students have to analyze the formulation critically and check the sufficiency of the data in the problem to choose the solving strategy to reach the solution (Allyon et al., 2016). They further claim the necessity of a high abstraction level and reflection in posing problems which allow students in reaching the reasoning phase that assists in mathematical knowledge building. The importance of problem-posing in mathematical knowledge building has been argued by many scholars (Stoyanova & Ellerton, 1996; Fernandez, 2013; Ayllon & Gomez, 2014; as cited in Allyon et al., 2016).

Problem-solving

A mathematical problem is a question that has to be solved by using mathematical ideas and concepts. Problem-solving is a process to reach a solution. When learners have to solve a problem, they have to examine the data, think and analyze critically for strategy and finally carry out the best strategy (Allyon et al., 2016). Dewey (1997) has kept five steps of problem-solving in his book "*How We Think"*. His five steps are defining the problem, brainstorming for the idea, evaluating all the solutions, implementing the best solution, and reviewing the result. Similarly, Polya (2004), in his book "How to solve it?" specified Heuristics techniques in problem-solving. His technique consists of four steps which are understanding the problem, making planes, carrying out the plan, and looking back. The second step of Dewey's "Brainstorm for idea" seems the same as the second step of Polya's "Make Planes" both of which advocate for the creativity of the problem solver. Similarly, the last step of Deway's "review the result" and the last step of Polya's "Looking back" hint to the solver that the selected plan or procedure might or might not take to the correct answer. In case of a wrong answer, the solver has to revisit and create the new solution method. Designing the different solution procedures in mathematics is based on reasoning skills of mathematics students.

Scholars have viewed problem solving form various angles. Polya (1965) have taken problem solving as a barometer of mathematical knowledge (as cited in Allyon et al., 2016). Brown (1987) has said problem solving generates new knowledge. Similarly, Koestler (1964) has linked problem solving with the creativity. According to him problem solving sparks the creativity. Garcia (1998) has also related creativity with the problem solving. Carrillo (1996) has taken problem solving as one of the ideal frameworks for meaningful learning because this framework helps in the development of critical and open attitude (as cited in Allyon et al., 2016).

According to Silver (1994) creativity is seen in the interplay between problem posing and problem solving. It is the interplay of formulating, attempting to solve, reformulating and finally solving the problem. To pose a problem and solve a problem, students have to think and analyze critically. Hence it can be said that problem posing and problem solving enhances the creative thinking and creative reasoning of the students.

Adversity Quotient, Argument Driven Inquiry and Reasoning Ability

The concept of Adversity Quotient (AQ) was first introduced by Paoul Stoltz in 1997. Adversity Quotient is the science of resilience and it measures how one respond to adversity. Shivaranjani (2014, as cited in Naimnule et al., 2020) describes how good someone's ability and struggle are in solving a problem. Quitters (Low AQ), campers (Middle AQ) and climbers (High AQ) are the three categories of AQ (Stoltz, 1997). Climbers easily face the difficult problems while quitters feel very hard to face the difficult problems (Phoolka & Kaur, 2012). According to Hendriana (2017) claims that quitter students are accustomed to solving problems by completing algorithm guided by teachers under direct learning method. Such direct learning methods are less innovative and the quitters' ability is limited to solving routine problems. As a result, the creative reasoning ability of quitters is low as compared to campers and climbers.

Suryadi and Santosh (2017) conducted a research study to measure the effect of self efficacy and adversity quotient of grade IX students regarding achievement in mathematics in Jakarta, Indonesia. With the survey method they found that both the internal factors self efficacy adversity quotient significantly affects the achievement in mathematics. According to Bandura (2010) self-efficacy is concerned with person's belief in his/her capabilities to produce given attainment. Self-efficacy and adversity quotient evoke students to struggle with the problem. In our belief both of them enhance the reasoning skills of the mathematics students as a result they effect in the achievement in mathematics

Argument Driven Inquiry (ADI) is a student centric learning process under guidance by the facilitator. Simpson et al. (2010) argued that Argument-Driven Inquiry (ADI) has seven stages which are: (1) Problem identification, (2) Designing methods and data collection, (3) Analyze data and develop initial arguments (4) Argumentation (5) Write a report (6) The process of blind peer-review (6) Revise and submit report. Hidayat et al. (2018) took mathematics teacher candidate in Kote, Cimahi, Indonesia as their population from whom they selected 60 mathematics teacher candidates as a sample. After the completion of their research work, they found types of adversity quotient affects the improvement of student's mathematical creative reasoning. They also found the improvement on the mathematical creative reasoning of students' who received argument driven inquiry. Rosidin et al. (2019) also carried out research study to determine the effect of Argument-Driven-Inquiry (ADI) learning model on students critical thinking skills based on difference in academic abilities, gender and personality type. They used the non-equivalent pretest-posttest control group design and one group pretest-posttest design. Their research was conducted on eight grade students of public and private schools. The finding of their research was that the ADI learning model was more effective in improving students' critical thinking skills than the conventional learning model. According to their finding gender and personality types did not show any significant difference in student's critical thinking abilities. Hence ADI learning model can be applied to all genders and personality to improve critical thinking skills. In terms of academic ability, the ADI model was found to be more effective in improving critical thinking skills of students who have a high academic ability than those who have low academic ability.

Adversity quotient is the capacity of students to fight with the difficulties that arise during the problem solving. Various studies have shown adversity quotient significantly affects the mathematical understanding, mathematical achievement and reasoning skills. Argument driven inquiry is a student centric method where students are allowed to solve problem by the exchange of ideas among them. Their learning is further accelerated by the probing questions. Research works on argument driven inquiry and creativity have revealed that argument driven inquiry is better option for increasing critical thinking ability as compared to conventional teaching method. It can be applied with students of different academic abilities, gender and personality.

Realistic Mathematics Education (RME) Approach Increases Creative Thinking Skills

Realistic Mathematics Education (RME) is an approach in mathematics education. This approach was developed in Netherland in 1971 by a group of mathematicians' form Freudenthal Institute of Utrecht University. RME approach uses real situation to construct mathematical concepts. Meaning making process is more important than the algorithmic process. Yuwono (2007) has pointed out understanding the contextual problems, discussing the problems and providing solutions to the problems as the main characteristics. Whereas Clements and Sarama (2013) have pointed out five characteristics of RME which are a) application of meaningful context, b) the development of model which lets the transformation happen form contextual to formal mathematics, c) the recreation of mathematics concepts by the students d) interaction between students and teachers and e) perception of mathematics as an integrated subject. These characteristics indicate that RME is a student centric approach. According to Wahyudi et al. (2017), in RME approach classroom is not considered as a place to transfer mathematical knowledge form teacher to students, but rather a place where students can reinvent mathematical ideas and concepts through exploration of real problems.

Lange (1996) claimed that RME approach has been adopted by most countries in the world such as England, Germany, Denmark, Spain, Portugal, South Africa, Brazil, United States of America, Japan and Malaysia. According to Lestari and Surya (2017) the RME-based textbook series "Mathematics in Context" has a considerable market share in the United States of America. Indonesia has also developed their version of RME as "Pendidikan

Mathematika Realistik Indonesia" which is abbreviated as PMRI (Lestari & Surya, 2017; Yuniarti, 2016).

Some of the research literatures have shown RME to be useful in enhancing the creative thinking skills in mathematics students. According to Saefudin (2012) RME can increase students' logical, critical and creative thinking. Lestari and Surya (2017) conducted experimental research. The aim of the research was to know whether RME approach is effective on ability of students' mathematical concept understanding or not. They took all students of grade VIII SMP Negeri 20, Medan, North Sumatera as a population. The sample was taken randomly form the population. The sample was divided into two groups with experimental group having 34 students and controlled group having 33 students. They followed "posttest only control design". They found ability of students' mathematical concept understanding who obtains learning topic using RME approach achieves individual mastery and classical mastery; and they confirmed that the ability of students' mathematical concept understanding who obtains learning topic using RME approach is better than the ability of students' mathematical understanding who obtains learning topic with lecture method.

RME approach has been adopted by many countries in teaching and learning mathematics. Real situations are used to construct the mathematical concepts in RME approach. That means students are provided an opportunity to learn with the contextual problems. Inside the classroom students reinvent mathematical ideas and concepts through exploration of the real problems. Students think, discuss and share their ideas and findings. The findings of the several researches on RME have shown that RME approach is better than lecture method in increasing the logical, critical and creative thinking and reasoning skills of students.

Creative Mathematically Founded (CMR) task Increases Creative Reasoning

Lithner (2008) identified two major reasoning skills in mathematical problem solving. First is algorithmic reasoning (AR) and second is creative reasoning (CR). The reasoning that is based on recall of previously learned formulas and algorithmic process in solving a familiar problem is AR whereas the reasoning that has to be used to construct the solution process in solving unfamiliar problem is CR. According to Lithher's framework (2008) a creative reasoning which consist of novelty, fluency, flexibility and plausibility is the creative mathematically founded reasoning (CMR).

CMR Task

Based on Lithner's reasoning type, two types of the mathematical task have been identified as AR tasks and CMR task by scholars. The task that can be solved by the use of algorithmic reasoning is called AR task and the task that demands creative mathematically founded reasoning is called CMR task. According to Norqvist (2016), a task is said to be CMR task if there is no provided formula or if the students have not done it previously. In such CMR tasks students have to base theirs on what they already know and should use this previously known knowledge to construct a new idea. For example, after teaching students the process and formula for finding the area of the triangle, if students are allowed to find the area of the parallelogram for the first time without teaching them the formula and process, the students may use the concept of finding the area of a triangle to find the area of the parallelogram. Here this type of problem can be considered as a CMR task where students have to construct their own solution process using creative mathematically founded reasoning.

Norqvist (2018) claimed that textbooks keep more tasks demanding algorithmic reasoning at the first part of the exercise and a few tasks demanding creative reasoning in the last part of the exercise. According to Sidedvall et al. (2015), students solve the selected tasks form the exercise book and often do not reach to the questions demanding creative reasoning. In 2012, Berqvist and Lithner studied mathematical reasoning in teachers' presentations while teaching mathematics. They noticed that teachers' presentations also were dominated by the algorithmic reasoning task based on procedural knowledge. Their presentations were focused on the solution process of how tasks should be solved rather than the concepts behind the procedures. But, Miller et al. (2016) have found conceptual instruction to be more beneficial than procedural instruction in promoting a more thorough understanding of procedures and concepts. In such a scenario Norqvist (2018) has felt the necessity of more conceptual tasks both in the textbooks and in teachers' instructions. Lithner (2008) claims that problem-solving has the potential to elicit mathematical reasoning. According to him, mathematical reasoning is the key variable in learning mathematics through problem-solving. Students use algorithmic reasoning (AR) in solving algorithmic problems and creative mathematically founded reasoning (CMR) in solving creative problems (Lithner, 2008). Algorithmic problems are routine problems that can be solved by copying previously used solution procedures. Creative problems are non-routine challenging problems. To solve them, students, have to create either the whole or a part of the solution themselves using CMR. The reasoning used by the student is called CMR if it is novel, plausible, flexible and mathematically founded.

On the basis of reasoning skills, two types of tasks have been identified by scholars as AR tasks and CMR tasks. Algorithmic and imitating reasoning are used to crack the AR task whereas creative mathematically founded reasoning is used to construct a part or the whole solution process of the CMR task. The researchers have shown that the students practicing the CMR task performed better than the students practicing the AR task. Hence it can be concluded that the CMR task increases the creative reasoning skill of mathematics students.

Findings and Conclusion

From the above discussion on the themes based on the available literature, we found that creative thinking and reasoning skills can be enhanced by engaging students in productive struggle in solving challenging and non-routine tasks. Problem-posing and problem-solving skills open the various avenues of the learner thereby enhancing their creativity. The adversity quotient is the capacity of a student to continue to struggle with the problem even when students are not able to reach the correct answer. From the available literature, it is found that the adversity quotient boosts the creativity of the students. Argument-driven inquiry is the student-centric approach to teaching and learning. After the review of some of the research work, we figured out that the argument-driven inquiry model is better than conventional teaching in developing creative thinking and reasoning skills in students. This model can be applied to all gender and personality types. We further noticed that like the ADI model, the RME approach is better than the conventional approach in enhancing the critical reasoning skills of mathematics students. We found many tasks present in the textbooks are routine type which could be solved using algorithmic reasoning by imitating either from the teacher or from the templates from the textbook. According to the existing literature, such tasks hinder the creativity of the students whereas CRM tasks enhance the creative thinking and reasoning skills of the students.

Based on the above literature and discussions finally, we conclude that productive struggle boosts creative reasoning, problem posing and problem-solving enhance creative thinking, adversity quotient with an argument-driven inquiry approach, RME approach, and CMR task are the ways to lead any Mathematics learners towards the better achievement along with the reasoning capacity in Mathematical ideas and concepts. After going through the many research articles and other research studies (Warshauer, 2015); Bullmaster-Day, 2015; Pasquale, 2016; Silver, 1997; Allyon et al., 2016; Hidayat et al., 2018; Harpen & Sriraman, 2013; Suryadi & Santosh, 2017; Lestari & Surya, 2017), we also conclude that reasoning activities in Mathematics learning can also develop the creative thinking in the learners which may give rise to the new ideas and can promote in the development of new concepts in mathematics. In this sense, we have denouement that a creative learner who has the reasoning capacity in Mathematics not only can solve the routine base problems of the textbook but can be able to solve the other mathematical problems and illusions found in his/ her academic carrier and in real life too in a creative manner.

The Implication of the Study

This study has pointed out some of the ways to enhance the creative thinking and creative reasoning of the students. The finding of this study might be helpful to mathematics teachers in designing their pedagogy which enhances the creativity of the students. This study might also evoke teachers, and teacher educators, to search for other possible ways to boost the creative reasoning skills among students. This study might also be helpful to textbook writers and curriculum developers. This study has figured out a few ways for improving creative thinking and creative reasoning skills. There might be many more ways to be uncovered. So, this study might be useful for future researchers and practitioners working in this field.

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