Mathematical Analysis of Hemodynamic Parameters of Blood Flow in an Artery

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Abstract
In blood rheology we study volume flow rate, blood pressure, velocity, viscosity and shear stress of blood. Cross-sectional area plays an important role for smooth flow of the blood. But some other parameter like composition of blood, length of vessel also affects in the flow rate and pressure of blood. Velocity and volume flow rate are derived by using Poiseuille's equation.This work presents a mathematical model of blood flow that was created using the N-S equations and computer simulation. Graphs are used to analyze the results.
Keyword: *Blood flow, N-S Equation, Hemodynamic, Cardiovascular System, Hypertension.*

1. Introduction

Blood is a fluid or liquid tissue or transport liquid mainly composed of 45% cells having different shapes and sizes and approximately 55% fluid plasma, which is an aqueous polymeric and ionic solution that contains 93% water and 3% particles like electrolytes, organic molecules, proteins, and trash. Furthermore, it is a freely flowing, opaque red liquid that is denser and more viscous than water [1]. The three main cell types found in blood: red blood cells(RBC), white blood cells(WBC), and platelets are suspended in plasma. Blood composition varies from person to per- son in terms of the numbers of different types of cells, which effects flow behavior and its viscosity. Calculating viscosity is difficult due to the change in size, non-uniform distribution of RBC, and shear thinned property of blood [2]. White blood cells combat infections, while platelets, which are found in the blood as tiny cells, aid in blood clotting and transfer oxygen to the tissues throughred blood cells. Blood is transported by blood vessels (arteries and veins) [3]. The heart pumped the blood as well as blood carries oxygen and nutrients to every cell of the body, so as waste product and carbon dioxide [5].

The Cardiovascular or circulatory system that consists the heart and vessels of the body. The heart uses the extensive, complex network of blood veins to transport oxygen and other essential components throughout our entire body [30]. This network also takes away items that our body doesn't require and transports them to organs that can dispose of the trash. The circulatory system or blood-vascular system are other names for the cardiovascular system. It is made upof the heart, a muscle pump, and a closed network of blood vessels known as arteries, veins, and capillaries [32].

According to Yildirim[6], the RBC around arteries travel to the center of the artery during theflow process, causing the hematocrit ratio to drop significantly close to the wall of artery andthe blood viscosity to not alter significantly with shear rate. The blood's viscosity is furtherdecreased by the high shear rate close to the arterial wall. Therefore, blood may be regarded asa Newtonian fluid for flow issues in major blood vessels [8]. The study of blood flow behavior, it's composition and adaptation according to the situation is called blood rheology.

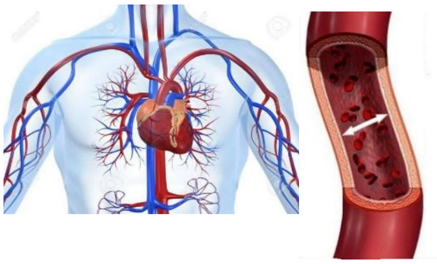


Figure 1: Cardiovascular system [34] and interior opening of an artery [35]

For the sake of human health, this investigation is crucial. The majority of scientific studies focus on how blood moves via arteries and veins. Understanding the factors responsible for high blood pressure is alsoone of the goal of this research. According to previous research, one of the causes of hypertensionis blood vessel narrowing[16].

Sharan and Popel [20] have studied the blood flow in narrow tubes with increased effective viscosity. In this article, no sleep condition is assumed at the wall, and mathematical relationsare derived for volume flow rate, viscosity, pressure drop, and wall shear stress. Medvedev and Fomin [14] have studied the blood circulation model of both large and small vessels, with bloodas a liquid-solid mixture. In this article, effective blood viscosity, pressure, and volume flow rate are calculated using an empirical formula. Roux et. al. [12] have discussed the endothelial layer sensed fluid shear stress and calculated various mathematical formulae for shear stress, pressure, viscosity, and volume flow rate.

The simulations in this paper are based on a numerical model of blood flow and blood pressure. The governing equations that reflect this problem will be derived using N-S equations in orderto simulate it. Some assumptions are made in order to create the blood flow and blood pressure model. Among them is the fact that blood vessels are cylindrical, flexible, and have circular cross sections. Blood is regarded as a Newtonian fluid subject to the continuity and N-S equation [13].

2. Mathematical Model

2.1 Poiseuille's equation

It state that the rate of flow Q of any liquid through a narrow tube is given by[15]

$$Q = \frac{\pi R^4}{8L\vartheta} P \qquad (1)$$

Where,

R = radius of tube, ϑ= viscosity of fluid, P = pressure and L = length of tube.

Let a fluid with viscosity ϑ is flowing in streamline motion through a horizontal tube with radius R and length L. When the condition becomes stable, the velocity of flow along the cylindrical axis having radius x be v. Now, in accordance to the Newton's law of viscous flow, the backgrounddragging force (F_d) on the liquid shell with surface area of cylinder A_1 is given by [22]

$$F_d = \vartheta A_1 \frac{dv}{dx} = 2\pi\vartheta x L \frac{dv}{dx} \quad (2)$$

If P is the difference of pressure between the two ends, then the force (F_a) accelerating the liquidis given by

$$F_a = P \times A_2 = \pi P x^2 \quad (3)$$

Where,

$A_2 = \pi x^2$ is the area of opening end of the cylinder. Now, for the steady flow of liquid,

$$F_d = -F_a$$

$$Or,\ 2\pi\vartheta x L \frac{dv}{dx} = -\pi P x^2$$

$$Or,\ dv = -\frac{P}{2\vartheta L} x dx$$

On integration

$$v = -\frac{Px^2}{4\vartheta L} + C_1 \quad (4)$$

Now,

Using boundary condition $v = 0$ at x = R, then $C_1 = \frac{PR^2}{4\vartheta L}$ and equation 4 becomes

$$v = -\frac{Px^2}{4\vartheta L} + \frac{PR^2}{4\vartheta L} = \frac{P}{4\vartheta L}(R^2 - x^2) \quad (5)$$

The amount of fluid Q flowing through the entire tube per unit time is [23]

$$Q = \int_0^R 2\pi x v dx = \int_0^R 2\pi x \frac{P}{4\vartheta L}(R^2 - x^2)dx = \frac{P\pi R^4}{8\vartheta L} \quad (6)$$

This is the *Poiseuille's equation*. The arterial blood flow is frequently computed using Poiseuille's equation.

2.2 Cardio vascular system equation

The velocity components in the x, y, and z directions are commonly referred to as u, v, and w, respectively. Let, ρ be the blood density, P be the blood pressure, and ϑ be the blood's dynamic viscosity. The N-S equation in Cartesian coordinate system disregarding the directionof gravity within the body is [11]

$$-\frac{\partial P}{\partial x} + \vartheta\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) \quad (7)$$

$$-\frac{\partial P}{\partial y} + \vartheta\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) = \rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) \quad (8)$$

$$-\frac{\partial P}{\partial z} + \vartheta\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) \quad (9)$$

N-S equations in Cartesian coordinates are represented here by equations 7, 8, and 9. An artery segment's axis serves as the z-axis in a cylindrical coordinate system (r, z, t), while the radial and circumferential directions serve as the r and t axes, respectively. Blood is thought of as an in- compressible Newtonian fluid with axially symmetric flow [24], and the artery channel is designed to be a circular, rectilinear, de-formable, thick shell of an isotropic, in-compressible material. It should not move longitudinally. Changing the variables in the Cartesian equations under the assumption that there is no tangential velocity and no x-y components of velocity will lead to the following system of equations [18]

$$\frac{\partial u}{\partial t} + w\frac{\partial u}{\partial r} + u\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \vartheta\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2}\right) \quad (10)$$

$$\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial r} + u\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \vartheta\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} + \frac{w}{r^2}\right) \quad (11)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rw) + \frac{\partial u}{\partial z} = 0 \quad (12)$$

Where,

$u(r, z, t)$ represents component of axial flow towards the z-direction and $w(r, z, t)$ representscomponent of the radial flow. Now the equation of continuity is written as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho w)}{\partial z} = 0 \quad (13)$$

We create a new variable called η to represent the radial coordinate as $\eta = \frac{r}{R(z,t)}$

Where,

$R(z, t)$ stands for the blood vessel's inner radius. Then,

$$\frac{\partial u(r,z,t)}{\partial t} = \frac{\partial u(\eta,t)}{\partial t}.\frac{\partial \eta}{\partial t} + \frac{\partial u(\eta,t)}{\partial t}.\frac{\partial t}{\partial t} = -\frac{\eta}{R}\frac{\partial u(\eta,t)}{\partial t}.\frac{\partial R}{\partial t} + \frac{\partial u(\eta,t)}{\partial t},$$

Now,

Using the new coordinate system (η, z, t), 10, 11 and 12 equations can be written as

$$\frac{\partial u}{\partial t} + \frac{1}{R}\left(\eta\left(u\frac{\partial R}{\partial z} + \frac{\partial R}{\partial t}\right) - w\right)\frac{\partial u}{\partial \eta} + u\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{\vartheta}{R^2}\left(\frac{\partial^2 u}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial u}{\partial \eta}\right) \quad (14)$$

$$\frac{\partial w}{\partial t} + \frac{1}{R}\left(\eta\left(u\frac{\partial R}{\partial z} + \frac{\partial R}{\partial t}\right) - w\right)\frac{\partial w}{\partial \eta} + u\frac{\partial w}{\partial z} = \frac{\vartheta}{R^2}\left(\frac{\partial^2 w}{\partial \eta^2} + \frac{1}{\eta}\frac{\partial w}{\partial \eta} + \frac{w}{\eta^2}\right) \quad (15)$$

$$\frac{1}{R}\frac{\partial w}{\partial \eta} + \frac{w}{\eta R} + \frac{\partial u}{\partial z} - \frac{\eta}{R}\frac{\partial R}{\partial z}\frac{\partial u}{\partial \eta} = 0 \quad (16)$$

The aforementioned equation system is a hemodynamic model [16].

Once more, it is assumed that the polynomial expression for the axial direction velocity profile $u(\eta, z, t)$, is as follows:

$$u(\eta, z, t) = \sum_{k=1}^{N} q_k(\eta^{2k} - 1) \quad (17)$$

And,

It is assumed that the redial direction velocity profile $w(\eta, z, t)$, has the following polynomialexpression:

$$w(\eta, z, t) = \frac{\partial R}{\partial z}\eta w + \frac{\partial R}{\partial t}\eta - \frac{\partial R}{\partial t}\frac{1}{N}\eta\sum_{k=1}^{N}\frac{1}{k}(\eta^{2k} - 1) \quad (18)$$

Where,

q(z,t) is an additional variable that will be decided later [21].

For simplicity the equations 17 and 18, choose $N = l$, then

$$u(\eta, z, t) = q(z, t)(\eta^2 - 1) \quad (19)$$

And,

$$w(\eta, z, t) = \frac{\partial R}{\partial z}\eta w + \frac{\partial R}{\partial t}\eta - \frac{\partial R}{\partial t}\eta(\eta^2 - 1) \quad (20)$$

The N-S equations acquire the following forms to derive the dynamic equations of $q(z,t)$ and $R(z, t)$,using the continuity equation, the radial coordinate, and the equations of axial and radial velocity profile as:

$$\frac{\partial q}{\partial t}-\frac{4q}{R}\frac{\partial R}{\partial t}-\frac{2q^2}{R}\frac{\partial R}{\partial z}+\frac{4\vartheta}{R^2}q+\frac{1}{\rho}\frac{\partial P}{\partial z}=0 \qquad (21)$$

$$2R\frac{\partial R}{\partial t}+\frac{R^2}{2}\frac{\partial q}{\partial z}+q\frac{\partial R}{\partial z}=0 \qquad (22)$$

Where,

R represents the radius of the blood vessels. Now, introduce the cross-sectional area $S(z, t)$ of the blood vessel as

$$S=\pi R^2 \qquad (23)$$

Additionally, blood flow rate is provided as a surface integral of u and $\partial\eta$ [19] is defined as,

$$Q=\iint_S u\,\partial\eta=\frac{1}{2}\pi q R^2 \qquad (24)$$

From equations 21, 22, 23 and 24, we get

$$\frac{\partial Q}{\partial t}+\frac{3Q}{S}\frac{\partial S}{\partial t}-\frac{2Q^2}{S^2}\frac{\partial S}{\partial z}+\frac{4\pi\vartheta}{S}Q+\frac{S}{2\rho}\frac{\partial P}{\partial z}=0 \qquad (25)$$

$$\frac{\partial S}{\partial t}+\frac{\partial Q}{\partial z}=0 \qquad (26)$$

The following is a straightforward differential equation that is created by combining the equations25 and 26,

$$\frac{\partial Q}{\partial t}-\frac{3Q}{S}\frac{\partial S}{\partial t}-\frac{2Q^2}{S^2}\frac{\partial S}{\partial z}+\frac{4\pi\vartheta}{S}Q+\frac{S}{2\rho}\frac{\partial P}{\partial z}=0 \qquad (27)$$

Equation 27, is also known as the cardiovascular system equation when discussing the cross- sectional area of blood vessels. By making various assumptions, we may obtained the model equation of the blood flow and blood pressure. By resolving the governing equations 25 and 26, the answers to the arterial cross-sectional area and its related blood flow may now be found [25].

2.3 Modeling equation of the blood flow rate

Blood flow and blood flow rate refer to the amount of blood moving continuously through the circulatory system in a certain amount of time. The cross-sectional area of the blood vessel does not change with time; hence, the rate of change in cross-sectional area with respect to distance does not exist. The pressure gradient over the length of the blood artery is also assumed to be constant when developing the model equation of blood flow. Equation 27, which is based on the above assumptions, is transformed into the model equation for blood flow rate [17]:

$$\frac{\partial Q}{\partial t}+\frac{4\pi\vartheta}{S}Q+\frac{S}{2\rho}\frac{\partial P}{\partial z}=0 \qquad (28)$$

The mathematical representation of the blood flow rate in one dimension is shown here.

2.4 Modeling equation of the blood pressure

Because of the pressure created by the heart's contraction and the muscles that surround our blood vessels, blood flows through our circulatory system. Blood pressure is a gauge for this force. One of the key indicators of physical fitness is blood pressure. Poiseuille's equations $Q=\frac{\pi R^4}{8L\vartheta}P$, Where L is the length of the blood vessel, which establish the relationship between blood flow rate and blood pressure, using this equation on equation 28 and solving we get the equation that represents the model equation of blood pressure [28]

$$\frac{dP}{dt}+\frac{4\vartheta}{R^2}P+\frac{4L\vartheta}{\rho R^2}\frac{\partial P}{\partial z}=0 \qquad (29)$$

3. Result and Discussion

We examined and analyzed various results for the intended model and

Poiseuille's equation inthis section of the paper. The previous efforts in this field can be used to establish the necessary boundary condition and the parameter values to solve this equation, such as; pressure gradient, $\frac{\partial P}{\partial z}$ = 100 to 40 mm Hg, initial value of Q = 1 to 5.4 liter/minute, Kinematic viscosity of blood ϑ= 0.035 cm^2/s, density of blood, ρ= 1.043 to 1.06 g/cm^3 [29].

3.1 Analysis of blood flow for different pressure gradient and viscosity using Poiseuille's equation

Figure 2A depicts the blood flow fluctuation for various pressure gradients along with the radius (ranging from 0.13 cm to 0.31 cm) as determined by using Poiseuille's equation simulation. Here, the blue line represents blood flow with a pressure gradient of 40 mm Hg, the black line representsblood flow with a gradient of 80 mm Hg, the red line represents blood flow with a gradient of 120 mm Hg, and the pink line represents blood flow with a gradient of 160 mm Hg, where the y-axis denotes volumetric blood flow rate and the x-axis denotes radius size. This outcome means that blood flow increases along with an increase in pressure gradient. Blood flow is also boosted by an increase in the cross-sectional area or blood vessel radius.

The outcome of simulating Poiseuille's equation for various viscosity and varied radius sizes is depicted in Figure 2B. Here, the blue line represents the blood flow for a viscosity of 0.03 cm^2/sec,the black line for a viscosity of 0.035 cm^2/sec, the red line for a viscosity of 0.04 cm^2/sec, andthe pink line represents the blood flow for a viscosity of 0.045 cm^2/sec, where y is the volumetric blood flow rate and x is the radius (from 0.13 cm to 0.31 cm). From Fig. 2B, it is clear thatthere has been a little rise in the blood's volumetric flow rate as blood viscosity has decreased.

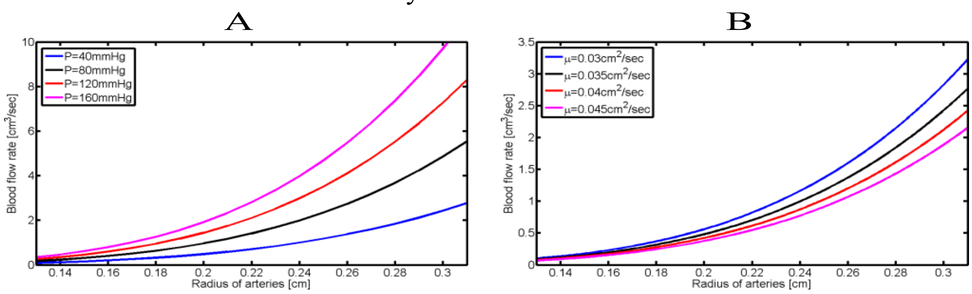


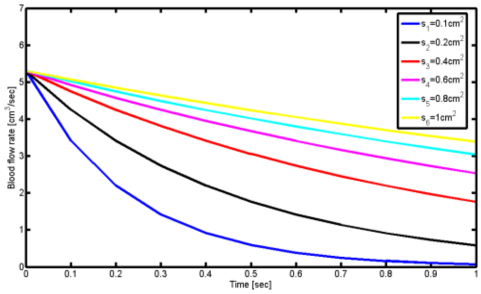
*Figure 2: Variation of blood flow for different **A:** pressure gradient, **B:** blood viscosity*

3.2 Analysis of blood flow for different cross-sectional area, blood viscosity, and pressure gradient

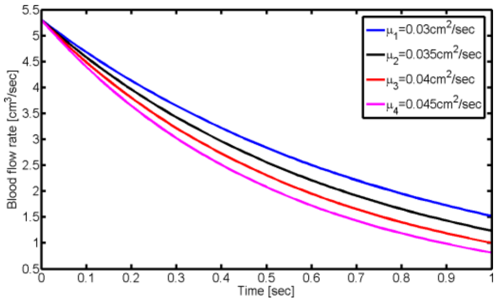
By using computer software to simulate the model equation of blood flow for various cross-sectional areas over time, we can arrive at the result shown in Figure 3**C**. Here, the blue line represent blood flow for vessels with a cross-section of 0.1 cm^2, the black line represents blood flow for vessels with a cross-section of 0.2 cm^2, the red line represents blood flow for vessels with a cross-section of 0.4 cm^2, the pink line represents blood flow for vessels with a cross-section of 0.6 cm^2, the cyan line represents blood flow for vessels with a cross-section of 0.8 cm^2, and the yellow line representsblood flow for vessels with a cross-section of 1 cm^2, where the x-axis denotes time (from 0 sec to1 sec) and the y-axis the volumetric blood flow rate. Here, we see that the rate of blood flow rose along with an increase in blood vessel cross-section. When the vessel cross-section is 0.1 cm^2 and the vessel cross-section is 1 cm^2, the difference in blood flow rate is visible in the Fig. 3**C**.

By solving the model equation of blood flow using computational software and simulating differentblood viscosity with time, we obtain the result shown in Figure 3**D**. Here, the blue line representsviscosity 0.03 cm^2/sec, black line represents viscosity 0.035 cm^2/sec, red line represents viscosity 0.04 cm^2/sec and the pink line represents viscosity 0.045 cm^2/sec where the y-axis represents

C



D



E

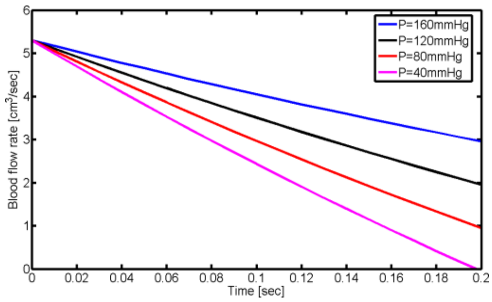


Figure 3: Variation of blood flow for different **C:** cross-section area, **D:** blood viscosity, and **E:** pressure gradient volumetric blood flow rate and the x-axis represent time (from 0 sec to 1 sec). Here we observethat if the viscosity of the blood vessel increased, the flow rate of blood also increased. When the viscosity of blood is 0.03 cm^2/sec and the viscosity of blood is 0.045 cm^2/sec, we can see adifference in blood flow rate. Figure 3**E** describes the volume flow rate along with different pressure gradients. The blue line represents a pressure gradient of 160 mm Hg, the black line a pressure gradient of 120 mm Hg, thered line an 80 mm Hg pressure gradient and the pink line a 40 mm Hg pressure gradient, wherethe y-axis represent volumetric blood flow rate and the x-axis represents time. Here we observe that if the pressure gradient of blood is increased, the flow rate of blood also increases. We can observe that difference in blood flow rate when the pressure gradient of the blood is 40 mm Hg and the pressure gradient of the blood is 160 mm Hg.

3.3 Analysis of blood pressure for different cross-section of vessel and viscosity

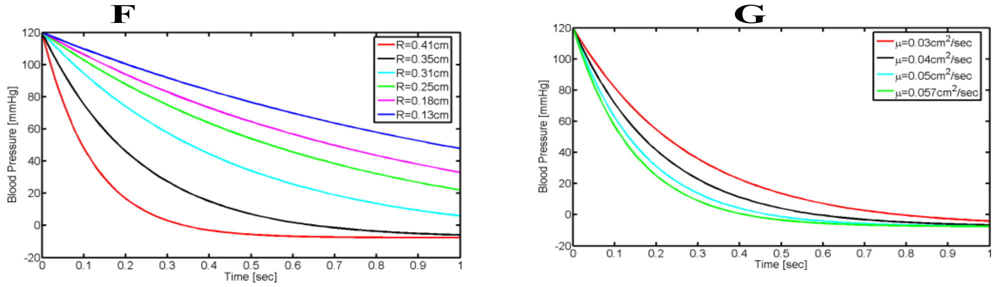


Figure 4: Variation of blood pressure for different ***F*** *cross section of vessel,* ***G*** *viscosity*

The blood pressure model equation can be solved by using computer software to simulate different cross-sections over a time range of 0-1 second. The result is shown in Figure 4**F**. Here, the indigoline represents blood pressure in a vessel with a radius of 0.13 cm. Similarly, the pink line, green,blue, black, and the red line are used for the radii of 0.18 cm, 0.25 cm, 0.31 cm, 0.35 cm, and 0.41 cm respectively. The x-axis in this graph represents time, and the y-axis represents blood pressure. Here, we see that the blood pressure decreases as the cross-sectional area increases. It concludes that pressure will rise as the radius decreases.

The blood pressure model equation can be solved by utilizing computational software and simulating various blood viscosity with time (from 0 sec to 1 sec). The result is shown in Figure 4**G**. Here, the red line depicts blood pressure in a vessel with a blood viscosity of 0.03 cm^2/sec, the black line for 0.04 cm^2/sec, the blue line for 0.05 cm^2/sec, and the green line for 0.057 cm^2/sec, where time is represented by the x-axis and blood pressure is by the y-axis. Here, we see that the blood pressure decreases as blood viscosity increases. Therefore, we see that the blood pressure will decrease as the viscosity increases.

4. Conclusion

This research examines an easy mathematical model that can accurately depict blood flow in the data. In this investigation, we looked at how blood flow rate and blood pressure were affectedby blood viscosity, pressure gradient, and arterial cross-sectional area. We found that the blood flow rate through the arteries affects blood pressure and is affected by a slight variation in the cross-sectional area. Additionally, variations in blood viscosity play a significant influence on hypertension. Although several assumptions have been made, the model can still be judged tobe realistic because it can demonstrate how changes in cross-sectional area, blood viscosity, and pressure gradient affect the blood flow rate. The model also demonstrates how the cross-sectionalarea and blood viscosity both affect blood pressure. The simulation result of the proposed model and Poiseuille's equation are in line. Finally, the graphical visualizations help to confirm the accuracy of the model that was proposed for this study.

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