Received Date: 22nd October, 2022
Accepted Date:23rd April, 2023

Realization of the TOW-THOMAS Biquad Active Filter

Er. Sagun Manandhar

Assoc. Professor / Head, Department of Electronics, Communication & Information Engineering. Kathmandu Engineering College.
E-mail: sagun.manandhar@kecktm.edu.np

Abstract— **The Tow-Thomas Biquad Circuit provides the filter designers with a valuable building block for building higher order active filters. It is a flexible circuit structure in which the transfer function properties are easily manipulated by modifying the passive RC elements that connect the operational amplifiers. It can be easily defined in its low-pass and band-pass operations. Above all, we can easily fabricate the possible filters inside a single chip with very slight modification of the standard Tow-Thomas Circuit.**

***Keywords* — Tow-Thomas Filter, Biquad, Ring-of-Three Filter, Active Filter, State Variable Circuit**

I. Introduction

In today's modern world, any complex communications and electrical system contains high order active filters. These higher order filters provide the necessary characteristics that match the necessary specifications for any filter. It would be a tedious and error-prone task of designing active networks for higher order filters if these were not built in a modular fashion. The introduction of low order, low-sensitivity and stable filter circuits has provided network and system designers with the necessary building blocks. These lower order filters are then cascaded in order to build higher order filters. Tow [1] enumerates the benefits of this type of active R-C filter design:

1. Low Sensitivity Realization: the general properties of the circuit remain stable even if the passive R-C elements are modified.
2. Building Block Approach: high-order transfer functions can be realized by cascading second-order functions.
3. Fixed Structure: each second-order block will be identical to each other thus making the structure of the circuit similar across the whole network and thus further make such a network easier to manufacture.

An example of such is the modular circuit realized by J. Tow [1] and L. C. Thomas [3]. This circuit is called the Tow-Thomas Biquad Circuit or the Ring-of-Three Filter.

II. Electrical Performance

The electrical performance of the Biquad was defined thoroughly by L. C. Thomas in his series of papers concerning the Biquad [3][4]. Presented below is the summary of Thomas' work.

A. Biquad Sensitivity

Tow [1] has shown that the sensitivity of the natural modes of the Biquad to its RC elements is comparable to a passive LC case. The significance of this result is that the precision tuning with the Biquad is possible. The values of the open-loop gain $\mu_0$and the normal selectivity of the transmission poles Q_{p0} for very high precision realizations can be obtained with$\mu_0 \geq 100Q_{p0}\mu_0 \geq 100Q_{p0}$. This will result in very minimal changes to Q_{p0}. A more practical ratio would be $\mu_0 \geq 20Q_{p0}\mu_0 \geq 20Q_{p0}$where there will be at most a 5 percent change inQ_{p0}.

B. Q_p Enhancement Caused by Finite Gain-Bandwidth

The value of the resonant frequency ω_0 can be set to high values without making the value for the normal selectivity of the transmission pole $Q_{p0} \gg 0Q_{p0} \gg 0$ which can cause it to become unstable.

C. Noise, Distortion and Signal Level

The Biquad has its own low-noise pre-amplification stage which is the inverting amplifier stage$(\frac{\omega}{R_3})$. The gain is simply determined by adjusting the resistor value of R_4.

D. Temperature Effects

The temperature effects of the Biquads are dependent only on the passive elements of the circuit. An upper limit of value of Q_{p0} with temperatures greater than $50°C$ has an order of magnitude of around 200. If a higher value forQ_{p0} is desired, better mechanical resonators must be utilized or the environmental conditions (particularly temperature conditions) must be controlled.

III. The Biquad

The Circuit shown in figure 2 is the schematic diagram of a Tow-Thomas Biquad Circuit which shall be referred to as the Biquad.

The Biquad exhibits a second-order transfer function that can be used to build higher order circuits. The Biquad is composed of three important parts: a lossy integrator, an inverting integrator and an inverting amplifier. The open-loop operational amplifier H_1 has a feedback loop $R_1C_1$with a R_4 resistor can be realized as a lossy-integrator structure. This structure is used to achieve the low-pass and band-pass frequency response states. The second operation amplifier H_2 has a feedback capacitor C_2 and input resistor R_2 can be realized as an integrator. The third operational amplifier H_3 is simply setup as an inverting amplifier with resistances *R*. This stage provides low noise pre-amplification of the entire circuit. The main feedback loop is composed of a resistor R_3. The inverting amplifier can be removed to reduce production cost if needed. Doing this would mean the replacement of the second stage inverting integrator circuit with a balanced time-constant (BTC) integrator or a resistance-bridge (RB) integrator [3]. In addition, this structure provides us with a high-impedance purely resistive input.

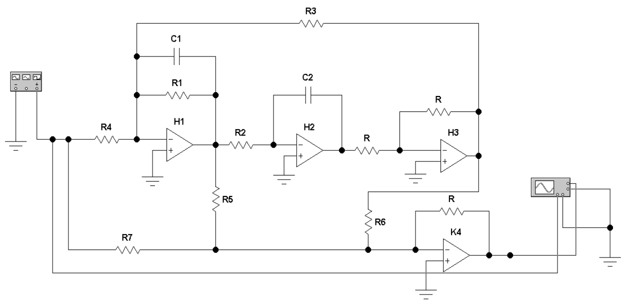

Figure 1. Tow Thomas Biquad Circuit

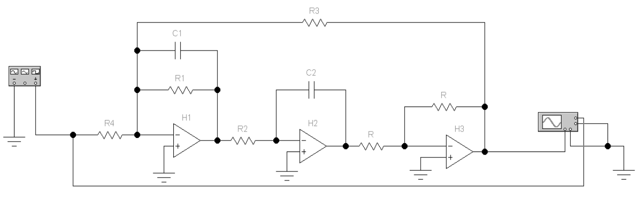

Figure 2. Realized Tow Thomas Biquad Circuit

The fundamental aspect of the Biquad circuit is its flexibility. The transfer function in Equation 1 is related directly to the passive RC elements which interconnect the operational amplifiers. This is the general transfer function for the entire circuit presented in Figure 1 were $T(S)$ refers to the transfer function of the Biquad ω_z and ω_p are related to the frequencies at which loss peaks and transmission poles occur. Q_{z0} and Q_{p0}refer to the normal selectivities of the zeros and the poles. A band-pass transfer function is defined as $\frac{V_1}{V_{in}}$ in Equation 2 and $\frac{V_2}{V_{in}}$ in Equation 3 for low-pass [3]. This type of transfer function is also generally referred to as the operational amplifier biquad realization.

$$T(S) = \frac{V_{out}}{V_{in}} = \frac{m(S^2 + \frac{\omega_z}{Q_{z0}}S + \omega_z^2)}{(S^2 + \frac{\omega_p}{Q_{p0}}S + \omega_p^2)} \quad ---(1)$$

$$T_{BP}(S) = \frac{V_1}{V_{in}} = \frac{\frac{1}{R_4C_1}S}{(S^2 + \frac{1}{R_1C_1}S + \frac{1}{R_2R_3C_1C_2})} \quad --(2)$$

$$T_{LP}(S) = \frac{V_2}{V_{in}} = \frac{\frac{1}{R_2R_4C_1C_2}S}{(S^2 + \frac{1}{R_1C_1}S + \frac{1}{R_2R_3C_1C_2})} \quad ----------(3)$$

The values for the constants in the resulting biquadratic transfer functions can be determined by modifying the values of the RC elements in the circuit. The value for the resonant frequency ω_p can be determined by Equation 4. The value of the normal selectivity of the transmission pole Q_{p0} is given in Equation 5. The value for the gain can be taken from Equation 6.

$$\omega_p = \frac{1}{R_3C_1} = \frac{1}{R_2C_2} \quad ----(4)$$

$$Q_{p0} = \frac{R_1}{R_3} \quad ---------(5)$$

$$G = \frac{V_1}{V_{in}}\bigg|_{\omega_p} = \frac{V_2}{V_{in}}\bigg|_{\omega_p} = \frac{R_1}{R_4} \quad ------(6)$$

The ability to determine some characteristics of the resulting biquadratic transfer function enables us to construct an inherently stable network. This is because the value for Q_{p0} is always positive since it is determined by the values of R_1 and R_3>0.

IV. Orthogonal Tuning

In order to maximize the potential of Biquads, the orthogonal tuning is used to choose the values of the R & C elements in the circuit in order to define the resulting bi-quadratic transfer function of the circuit.

1. First, we set $R_2 = R_3 = R$ and $C_1 = C_2 = C$ to make $\omega_p = \frac{1}{RC}$
2. R_3 may be adjusted to determine the value of Q_{p0}
3. R_4 may be adjusted to determine the value of *G* without affecting Q_{p0} or $\omega_o r$ then the value of

$$|A(peak)| = \frac{R_1}{R_4}$$

The following orthogonal tuning is based on the equations: 4, 5 and 6. Tow also proposes another compatible tuning method described in his paper [1].

V. Building High-Order Active Filters

High-order active filter can now be constructed by serially cascading the Biquad in order to realize the necessary transfer function. Tow [2] discusses a simple yet elegant method for building high-order active filters.

A. Obtaining the Transfer Function

The first step in building high-order active filters is to obtain the necessary transfer function of the designed filter. The methods would vary from filter to filter. Each type such as the Butterworth, Bessel and Chebychev filter would have their own necessary computations. But, in general, it would be necessary to derive the transfer function from the specifications of the problem. Given the specifications of the problem the appropriate order of the filter can be determined. Thus, from this the designer can then select the appropriate filter type and look or compute for the constants with respect to their transfer functions. The transfer functions are typically broken up into second-order equations and an optional first-order equation.

B. Realization of Active Filter

In order to realize the circuit completely, the appropriate building blocks must be used. The second-order equations can be realized with the Biquad. The first-order equations can be realized with a simple first-order active filter. The Biquad hence will represent a portion of the circuit's transfer function. The equations 4, 5 and 6 can be determined from the second-order transfer function. The orthogonal tuning presented in the section above can then be used to select the value for the RC elements. This procedure is then performed for each section of the transfer function. The resulting Biquads are then cascaded to produce the necessary high-order active filter. It can be noted that the procedure outline above can be utilized for other second-order circuits such as the Sallen-Key Biquad other than the Tow-Thomas Biquad.

VI. Conclusions

On the basis of sensitivity, economics and ease of trouble shooting and tuning, the Biquad circuit is an ideal building block for realizing higher order active filters. This is brought about by cascading the Biquad which is capable of realizing a second-order transfer function. This allows the circuit designer to a similar structure for each building block of the circuit.

The building block approach is also a cost-effective approach to design higher-order circuits since each section that can be easy manufactured and fabricated in a single chip. These different sections can then be cascaded to build the necessary active filter. Since the circuit is modular, troubleshooting can be made simpler by partitioning the problem to the different sections. Orthogonal tuning can also be made simply by adjusting the RC elements of each Biquad without affecting the rest of the circuit.

VII. References

[1] J. Tow, "Active RC Filters - A state-space realization", Proc. IEEE (Letter), Vol. 56, 1137-1139, Jun. 1968.

[2] J. Tow, "A step-by-step active filter design", *IEEE Spectrum* Vol. 6, 64-68, Dec. 1969.

[3] L. C. Thomas, "The Biquad : Part I - Some Practical Design Considerations", *IEEE Transactions on Circuit Theory* CT-18(3), 350-357, 1971.

[4] L. C. Thomas, "The Biquad : Part II - A Multipurpose Active Filtering System", *Transactions on Circuit Theory* CT-18(3), 358-361, 1971.