Kaumodaki : Journal of Multidisciplinary Studies
[A Peer-Reviewed, Open Access Journal; Indexed in NepJoL]
ISSN : 2822 - 1567 (Print); ISSN : 2822 -1583 (Online)
Published by Research Management Cell, Shree Vinduwasini Sanskrit Vidyapeeth (Campus)
Nepal Sanskrit University, Pokhara, Nepal
https://ejournal.vsc.edu.np

Role of Earliest Arrival Flow in an Evacuation Network

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Article History : *Submitted 10 Oct. 2023; Reviewed 22 Oct. 2023; Accepted 02 Nov. 2023*
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DOI : *https://doi.org/10.3126/kdk.v4i1.64568*

Abstract

Traffic route guidance, destination optimization, and optimal route choice are some of the approaches to accelerate the evacuation process of transit-based evacuation problems. Their effectiveness depends upon the evacuee arrival patterns at the pickup locations and their assignment in an appropriate way to the resources in the network. The arrival patterns of the evacuees at different pickup locations might be a constant or random variable, deterministic or stochastic, time-dependent, or flow-dependent, or can be categorized either way. In this work, we are focused on the collections of evacuees at the pickup locations from the danger zone in the earliest arrival flow patterns, which can be further assigned to the transit vehicles in the integrated evacuation network by treating such pickup locations as the sources for the subsequent process to minimize the overall network clearance time from the danger zone to the safety.

Keywords: Evacuation, evacuation network, earliest arrival flow, transform network, time complexity

Introduction

The massive loss of human life and the socio-economic damage caused by different natural or man-made disasters draw increasing attention from society and researchers toward disaster management. The great loss of people in disasters is due to the lack of proper planning rather than the disaster itself. Mostly, the evacuation planning problems are based on the network model.

A network $N = (V, A)$ consists of a finite set of nodes and edges. Each node corresponds to the intersection of streets and the edges connect a pair (i, j) of nodes, corresponding to roads or the streets in the region. Commodities flow from node to node, transported by edges of a network having a capacity u_a, which restricts the amount it can transverse. The locations where evacuees are situated initially are the source nodes denoted by s and the safe locations where the evacuees are to be transported are sink nodes denoted by t. Intermediate pickup locations are Y. The transit time is the amount of time it takes for the flow to travel through the edge i.e., $\tau_a \; \forall \, a \in \; A$.

Consider a flow over time evacuation network as in Figure 1, $N = (V,\; A, u,\; \tau,\; s,\; t,\; T)$ where $u,\; \tau,\; s,\; t,$ and T represent capacity, transit time, set of sources, set of sinks, and the time horizon, respectively. Then its transformed network $N' = (V, A', u', \tau', s, t, T)$ consists of the modified arc capacities and symmetric transit times,
$u'_a = u_a + u_{a'}$ whereas $\tau'_a = \tau_a$ if $a \in A$ and $\tau'_a = \tau_{a'}$ if $a \notin A$. ………… (1), where an arc $a' \in A'$ is in the transformed network, if $a \vee a' \in A$ in the network N. The remaining data structure remains the same.

The flow of evacuees from the source to the sink over time is a non-negative function f on $A \times \mathbb{R}_{\geq 0}$ for given time T= {0, 1, 2, …, T-1} satisfying the flow conservation and the capacity constraints, (1-3). The inequality flow conservation constraints allow waiting for flow at intermediate nodes. However, the flow conservation constraints fore that the flows entering an intermediate node must leave it again immediately.

$\sum_{a\epsilon\, A_i^{in}} \sum_{\sigma=\tau_a}^{T} f\;(a, \sigma - \tau_a) - \sum_{a\epsilon\, A_i^{out}} \sum_{\sigma=0}^{T} f\;(a, \sigma)\; = 0, \forall\, i \in V \backslash (S \cup Y)$ (1)

$\sum_{a\epsilon\, A_i^{in}} \sum_{\sigma=\tau_a}^{\theta} f\;(a, \sigma - \tau_a) - \sum_{a\epsilon\, A_i^{out}} \sum_{\sigma=0}^{\theta} f\;(a, \sigma)\; \geq 0, \forall\, i \in V \backslash (S \cup Y), \theta \in T$ (2)

$0 \leq f\,(a, \theta) \leq u_a \;\forall\, a \in A, \theta \in T$ (3)

The sets of outgoing and incoming arcs for the node $i \,\epsilon\, V$ are denoted by, $A_i^{out} = \{a = (i,j)\,\epsilon\, A\}$ and $A_i^{in} = \{a = (j,i)\,\epsilon\, A\}$, respectively. Not stated otherwise, for all pickup locations $y \,\epsilon\, Y$ and $s \,\epsilon\, S$, we assume that $A_i^{out} = A_i^{in} = \phi$ in the case without arc reversals. However, the flow value to $\upsilon_f\,(s) > 0$, and $\upsilon_f\,(y) < 0$, respectively, where $\sum_{i\,\epsilon\, V} \upsilon_f\,(i) = 0$.

In such network, the flow reached to Y for all time setup up to $\theta' \in \mathbb{R}_{\geq 0}$ is given by,

$$|\nu_f|_{\theta'} = \sum_{\theta=1}^{\theta'} |\;(\nu(Y, \theta)| \qquad (4)$$

For the given time bound T, its value becomes $|\nu_f| = \sum_{\theta=1}^{T} |\;(\nu(Y, \theta)|$ (5)

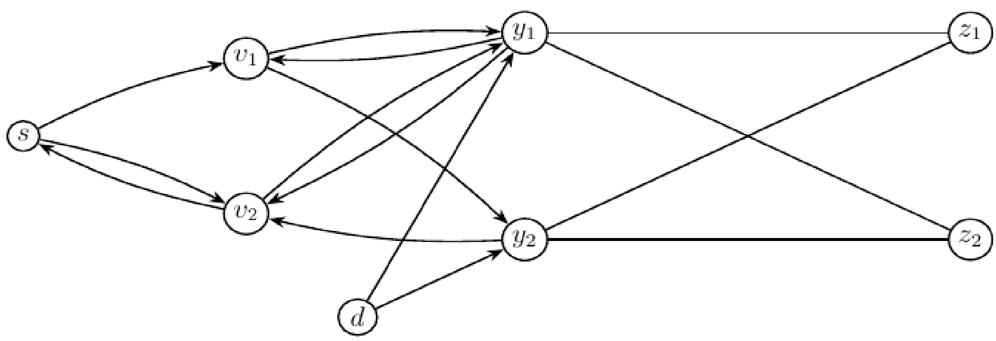

Figure 1: A simple embedded evacuation network to collect evacuees.

For such problems, not only the amount of flow transmitted but also the time needed for the flow plays an important role and are named as flows over time problems. Such a plan is dependent upon many sources, sinks, and different parameters on the arcs and nodes, like constant, time-dependent, or flow-dependent capacities, etc. The flow may be auto-based or transit-based and the time may be discrete or continuous. Such problems arise in many applications of evacuation planning problems. There has been a fair amount of work in this area, as referred to by the authors in [1]-[7], [9] and the references therein.

In this paper, we seek to focus on the collections of the evacuees at the pickup locations from the danger zone in the earliest arrival flow patterns that are better suited for the evacuation scenarios as they send the maximum number of evacuees as much as possible at every point in time within T. For this, the danger zone is connected to the pickup locations via intermediate nodes. This part of the network is not fully connected as the source has only outgoing arcs. Furthermore, vehicles do not return either to the depot or to the danger zone. However, it can be extended to an integrated network for the integrated approach to minimize the overall network clearance time.

Here, Section 2 deals with its literature, the earliest arrival flow in Section 3. Finally, Section 4 concludes the paper.

Literature Review

The pioneering work of Ford and Fulkerson [10] opened the wide horizon of dynamic network flow problems by focusing their work on maximum dynamic flow problems, which gives the optimal maximum dynamic flow from the source to the sink in the given time horizon T. Quickest flows, as the variant of maximum dynamic flows, are for the minimization of time-horizon to transfer a given amount of flow value from initial position to the final destination along with the paths in the network and are similar to the evacuation of spectators from the stadium. Earliest arrival flow is a flow over time that simultaneously maximizes the amount of flow arriving at sinks before time T for all θ in every time unit including the time horizon T. Note that it is not entirely clear that such a flow can generally exist in every network, though the earliest arrival flows do exist for a single source single sink network by Gale [11].

A prominent bus-based evacuation problem model is proposed by Bish [8] to minimize the time of evacuation in case of short notice using many homogeneous buses satisfying all evacuee demands and without violating both shelter and vehicle capacity constraints. In this, the number of evacuees at the demand node might be greater than the capacity of a bus and demand the split delivery within the pickup locations. Contrary to this, Goerigk et al. [13] assumed that the number of evacuees at every source is known in terms of the integral multiples of the bus loads and that does not demand the split delivery service in the network.

Most of the evacuation planning problems in the literature used a homogeneous fleet of vehicles to collect the diverse types of evacuees, and each pickup location may have multiple visits. Most of them the evacuee arrival pattern is silent. However, authors in [5] and [7] have presented the earliest arrival pattern in an integrated evacuation network to achieve the minimum clearance time in the network considered. They have considered the embedding of two different sub-networks as the collection network and the assignment network. In the collection network, evacuees are shifted to the pickup locations from the sources for the evacuation. Such pickup locations are taken as the sources for the subsequent evacuation process to minimize the overall network clearance time where the transit vehicles are provided from the bus depot to the assignment sub-network of that embedding. In such integrated network, the earliest arrival flow pattern during the collection of evacuees at primary sub-network respects the partial lane reversal strategy, whereas the better assignment of transit-vehicles in such embedding are based on dominance relations concerning the evacuation duration.

Evacuees were supposed to gather themselves from their residents depending on the disaster scenario to the nearby pickup locations. In a case study of the hypothetical endangered region of a highly populated and dense area of Kathmandu having a large transit-dependent population, evacuees were supposed to gather randomly from their residents to the nearby pick-up locations. The excess exterior points of such considered regions were taken as the pick-up locations and the available open spaces as the sinks. The evacuees were supposed to be brought at such sinks from the pick-ups by using buses of uniform capacity as the multi-source and multi-sink network. Different empirical conclusions were drawn and showed that the domain of optimal solutions remains on many buses with higher capacity and speed irrespective of the population chosen where the choice of many sources and sinks does not play a significant role [14]. In literature, very fewer priorities are given for the collection patterns of the evacuees at the pickup locations from the danger zone and are our concern for what we are dealing with the earliest arrival flow patterns in our collection sub-network.

Here, our concern is about such a better-suited arrival pattern in an evacuation network.

The Earliest Arrival Flow

The arrival of evacuees at the pick-up locations can be categorized as random or constant arrival, deterministic or stochastic, time-dependent, or flow-dependent, and so on. As mentioned earlier, most of the evacuations planning problems were silent about the arrival patterns at the pickup location. But when preparing for an evacuation, it is uncertain how much time it will take to enact it and hence it is preferential to plan within the available time horizon to execute the maximum flow possible as in [12], i.e., the transversal of a maximum number of evacuees in each possible time unit, which is offered by the earliest arrival flows. In the earliest flow pattern, it maximizes the flows of evacuees simultaneously at each instance within the given time horizon and is better suited for evacuation planning problems. Such flow patterns are more appropriate to achieve the minimum network clearance time in the integrated evacuation network. Here, the network clearance time is the effective duration of the evacuation planning process from the beginning to the evacuation of the last evacuee or the evacuee's caring vehicle in the evacuation network.

Earliest arrival flow will simultaneously maximize the flow that reaches the sinks in every time unit, but such flows do not exist in arbitrary networks either in constant or/and variable time or in zero transit time. For the special case of the flow over time with zero transit times, some characteristic networks always permit the earliest arrival flow pattern regardless of their capacities, supplies, and demands, [15].

Theorem 1 [15]. There exists the earliest arrival transshipment for zero transit times in a single source multi-sinks network where the depth is at most two. Hence, we have:

Lets denote the source and y is the sink. Here, each of the networks as illustrated in Figure 2 has a depth less or equal to two, and hence there exists the earliest arrival flow.

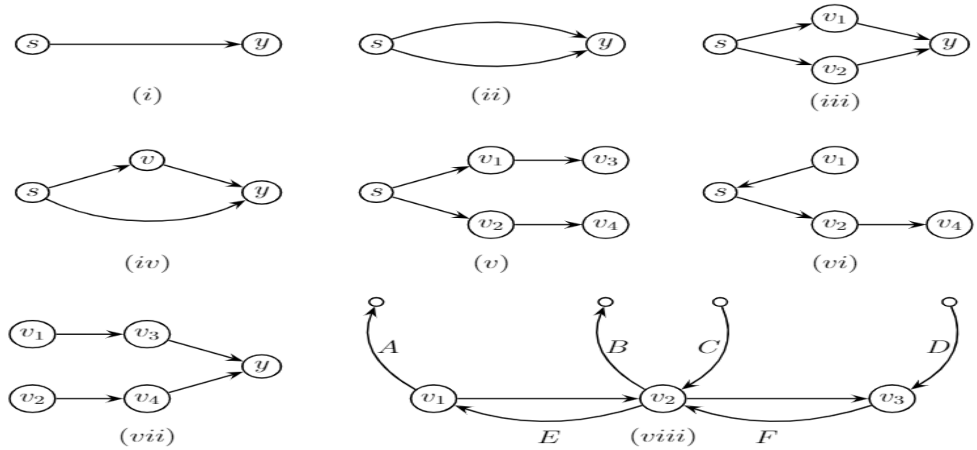

Figure 2. The nature of different networks in which the earliest arrival flows exists.

In fact, for the special case of the flow over time with zero transit times, some characteristic networks always permit the earliest arrival flow pattern regardless of their capacities, supplies, and demands, for a single source with multiple sinks as in [15] and their depth are bounded by two. But this need not be held in multi-source multi-sink networks.

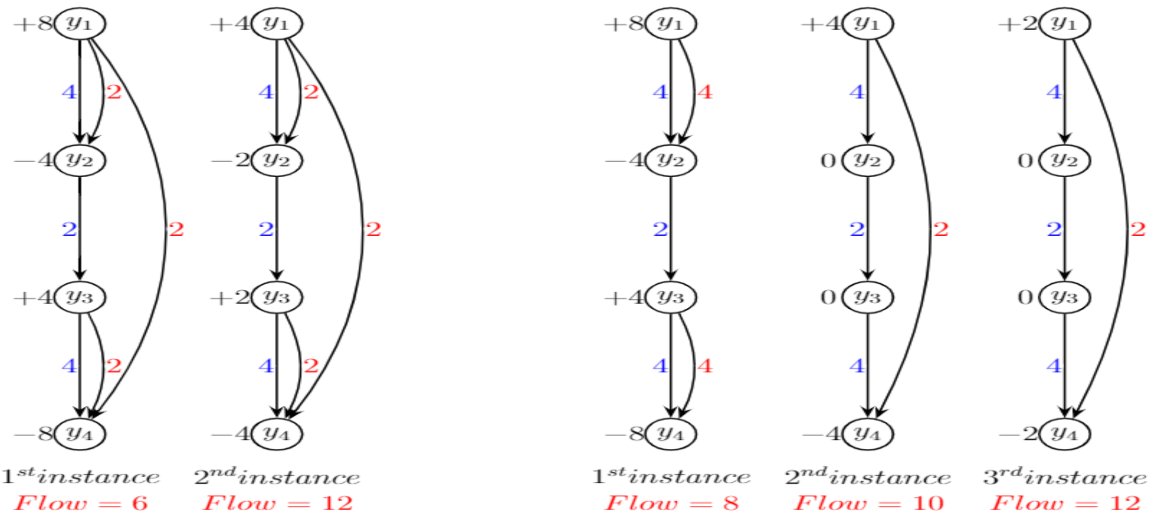

Figure 3. Non-existence of the earliest arrival flow in a multi-source multi- sink network.

Example 1: In the adjoining Figure 3, let the capacities be $u(y_1, y_2) = u(y_3, y_4) = 4, u(y_2, y_3) = 2$ whereas the balances be $\mu(y_1) = 8,\ \mu(y_2) = -4,\ \mu(y_3) = 4,\ \mu(y_4) = -4$. Here two different settings of flows are possible in different instances with the same amount of flow in aggregate but not the same in each time unit. For this, let us send two units each from(y_1, y_2), (y_1, y_4), and (y_1, y_4) . Then, we will get 6 units of flow in the first instance as in left-half of Figure 3. However, it can balance all the nodes in the second step with the same repetition of flows with the total of 12 units in second instance. But in another setting, consider 4 units of parallel flows on(y_1, y_2), and (y_3, y_4) will balance completely the y_2 and y_3 with the total flows of 8 units in the first instance itself, but the remaining 4 units can only be sent in two different time steps which needs three time steps for the total flow of the same 12 units, as in right-half of Figure 3. Hence, no earliest arrival flow does exist in such a network even in zero transit times.
Now, we are presenting the earliest arrival evacuee algorithm:

Algorithm 1: Earliest arrival evacuee algorithm
Input: A flow over-time network $N = \{s, V, A, u_a, \tau_a, Y\}$ with $\tau_a = 0 \quad \forall\, a \in A$.

1. Construct a transformed network N' as in Equation 1.
2. Determine the maximum number of evacuees at every possible time instance at each Y from s as in [15].
3. For each $\theta \in T$ and reverse $a' \in A$ up to capacity $c_a - u_a$ if and only if $c_a > u_a$ And u_a is replaced by 0 whenever $a \notin A$ in N where c_a denotes the capacity of the static s-t flow value $\forall\, a \in A, \forall$ N.
4. For each $\theta \in T$ and $a \in A$ if a is reversed to capacity $\kappa_a = -c_{a'} + u_a$ and $\kappa'_a = 0$. If neither a nor a' is reversed, then $\kappa_a = -c_a + u_a$ where κ_a is saved capacity of a

Output: Earliest arrival flow of evacuees at Y with $\tau_a = 0 \ \forall a \in A$.

Theorem 2. The earliest arrival evacuee algorithm designed in above Algorithm 1 sends the evacuees at earliest arrival time to Y at each time and saves the unused capacity.
Proof: The construction of a transformed network 'N' for a given network in Step 1 is feasible. Steps 2 and Steps 4 are feasible. As there is no cycle in Step 2, the flow is either in a or in a' but not simultaneously in both. Moreover, such a flow is not greater than the modified capacities of each arc in the transformed network. Hence, Step 3 is also feasible. Thus the earliest arrival flow as in Algorithm 1 is feasible.

Now, we show that Algorithm 1 gives an optimal solution. In N, the maximum number of evacuees reached to every pickup location in Y at every possible time unit with zero transit times on each arc is computed by Equation 4. As in Theorem 1, there are some characteristic networks with depth less or equal to two in which earliest arrival flow exists and is so in this network provided by Algorithm 1. The capacities of the arcs not used by the flow in such transformed network are recorded in Step 4 and can be used for logistics. This completes the proof.

Theorem 3. The earliest arrival evacuee flow problem having zero transit times with partial arc reversal capacity follows the principle of temporary repeated flows and can be solved in polynomial time complexity using earliest arrival evacuee algorithm designed by Algorithm 1 in the evacuee collection sub-network of the embedding.
Proof: As Steps 1,3, and 4 of the earliest arrival evacuee algorithm given by Algorithm 1 are solved in linear time and its time complexity is dominated by the time complexity of computation of the earliest arrival evacuees at the pickup locations Y with zero transit times on each arc as in [15] and is solved in polynomial time. Thus, the earliest arrival evacuee problem can be solved in polynomial time complexity in the evacuee collection sub-network.

The flow overtime problem having zero transit times that reached to each of the pickup locations determines the maximum number of evacuees at every possible time instance from the beginning in the evacuee collection sub-network as in [4] and in [15]. That means the earliest arrival evacuees at such pickup locations Y from the source s with zero transit times on the transformed network follows the principle of temporary repeated flows which is equivalent to the solution with arc reversal capability on the original network. This completes the proof.

Conclusions

Earliest arrival flows need not exist in the networks with multiple sinks for general transit times. Under some characterizations, some of the specific networks always permit the earliest arrival transshipments regardless of all choices of the capacities and balances in zero transit times.

Here, we are focused on the new and better-suited form of arrival patterns of evacuees in the earliest arrival flow patterns which will maximize the arrival of the evacuees at every possible instance at the pickup locations on zero transit time from a source and are considered as the supplies for the subsequent evacuation process to have the complete evacuation. However, due to asymmetric network structure, complex traffic congestions, and heterogeneous types of vehicles available for the assignment such solutions and solution strategies also have some limitations and complete solutions for such real-life problems are always challenging.

Acknowledgments

I want to acknowledge the University Grants Commission, Nepal for the partial research support, a small RDI.

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