Quickest Transshipment in the Prioritized Evacuation Network

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Abstract
Flow maximization, time minimization, and cost minimization are three main aspects of mathematical optimization problems. The evacuation planning problems are about flow maximization and/or time minimization problems in different dynamic evacuation networks. The quickest transshipment problem in such a network is to send exactly the right amount of flow out of each source and into each sink in the minimum overall time. In evacuation planning problems, the term flow stands for either the evacuees or the evacuees carrying vehicles. Here, we use the quickest transshipment strategy in a prioritized evacuation network. It consists of a collection and an assignment sub-network as the primary and secondary sub-networks, respectively. Pick-up locations are prioritized in the collection network. By treating such pick-up locations as sources, the available set of transit buses is assigned in the assignment sub-network to shift the evacuees to the sinks to achieve the quickest transshipment. Such an evacuation planning strategy is better suited for the simultaneous collection, assignment, and evacuation process in the prioritized evacuation network.
Keywords: Evacuation planning, evacuation network, lexicographic flow, prioritized network, transshipment

Introduction

Problems with dynamic networks were introduced by Ford and Fulkerson [1, 2]. The quickest transshipment strategy corresponding to the evacuation planning problems is modeled in dynamic networks where each arc has a nonnegative flow capacity and integer transit time. It demands a minimum time limit such that all supplies can be sent to the sinks (safe places) from the sources (disastrous zones). There has been a fair amount of work regarding its different aspects, including the quickest transshipment, as referred by [3-9]. These problems are handled from different perspectives: transit-based, car-based, and pedestrian movements. The transit-based planning problems are to minimize the evacuation duration by routing and scheduling a fleet of vehicles, say buses.

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The NP-hard multi-depot, multi-trip bus-based evacuation planning problem (BEPP) was introduced and analyzed prominently in [10]. However, if there is only one bus depot, assuming that the bus pickups the same number of people that equals its capacity, the author in [11] has also proposed the BEPP for the evacuation of a region from a set of collection points to a set of capacitated sinks.

A solution to the maximum flow problem sends the maximal amount of flow from the sources to the sinks for the fixed integer time horizon T. Lexicographically maximum (Lex-max) flow problem with many sources and many sinks was introduced and many efficient algorithms were presented from different aspects in [12, 13]. In such a problem, the terminals (sources and/or sinks) are ordered with a certain priority for a Lex-max flow respecting the priority. The partial contraflow with path reversals leads to a significant improvement in increasing the flow values, decreasing the evacuation time, and utilization of the unused capacities of paths for humanitarian logistics and vehicle movements [14]. Such problems have been studied with the help of the Lex-max dynamic flow problem by applying the minimum cost flow computations as a tool [15]. An algorithm to find the universally quickest transshipment has been presented [16]. They also used the minimum cost flow computations.

Here, evacuees are collected from the disaster zone to the pickup locations of the collection sub-network in minimum time as the quickest transshipment by using the Lex-max flow approach. Considering such pickup locations as the sources, available transit buses are also assigned in the network to evacuate the evacuees safely to the sinks to achieve the minimum clearance time. Such a strategy is better suited for simultaneous evacuation in an integrated network.

The structure of the paper is designed in six sections: an explanation of an integrated evacuation network is in Section 2. Collection of evacuees at the collection sub-network is in Section 3. The assignment of vehicles in the assignment sub-network is in Section 4. An integrated solution approach is in Section 5. Section 6 concludes the paper.

2. An Integrated Evacuation Network

Consider two separate dynamic networks N_1 and N_2 as collection and assignment sub-networks, respectively, to form an integrated evacuation network $N = \mathrm{N}_1 \cup \mathrm{N}_2$. Here, N_1 is a directed two-way network and N_2 is the mixed network having directed one-way arcs and undirected edges.

2.1. The collection sub-network

Let $N_1 = (S, V, A, u_a, \tau_a, Y)$ be a collection sub-network, $S = \{s_1,\ s_2, \dots, s_n\}$, $V = (v_1, v_2, \dots, v_n)$ and $Y = (y_1, y_2, \dots, y_n)$ denote the sets of sources, auxiliary nodes, and pickup locations, respectively. The set of arcs joining any two nodes in N_1 is denoted by A where the capacity and transit time are denoted by u_a and τ_a, respectively. The capacity $u_a\colon A \to \mathbb{Z}_{\geq 0}$ restricts the amount of flow on the arc and the transit time $\tau_a\colon A \to \mathbb{Z}_{\geq 0}$ represents the time required for the flow to transverse through the respective arc. During evacuee arrival at the pickup location Y from S, the set of pickup locations works as the set of sinks.

2.2. The assignment sub-network

Let $N_2 = (d, Y, E, \tau_e, Z)$ be an assignment sub-network, where d and $Z = \{z_1, z_2, \dots, z_n\}$ are the bus depot and sink, respectively. A set of transit-buses $B = \{b_1, b_2, \dots, b_n\}$ with uniform capacity are located initially at the bus depot d and are assigned as required, during the evacuation procedure. Buses do not return to d even after the completion of the evacuation process as it is risky to return to it under such threats. So, it does not play a significant role further in the system. The set E consists of the one-way arcs $e = (d, y)$ with $y \epsilon Y$ and the undirected edges $e = [y, z]$, with $y \in Y$, $z \in Z$. Transit times of the respective arcs and edges are denoted by $\tau_e \in \mathbb{Z}_+$ as τ_{di} and τ_{ij}, respectively.

2.3. An Embedding of the Integrated Network

In an embedding $N = N_1 \cup N_2$, capacity, transit times, and other related attributes for N are carried over from their respective sub-networks, as in Figure 1. The node Y works as the sink concerning N_1 but as the supply for N_2. Transit buses are assigned in N_2 from d, which are sufficiently closer to it, on a first-come-first-serve basis, i.e., the evacuees collected earlier will be assigned earlier to the appropriate sink and will be continued till the supply is available at Y respecting the capacity of the sink. For details, we refer to [7-8].

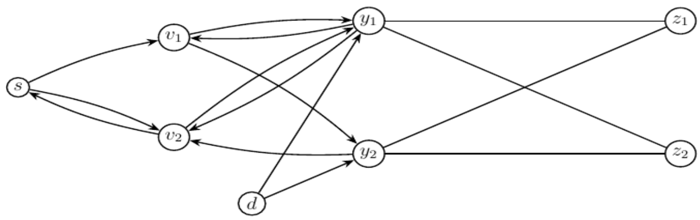

Figure 1. An integrated evacuation network

3. Collection of Evacuees

Mostly, the evacuees collected at pickup locations are to wait a long time for being assigned to transit buses in the network. On the other hand, the nature of the pickup locations may differ. So, the network is better to be prioritized. If any two or more pickup locations are with the same priority, they can be ordered either way. For such formulations, the lexicographic approach is better suited. Such an approach can be applied to achieve the quickest transshipment with several sources and sinks, provided with the given supply and demand [15].

The $s - y$ flow of evacuees over time is a non-negative function f on $A \times T$ for given $T = \{0,1, \dots, T\}$ satisfying the flow conservation and capacity constraints (1-3). The inequality flow conservation constraints allow it to wait for flow at intermediate nodes, however, the equality flow conservation constraints force that flows entering an intermediate node must leave it.

$$\sum_{a \epsilon A_i^{in}} \sum_{\sigma=\tau_a}^{T} f\ (a, \sigma - \tau_a) - \sum_{a \epsilon A_i^{out}} \sum_{\sigma=0}^{T} f\ (a, \sigma)\ = 0, \forall\, i \in V \backslash (S \cup Y)\ (1)$$

$\sum_{a\epsilon A_i^{in}} \sum_{\sigma=\tau_a}^{\theta} f\ (a, \sigma - \tau_a) - \sum_{a\epsilon A_i^{out}} \sum_{\sigma=0}^{\theta} f\ (a, \sigma)\ \geq 0, \forall\, i \in V \backslash (S \cup Y), \theta \in T$ (2)
$0 \leq \mathrm{f}(\mathrm{a}, \theta) \leq u_a\ \forall\ a \in A, \theta \in T$ (3)

The sets of outgoing and incoming arcs for the node $i\ \epsilon\ V$ are denoted by, $A_i^{out} = \{a = (i, j)\ \epsilon\ A\}$ and $A_i^{in} = \{a = (j, i)\ \epsilon\ A\}$, respectively. Not stated otherwise, for all $y\ \epsilon\ Y$ and$s\ \epsilon\ S$, we assume that $A_i^{out} = A_i^{in} = \phi$ in the case without arc reversals. However, for the source node s and sink node y, we get the flow value $\upsilon_{\mathrm{f}}(\mathrm{s}) > 0$ and $\upsilon_{\mathrm{f}}(\mathrm{y}) < 0$, respectively, where $\sum_{i\ \epsilon\ V} \upsilon_{\mathrm{f}}(\mathrm{i}) = 0$.

Problem 1.

Given a collection sub-network $N_1 = (S, V, A, u_a, \tau_a, Y)$ with supplies at S, demands at Y auxiliary nodes V, arc capacity u_a, and arc transit time τ_a for $a\ \epsilon\ A$. The quickest partial arc reversal transshipment problem is to find the quickest arrival of evacuees at Y with partial arc reversals capability.

Let the reversals of an arc $a = (i, j)$ be denoted by $a' = (j, i)$. The transformed network of N_1 is with modified arc capacities and constant transit times,
$u_{\bar{a}} = u_a + u_{a'}$ and $\tau_{\bar{a}} = \tau_a$ if $a \in A$ and is $\tau_{a'}$ for $a \notin A$ (4)

Here, an edge $\bar{a} \in \bar{A}$ in transformed network $\overline{N_1}$ if $a \vee a' \in N_1$. Concerning the auxiliary reconfiguration, it is allowed to redirect the arc in any direction with the modified increased capacity but with the same transit time in either direction. The remaining graph structure and data are unaltered.

Consider the set of sources and sinks to be prioritized as $\{s_1, s_2, \dots, s_n\}$ and $\{\mathrm{y}_1, \mathrm{y}_2, \dots, \mathrm{y}_\mathrm{k}\}$, respectively.

Let s^* be the super source connected to such s_i with arcs having infinite capacity and zero transit time.

Let the sinks in N_1 be prioritized with the highest priority to the nearest by determining their shortest distances concerning s^*. The lex-max flow within the specified time horizon T entering the sinks Y in N_1 in that order can be computed in polynomial-time, as in Algorithm 1, based on [15]

As a *mutatis mutandis* to the results in [15], we are here with two more result

Algorithm 1. Lex-max dynamic flow of evacuees in $\boldsymbol{N_1}$.
Input: A dynamic collection network $N_1 = (S, V, A, u_a, \tau_a, Y)$. Let $V = V \cup \{s^*\}$ and $A^{\delta+1} = V \cup \{(s^*, s_i)\}$: $s_i \in S$, where $u_a(s^*, s_i) = \infty$ and $\tau_a(s^*, s_i) = 0$ for $\{s^*\}$ be the super source. Let $N_1^{\delta+1}$ denotes the resulting network with $g_0^{\delta+1} = 0$ be the zero flow and the set of chains be $\Gamma^{\delta+1} = \phi$. 1. For $i = \delta, \dots, 1$, set the arc be $A^i = A^{i+1}$, a. If s_i is sink, add the arc $s_i s^*$ with $u_a(s^*, s_i) = \infty$ and $\tau_a(s^*, s_i) = -(T+1)$. Then get f^i = minimum cost circulation in the resulting network using τ_a as the arc cost. b. If s_i is source, delete the arc $s_i s^*$ from A^i and get f^i = minimum cost maximum $s^* s_i$ – flow in the resulting network using τ_a as the arc cost. 2. Update the dynamic flow $g^i = g^{i+1} + f^i$.

3. Let Δ^i = standard chain decomposition of f^i, the chain decomposable set, $\Gamma^i = \Gamma^{i+1} + \Gamma^i$.
4. Finally, return $\Gamma = \Gamma^1$.

Theorem 1. Algorithm 1 constructed for the Lex-max dynamic flow in N_1 gives the feasible solution for the Lex-max number of evacuees at Y.

Theorem 2. For any Lex-max dynamic flow problem, such a flow can be computed via k minimum cost flow (MCF) computations in $O(k(MCF)(m,n))$, where $O\big(k(MCF)(m,n)\big) = O(m \log n\ (m+n))$ [16].
The Lex-max flow problem in dynamic networks by applying the MCF computations is presented in [15]. Now, we are presenting the quickest partial arc reversal transshipment algorithm to get the quickest arrival of evacuees at Y in N_1 corresponding to the arc reversal capability.

Theorem 3. Algorithm 2 constructed gives the optimal solution for the quickest arrival of evacuees at Y and saves the unused capacity. [7, 8]

Algorithm 2. Quickest partial arc reversal transshipment algorithm. [7, 8]
Input: A dynamic collection network $N_1 = (S, V, A, u_a, \tau_a, Y)$, with the supply and demand. Construct a transformed dynamic collection network $\overline{N_1}$ as in Equation (4). Solve the quickest transshipment problem [15] in the transformed network of Step 1. For each $\theta \in \mathrm{T}$ and reverse $a' \in A$ up to capacity $c_a - u_a$ iff $c_a > u_a$, u_a replaced by 0 whenever $a \notin A$, in N_1, where c_a denotes the static $s -$ y flow value in each $a \in A$. For each $\theta \in \mathrm{T}$ and $a \in A$, if a is reversed, $k_a = u_{\bar{a}} - c_{a'}$ and $k_{a'} = 0$. If neither a nor a' is reversed, $k_a = u_a - c_a$ where k_a is saved capacity of a, [14]. Output: The quickest arrival of evacuees at Y in N_1 with partial arc reversal.

Theorem 4. For the quickest partial arc reversal transshipment in N_1, the quickest evacuee arrival problem can be computed in polynomial-time complexity via k minimum cost flow (MCF) computations in $O(k(MCF)(m,n))$ time, where $MCF(m,n) = O(m \log n\ (m + n \log n))$. [7,8]

4. Vehicle assignments

Here, we investigate the BEPP for an integrated evacuation planning approach.

4.1. Bus-based Evacuation Planning Problem

Let $i \in Y, j \in Z$ with τ_{ij} as the source-sink travel times. Let τ_{di}, l_i, μ_j be the depot-source travel times, the number of evacuees, and sink capacities, respectively. Then the BEPP is to find a tour plan to minimize the maximum travel times overall buses such that all the evacuees be transported to the sink.

Let the number of evacuees at every source be known. Assume Q be the uniform bus capacity, as a unit. The movement between the pickup locations Y is ignored and the same situation in between the sinks Z. The set of tours of the buses cannot be changed anymore after they start. Let $\sum_{i\in Y} l_i$ and $\sum_{j\in Z} \mu_j$ be the total number of evacuees and the total sink capacity, respectively. The maximum number of rounds R for the evacuation is given by $\sum_{i\in Y} l_i$. The nonnegative travel cost of τ_{ij} on each edge $e = (i, j) \in E$ are taken symmetric and satisfies the triangle inequality. The variables τ_{to}^{br} and τ_{back}^{br} give travel time for the bus b within the round r from source to sink, and from the sink to the next source, respectively. Let T_{max} be the total duration of the evacuation. The problem can be formulated to minimize $T_{\max}$ such that,

$$T_{max} \geq \sum_{i\in Y} \sum_{i\in Z} \tau_{di}\ x_{ij}^{b1} + \sum_{r\in R} \tau_{to}^{br} + \sum_{r\in R} \tau_{back}^{br}, \forall\, b \in B, \tag{5}$$

$$\tau_{to}^{br} = \sum_{i\in Y} \sum_{i\in Z} \tau_{ij}\ x_{ij}^{br}, \forall\, b \in B, r \in R, \tag{6}$$

$$\tau_{back}^{br} \geq \sum \tau_{ij} \left[\sum_{k\in Y} x_{kj}^{br} + \sum_{l\in Z} x_{il}^{b,r+1} - 1 \right], \forall\, b \in B, r \in R-1, \tag{7}$$

$$\sum_{i\in Y} \sum_{i\in Z} x_{ij}^{br} \geq \sum_{i\in Y} \sum_{i\in Z} x_{ij}^{b,r+1}, \forall\, b \in B, r \in R, \tag{8}$$

$$\sum_{i\in Y} \sum_{i\in Z} x_{ij}^{br} \leq 1, \forall\, b \in B, r \in R-1, \tag{9}$$

$$\sum_{i\in Y} \sum_{i\in Z} \sum_{r\in R} x_{ij}^{br} \geq l_i, \forall\, i \in Y, \tag{10}$$

$$\sum_{i\in Y} \sum_{b\in B} \sum_{r\in R} x_{ij}^{br} \leq \mu_j, \forall\, j \in Z, \tag{11}$$

$$x_{ij}^{br} \in \{0,1\}, \forall\, \tau_{to}^{br}, \tau_{back}^{br}, T_{max} \in \mathbb{R}. \tag{12}$$

Constraint (5) needs T_{max} to be greater than or equal to the maximal travel cost subject to all bus movements and is to be minimized on $T_{\max}$. Constraints (6) and (7) are the measure of travel time for the bus b within the round r from source to sink, and from that sink to the next source, respectively. Constraint (8) tells that the tours are connected and can stop whenever they like. Constraint (9) allows a bus from a source to a sink per round. Also, (10) and (11) represent the bus and shelter capacity constraints, respectively. Constraint (12) represents whether the bus b travels from source i to sink j in round r. For details, we refer to [11].

4.2. An Integrated Evacuation Planning Approach

Transit buses having uniform capacity Q are assigned from d which are sufficiently nearer to Y in N_2 on a first-come-first-serve basis. Such an assignment begins only after $\alpha_1 \geq Q$ to α_1 be the number of evacuees who arrived at the highest pickup demand. For the subsequent assignments, the effective waiting instance ξ is almost negligible. However, waiting at pickup locations is comparatively better than waiting at the disaster zone itself. Buses are assumed to pick up their full capacities. For this, the potential demands of the pickup locations are adjusted to be the integral multiple of busloads. Let the potential demand of the pickup location $y_k \in Y$ be $\alpha(y_k)$. Then the demands can be adjusted to be $\alpha'(y_k)$ by using the following demand adjustment,

$$\alpha'(y_{k-1}) = \lfloor \{ \alpha(y_{k-1}) + \alpha(y_{k-2}) - \alpha'(y_{k-2}) + \cdots + \alpha(y_1) - \alpha'(y_1) \} / Q \rfloor . Q \tag{13}$$

But if the k^{th} pickup location is the last one with the least priority, then it is taken as,

$$\alpha'(y_k) = \alpha(y_k) + \alpha(y_{k-1}) - \alpha'(y_{k-1}) + \cdots + \alpha(y_1) - \alpha'(y_1) \tag{14}$$

With the constraints (5-12), the integrated problem can be reformulated to minimize $T_{\max}$ *such that:*

$$T_{max} \geq \xi + \sum_{r \in R} \tau_{to}^{br} + \sum_{r \in R} \tau_{back}^{br} \quad \forall\, b \in B. \qquad (15)$$

5. An Integrated Solution Approach

Here, we design an algorithm for the quickest transshipment in the integrated evacuation network.

Algorithm 3. Evacuation planning algorithm in an integrated network topology [7, 8].
Input: $N = (S, V, A, u_a, \tau_a, Y, d, u_e, \tau_e, Z)$, with given supply and demand. 1. $N_1 = (S, V, A, u_a, \tau_a, Y)$ with their pickup locations be Y. 2. Construct a priority on Y assigning the highest priority to the nearest. If any two or more are with the same priority, they are to be prioritized consecutively in either order. 3. Determine the arrival of evacuees at Y of N_1 from S using Algorithm 2. 4. Assign the transit buses from d to Y in $N_2 = (d, Y, u_e, \tau_e, Z)$ for the supplies obtained in Step 3, to the nearest sink Z, on the first-come-first-serve basis. 5. Begin the assignment with $\alpha_1 \geq Q$ for α_1 be the collection of evacuees at $y_1 \in Y$ and Q be the homogeneous transit bus capacity and is continued for the adjusted demands provided by Equation (13). 6. Stop, if all the supplies at each Y are fulfilled, respecting the capacity constraints of Z. 7. Otherwise, return to Step 4. Output: Quickest transshipment of evacuees finally to Z.

Theorem 7. Algorithm 3 constructed for the evacuation planning problem in an integrated network gives the quickest transshipment of evacuees in the integrated evacuation network. [7-8]

6. Conclusion

The quickest transshipment problem is to find the minimum clearance time to send a given amount of flow from multiple sources to multiple sinks network. The quickest transshipment problem in a dynamic network can be reduced to an equivalent Lex-max flow problem and is solved in polynomial-time complexity. Evacuees are collected at the prioritized pickup locations of the collection network following the quickest transshipment in the Lex-max flow approach and are assigned simultaneously to the homogeneous transit buses in the assignment network on a first-come-first-serve basis. The waiting delay at the pickup locations is almost negligible. On the other side, if there is some waiting, it is preferable to wait at such pickup locations rather than to be at the danger regions. The arc reversal capability is beneficial to improving the transshipment time of the evacuees in the integrated evacuation network where the saved unused arc capacities are useful for emergency facility locations and logistics. Such a strategy is better suited for the simultaneous evacuation process in the integrated evacuation network.

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