Route Dominance on Symmetric Type Transit-based Evacuation Network

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Abstract

The selection of dominant routes for transit-based evacuation planning problems mainly depends upon the structure and nature of the available network. To achieve efficient evacuation planning, the problem is not only on the selection of a route and a shelter for each route on the available network topology but also on the route-to-vehicle assignment and vice versa, which is more complex and challenging in real practice. The evacuation network adds complexity to the solution and impacts the problem. In this paper, the transit-based evacuation network has been revised from different perspectives focusing on the symmetric type networks with their dominant evacuation planning routes to have the minimum evacuation cost.

Keywords: Evacuation planning problem, transit-based network, vehicle assignment, route dominance

Background

The most fundamental necessity of human beings is saving lives which should be the fundamental objective of planning for emergency evacuation during different natural or man-made disasters. Most disasters cannot be predicted and are unavoidable, and the damages caused by them are severe if the evacuation strategies are not well-planned and well-implemented. Evacuation plans and mitigation strategies are followed by the response and recovery with necessary actions to normalize the situation. Bringing normalcy is one of the major challenges during a disaster. One has to idealize the real-life problem by making some simplified hypotheses and different assumptions for its mathematical formulation, though such models seldom represent all the existing characteristics of real-life situations.

In evacuation planning, auto-based evacuees and transit-based evacuees can be categorized as high and low-mobility populations, respectively. The former is supposed to withdraw from the hazardous area by using their own vehicles whereas the latter needs to be sent to the transit hubs for further evacuation. In large cities of developing countries, many people fall into the low-mobility population and are to be evacuated by using transit vehicles. The great loss of people during Hurricane Katrina was due to the lack of proper planning for the transit-based evacuees rather than the disaster itself, Litman (2006).

Evacuation models can be classified into two broad categories microscopic and macroscopic. The former emphasizes individual parameters like walking speed, physical ability, reaction time, and the interaction of each evacuee with other evacuees during the movement and are based mainly on simulation. Whereas, the macroscopic models treat the occupants as a homogeneous group and are taken account of their common characteristics only. So the macroscopic models are able to produce mainly the lower bounds of the evacuation time only whereas, the microscopic models work with a smaller degree of simplification than the macroscopic ones and the method of choice is mostly the simulation approach. These two aforementioned models can also be combined in an interactive solution process, where the output of one type of model is used as input to the other so that the output of both models remains stable and is named a sandwich method.

In this paper, different instances of transit-based evacuation planning (TEP) problems are revisited for the dominant evacuation routes with respect to evacuation cost (EC). Section 2 presents the state of the art of the problem. The formulation of the TEP problem is in Section 3. Section 4 includes the transit vehicle assignment on such a network. Their respective route dominances are in Section 5. Finally, in Section 6, it has been concluded.

Literature Review

Evacuation planning problems and their variants are based on vehicle routing problems (VRP). Such VRPs are for the efficient distribution of goods from different depots to the customers to design the least cost delivery whereas, in the TEP problems, the objectives are to minimize the duration of evacuation by routing and scheduling a fleet of homogeneous and capacitated vehicles like the buses, which were initially located at one or more depots. For more details about such problems and their variants, we refer to Adhikari & Dhamala (2020a), Adhikari & Dhamala (2020b), Dhamala & Adhikari (2018), Adhikari et al. (2020), Adhikari & Dhamala (2022), Adhikari (2023), and the references therein.

The performance and efficiency of the strategies for evacuation planning problems depend upon the nature of the road network, population density, the behavior of the population, and many other factors. It concentrates mainly to find the optimal use of vehicles and their appropriate assignment on the available network during their shifting from danger zones to safety as effectively as possible with utmost reliability, Dhamala & Adhikari (2018), and Adhikari (2020).

In real practice, the number of evacuees at each location can exceed the capacity of a bus. It demands the necessity for a split delivery (SD) service. Moreover, the number of available buses is insufficient to transport all the evacuees without multiple trips and each shelter has a capacity that limits the number of evacuees it can serve. Such situations also demand an SD service. In such situations, Bish (2011) proposes and analyzes two alternative models for the multi-depot, multi-trip, bus-based evacuation planning (BEP) problem, at which the first simultaneously identifies optimal route construction and assignment of the vehicle where the next identifies the optimal route assignment from a set of feasible routes. Goerigk et al. (2013) have developed a simplified version for the evacuation of a region from a set of collection points to a set of capacitated shelters with the help of buses in minimum time assuming that the bus pickups exactly the number of people that equals its capacity when visiting a source without SD service.

Simple examples of evacuation networks without and with SD service are shown in Figure 1, where D, $\{X_1, X_2, X_3\}$, and $\{Z_1, Z_2, Z_3\}$ stands for the bus depot, set of pick-up demands, and the sinks, respectively.

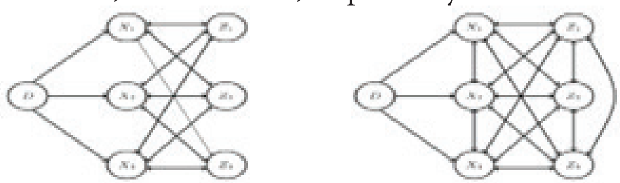

Figure 1. Transit-based evacuation networks: (a) without SD and (b) with SD.

Assuming that the number of evacuees is not known exactly, the BEP is extended to the robustness by Goerigk et al. (2014) in which the exact number of evacuees is not known in advance however a set of likely scenarios is known and after some time, such uncertainty will be removed. In such instances, it is to decide whether the buses are better to send right now as the here-and-now bus under uncertainty or to wait as a wait-and-see bus until the exact scenario becomes known.

Formulation of a TEP problem

For the formulation of such a problem, based on Bish (2011), consider the network to be N=(*V*,*E*) where V is the set of nodes/vertices and E is the set of arcs. N is composed of three subsets D, X, and Z where D stands for a depot at which buses are initially located and dispatched; X is a set of demand nodes representing pickup locations demanding evacuation services and Z is a set of shelter (sinks) where the evacuees are to be transported. Let B be the available transit-vehicles, say buses, each having a capacity Q, and the bus is initially located at the depot. Depot is only the initial location of the buses and does not play significant roles further in the evacuation process and the buses do not return to the depot after the completion of the evacuation process. In fact, it could be risky to return to the depot under a threat. Moreover, such depots may not be the best places to store the buses during threats. Let the demand node i has a demand α_i, $i \in X$ and shelter j have a capacity β_j, $j \in Z$. Then the arc (i, j) has a non-negative travel cost of τ_{ij} for $(i, j) \in E$. Travel cost is proportional to the travel time or distance; though we mainly use the term time, as our aim is to route and schedule the buses to minimize the evacuation duration. All costs in the network are taken symmetric and are supposed to satisfy the triangle inequality.

Let the number of evacuees at every source be known. Assume Q be the uniform bus capacity, as a unit. The movement between the pickup locations Y is ignored and the same situation in between the sinks Z. The set of tours of the buses cannot be changed anymore after they start. Let $\sum_{i \in Y} l_i$ and $\sum_{j \in Z} \mu_j$ be the total number of evacuees and the total sink capacity, respectively. The maximum number of rounds R for the evacuation is given by $\sum_{i \in Y} l_i$. The nonnegative travel cost of τ_{ij} on each edge e=(i, j) $\in E$ are taken symmetric and satisfies the triangle inequality. The variables τ_{to}^{br} and τ_{back}^{br} give travel time for the bus b within the round r from source to sink, and from the sink to the next source, respectively. Let T_{max} be the total duration of the evacuation. The problem can be formulated to minimize T_{max} s.t.

$$T_{max} \geq \sum_{i \in Y} \sum_{i \in Z} \tau_{di}\ x_{ij}^{b1} + \sum_{r \in R} \tau_{to}^{br} + \sum_{r \in R} \tau_{back}^{br}, \forall\, b \in B \quad (1)$$

$$\tau_{to}^{br} = \sum_{i \in Y} \sum_{i \in Z} \tau_{ij}\ x_{ij}^{br}, \forall\, b \in B, r \in R \quad (2)$$

$$\tau_{back}^{br} \geq \sum \tau_{ij} \left[\sum_{k \in Y} x_{kj}^{br} + \sum_{l \in Z} x_{il}^{b,r+1} - 1 \right], \forall\, b \in B, r \in R-1 \quad (3)$$

$$\sum_{i \in Y} \sum_{i \in Z} x_{ij}^{br} \geq \sum_{i \in Y} \sum_{i \in Z} x_{ij}^{b,r+1}, \forall\, b \in B, r \in R \quad (4)$$

$$\sum_{i \in Y} \sum_{i \in Z} x_{ij}^{br} \leq 1, \forall\, b \in B, r \in R-1 \quad (5)$$

$$\sum_{i \in Y} \sum_{i \in Z} \sum_{r \in R} x_{ij}^{br} \geq l_i, \forall\, i \in Y \quad (6)$$

$$\sum_{i \in Y} \sum_{b \in B} \sum_{r \in R} x_{ij}^{br} \leq \mu_j, \forall\, j \in Z \quad (7)$$

$$x_{ij}^{br} \in \{0,1\}, \forall\, \tau_{to}^{br}, \tau_{back}^{br}, T_{max} \in \mathbb{R} \quad (8)$$

Constraint (1) needs T_{max} to be greater than or equal to the maximal travel cost subject to all bus movements and is to be minimized on T_{max}. Constraints (2) and (3) are the measure of travel time for the bus *b* within the round *r* from source to sink, and from that sink to the next source, respectively. Constraint (4) tells that the tours are connected and can stop whenever they like. Constraint (5) allows a bus from a source to a sink per round. Also, (6) and (7) represent the bus and shelter capacity constraints, respectively. Constraint (8) represents whether the bus *b* travels from source i to sink *j* in round *r*. For details, we refer to Bish (2011).

Example 1. In this instance as in Figure 2, let the demands (3, 3, 1), capacities at the sinks are (3, 4, 3) with buses available is 3. The distance of the demands from the depot is (4, 3, 6) and the distances between X to Z are $\begin{pmatrix} 4 & 7 & 9 \\ 10 & 7 & 6 \\ 7 & 6 & 9 \end{pmatrix}$.

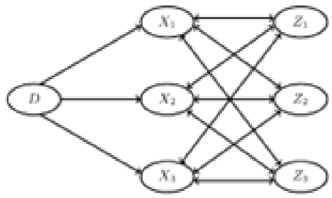

Figure 2. An instance of a BEP problem.

The BEP problem is formulated so as to choose the minimum evacuation cost of all possibilities. Here, the plan's critical path is for bus 3 or bus 1 as in Table 1, which shows its feasible solution with the evacuation cost of 25.

Table 1. A feasible solution to the BEP problem

Trip	1	2	3	Route assignment	EC
Bus 1	(1, 3)	(3, 1)		$\tau_1, \tau_{13}, \tau_{33}, \tau_{31}$	25
Bus 2	(2, 1)	(2, 3)		$\tau_2, \tau_{21}, \tau_{21}, \tau_{23}$	21
Bus 3	(1, 1)	(1, 2)	(2, 3)	$\tau_1, \tau_{11}, \tau_{11}, \tau_{12}, \tau_{22}, \tau_{23}$	25

Transit-vehicle assignment on the evacuation network

Let X and Z denote the pickup demands and sink respectively. Let them be connected by different paths having different capacities, and transit times. Let T be the travel time of the path, as the sum of the travel time of edges, C be the capacity of the path as the minimum capacity of the edges and ξ be the number of evacuees at the source or the pickup demands, then the EC is calculated as in Min & Neupane (2011) by $EC = T + \lceil \frac{\xi}{C} \rceil - 1$

As far as the quickest path is concerned, path X_i is said to be the quickest path if and only if

$$T_i + \left\lceil \frac{\xi}{C_i} \right\rceil - 1 \leq T_j + \left\lceil \frac{\xi}{C_j} \right\rceil - 1 \quad \forall\, j \in \{1,2,3,\ldots,k\} \backslash \{i\}.$$

Let { X_1^*,X_2^*,X_(3)^*,…,X_k^* } be k edge-disjoint paths from X to Z, with C_i and T_i be the capacity and transit times of paths X_i and ξ be the number of evacuees at X, then the combined evacuation cost (CEC) becomes

$$CEC(X_1^*, X_2^*, X_3^*, \ldots, X_k^*) = \left\lceil \frac{\xi + \sum_i^k C_i T_i}{\sum_{i=1}^k C_i} \right\rceil - 1 \quad (10)$$

Example 2. Consider three possible paths X_1, X_2 and X_1 in between demand node X and the sink Z with their respective travel time T and capacity C as in Figure 3. Suppose the evacuees at the demand nodes are 52 then the EC through these paths can be calculated by using Equation (9) which are 23, 20, and 17, respectively among them EC (X_3^*) be chosen as the quickest path. But if the next path X_1^* is added on the evacuation route then the CEC as in Equation (10) is CEC(X_1^*, X_2^*)=16, which is shorter than the current EC. Moreover, by adding the next path X_2^* their CEC becomes CEC (X_1^*, X_2^*, X_3^*)=13, further improved, which is smaller even than the route X_1^* with the longest travel time. Hence, the route X_1^* having longer travel time can be removed from the evacuation route, in such edge-disjoint paths since CEC (X_1^*, X_2^*)=11<CEC(X_1^*,X_2^*, X_3^*), and the EC be reduced even more by 2.

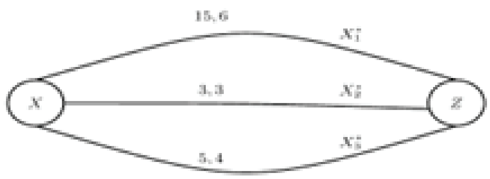

Figure 3. A simple evacuation network.

5. Route dominance in the transit-based evacuation system

The route p is said to dominate the route p', and is named the dominant route, if it does not have a longer evacuation duration, it doesn't have more cost for the demand nodes considered and every unreachable demand node on route p is also unreachable for p', among all the feasible routes on the evacuation network. If different paths lead to the same shelter and neither of them is better than the other overall criteria, then neither of the paths is dominating. The selection of such dominant routes mainly depends upon the nature of the available network. In the case of evacuation planning problems on the available network, the problem is not only on the selection of a route and the selection of a shelter for each route but also on the route-to-vehicle assignment and vice versa, which is more complex, in practice. During the vehicle assignment on the secondary network, a tour of vehicles are to be assigned in an optimal way. We are here to see their dominance with respect to their evacuation duration for different approaches to vehicle assignments in the evacuation network.

In the single objective optimization problem the superiority of a solution over the other solutions is easily determined by comparing their objective function values whereas, in multi-objective optimization problems, the goodness of a solution is determined by the dominance.

The solution x_1 is said to dominate the solution x_2 if, (i) solution x_1 is no worse than x_2 in all objectives, and (ii) solution x_1 is strictly better than x_2 in at least one objective. Hence, there are three possibilities that can be the outcomes of dominance between two solutions x_1 and x_2 of a bi-objective function, (i) solution x_1 dominates solution x_2, (ii) solution x_1 gets dominated by solution x_2, and (iii) neither of them dominates nor dominated by each other. Some of their properties are as follows:

- **Not reflexive:** The dominance relation is not reflexive, i.e., x_1 does not dominate x_1.
- **Not symmetric:** The dominance relation is not symmetric. In other words, if x_1 dominates x_2 then x_2 does not dominate x_1. So the dominance relation is asymmetric.
- **Not antisymmetric:** Since the dominance relation is not symmetric, it cannot be antisymmetric as well.
- **Transitive:** The dominance relation is transitive. That is, if x_1 dominates x_2 and x_2 dominates x_3 then x_1 dominates x_3.

Example 3. Consider a bi-objective optimization problem with six different solutions with respect to their different objective function values in their objective spaces as shown in Figure 4. Let the function f1 be maximized and the function f_2 be minimized. In such a situation, it is usually difficult to find a single solution that is better with respect to both objectives, however, we can choose the better one among them in terms of both objectives by using the principle of dominance. Here, solution x_1 dominates solution x_2 for both objectives. But, solution x_5 is better than solution x_1 with respect to f_1 whereas solution x_5 is no worse than solution x_1 in the next objective f_2. Hence, solution x_5 dominates solution x_1. However, x_3 and x_5 are non-dominated to each other, as either of them is better than the next in one objective but the worst in the next.

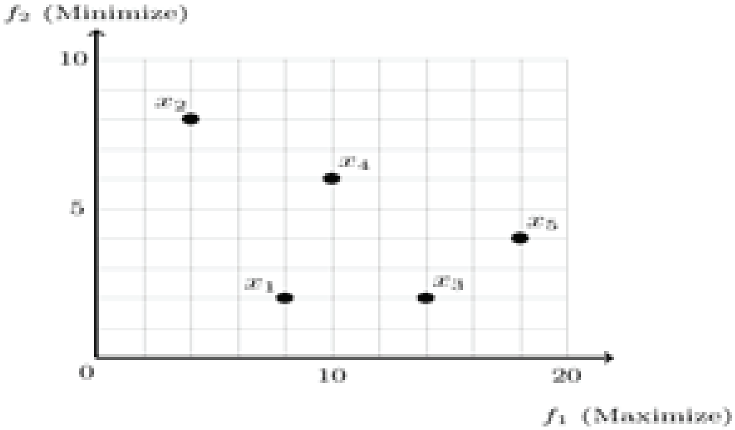

Figure 4. Solution space of a simple bi-objective function.

The nature of the evacuation network highly affects the proper choice of evacuation planning strategies and the resource assignment in the network. The problem itself is an NP-complete problem and its exact solution is imperative for that different heuristics are designed for better and approximate solutions. Here, we are presenting different instances of vehicle assignments in the different evacuation-type networks with the help of some examples.

Example 4.

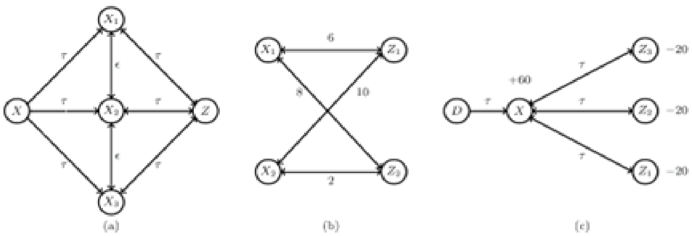

Figure 5. Different scenarios of vehicle assignment on evacuation routes.

(a) Consider a simple instance as in Figure 5 (a) where X and Z are simply the source and sink respectively, with X_1, X_2, and X_3 be three different pickup nodes which are supposed to be at the evacuation cost of τ from X and Z both. Let the pickups be near enough to each other at the evacuation cost (EC) of apart. The evacuation cost with and without SD in such given instances and their respective bounds, as the ratio of their EC without SD to with SD are shown in Table 2, where the vehicles were scheduled simultaneously from X to Z. Let |B| denote the number of vehicles. It is observed that the EC is 2τ for $|B|=3$ with and without SD in the network. This instance signifies that the evacuation route with split delivery need not be beneficial always.

Table 2. EC with and without SD showing the respective EC and bound.

$\lvert B\rvert$	EC with SD	EC without SD	Bound	Remarks
1	$4\tau+\epsilon$	6τ	1.5	Improved
2	$2\tau+\epsilon$	4τ	2	Improved
3	2τ	2τ	1	Not improved

(b) Consider the network as in Figure 5 (b). Let a vehicle that has picked up a full load of evacuees at X1 can still pick up a full load at X^2 and the sinks Z_1 and Z_2 are with sufficiently high capacity. If the sink closest to its last pickup is assigned, then the vehicle would be routed on X_1-Z_1-X_2-Z_2 with the evacuation duration 27, whereas the optimal solution would be on the route X_1-Z_2-X_2-Z_2 with evacuation duration 18. It means, sending each vehicle to its closest sink to the last pickup node is not always the appropriate route to be assigned for optimal routing.

(C) Consider the fixed demand at X to be 60 with the vehicles and shelters with a capacity of 40 and 20 respectively as represented in Figure 5(c). Then the evacuation cost (EC) for different vehicle assignment scenarios with |B| as the number of vehicles can be obtained in Table 3. It indicates that more vehicles seem to be beneficial in considering the EC to some extent.

Table 3. Table showing the route assignments and the EC

$\lvert B\rvert$	Vehicle route assignments	EC
1	D-X-Z_1-X-Z_2-X-Z_3	6τ
2	*(i)*D-X-Z_1,*(ii)* D-X-Z_2-X-Z_3	4τ
3	*(i)*D-X-Z_1,*(ii)* D-X-Z_2 ,*(iii)* D-X-Z_3	2τ

Example 5. Consider an evacuation network. Let it be with one, two, three, and four vehicles as in Figure 6 where vehicles start from the depot D assigned to the demands X_i and finally return to the depot. Let all the unmarked arcs be with length unity. The number of vehicles |B| assigned and the respective maximum route length τ are also shown in Figure 7. Their relationship is also shown in Figure 6 and also in Figure 7. Here, some useful results on BEP problems regarding the fleet size can also be drawn,

- For the min-max objective there is an optimal threshold fleet size. Increasing the fleet size beyond this threshold does not impact optimality.

- The evacuation cost does not always decrease in a convex manner with the number of vehicles.

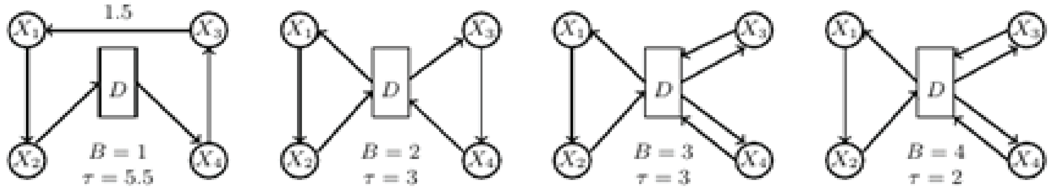

Figure 6. Evacuation network with a single depot and different demands.

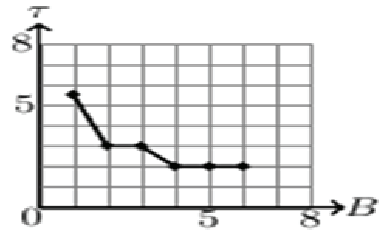

Figure 7. Relation between τ and |B|.

Conclusion

Evacuation planning strategies, and their operations may vary due to their applicable geographical scales, total affected population size, transportation resources and traffic capacity, evacuation objectives, and the time spans.

Evacuation scheduling, traffic route guidance, destination optimization, and the optimal route choice are some of the prominent approaches to accelerate the evacuation process on dominant evacuation route for the efficient evacuation on such symmetric network. However, an integrated optimal evacuation approach to have a single comprehensive solution to the problem is always challenging and somewhat lacking for real case scenarios as the evacuation network topology adds complexity to the solution and impacts on the problem. In real scenario, the evacuation network is almost asymmetric, and such assumptions are almost impracticable and are to be modified accordingly, which is one of the highly challenging issues in evacuation planning problems and strategies.

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