A SURVEY ON NETWORK CONTRAFLOW EVACUATION PLANNING PROBLEMS

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Abstract

Evacuation planning is becoming crucial due to an increasing number of natural and human-created disasters over last few decades. One of the efficient ways to model the evacuation situation is a network flow optimization model. This model captures most of the necessities of the evacuation planning. Moreover, dynamic network contraflow modeling is considered a potential remedy to decrease the congestion due to its direction reversal property and it addresses the challenges of evacuation route planning. However, there do not exist satisfactory analytical results to this model for general network. In this paper, it is tried to provide an annotated overview on dynamic network contraflow problems related to evacuation planning and to incorporate models and solution strategies to them developed in this field to date.

Keywords: Evacuation Planning, Dynamic Network Flows, Contraflows.

1. Introduction

Taking into account the last few decades, we have seen potential increment in natural disasters as well as human–created problems which cause massive destruction including the loss of human lives. Some worth-mentioning examples are recent (2015) earthquake in Nepal, Chichi Bam and Kashmir earthquakes in Taiwan, Iran and Pakistan, the tsunami in Indian Ocean and Japan, September-11 attack in the USA, and hurricanes like Rita and Katrina in 2005. Such hazardous scenarios have drawn the attention of academicians like mathematicians, computer scientists and management scientists to develop the efficient evacuation route planning. Besides, it is useful for the management of mass-meetings and to mitigate the traffic situation in busy traffic hours (60-miles-11-day long jam of China in 2010).It is a part of overall emergency management that includes prevention, planning, response and recovery ($PPRR$).

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Here, planning refers to the emergency evacuation planning i.e. carrying people from unsafe zone to safe zone as soon as possible. Prevention indicates locating the safe zones and developing awareness whereas response and recovery demand to implement the evacuation plans and to counsel affected people to return back into the normal situation as before the catastrophe.

Broadly speaking, many aspects (macroscopic, microscopic and mesoscopic) have been anticipated to model the evacuation scenarios, and still there is debate between researchers about the suitability of the approaches and models. The models incorporating the individual evacuee's behavior are known as microscopic models which are mainly based on simulation experiments and provide upper bound for evacuation time. This approach gives more reliable solution in many cases but is tedious since it consumes more time and needs more search space. Detail of this approach is omitted as it is not part of the topic here. The macroscopic models provide a good lower bound for the evacuation time and optimize the system i.e. this approach assumes cooperative behavior of the evacuees. Macroscopic solutions usually carried out by solving network flow problems. This approach is widely liked and studied due to the increased public attention, improved techniques and the computational efficiency for large-size networks. Network flow model (macroscopic) also represents transportation system, and therefore, the evacuation situation. Evacuation planning problems can be modeled as flow problems in dynamic networks.

In the case of emergency evacuation planning of a part of a city, for example, an evacuation network consists of roads or streets as arcs and that of intersections of the roads as nodes; unsafe place(s) where accident has occurred or going to be occurred soon can be taken as source(s) and the safe place(s) where the people are to be evacuated are assumed as sink(s). Sources and sinks are terminal nodes. Source nodes contain evacuees, and sink nodes wait them for shelter.

There are capacity constraints (maximum number of evacuees at a unit time) on the nodes and arcs. Moreover, arc travel time for an evacuee is assigned to each arc. The parameters on the constraints may be function of time and/or flow or constants. Given a detailed road map of a city, we can model the network. If we can find an optimal evacuation plan in a realistic flow model where each evacuee is supposed to be evacuated in a minimal time period, we are done. This minimal time period is the lower bound that an evacuee needs.

The contraflow problem for evacuation is simply a transportation network with arcs each having capacity and travel time requiring a reconfiguration identifying the ideal direction and reallocating the available capacity for each arc to minimize the evacuation time from source to sink. Achieving the optimal contraflow network model is a challenging task since one has to enumerate combinations of arc orientations and compare those combinations by calculating the evacuation time. Considerable time is required to evaluate each contraflow model candidate incorporating the dynamic traffic flow. The task is NP −complete, Kim et al, (2005). Therefore, there should be a proper balance between evacuation model and real situation.

2. Literature

Still there are of many optimization techniques in the literature to deal with contraflow problem for general graphs. Contraflow problems which appear in the earlier literature are concentrated on the feasibility and effectiveness by simulation or numerical analysis. In their technical report Tuydes and Ziliaskopoulos (2004) presented a mesoscopic contraflow network model based on dynamic traffic assignment method though they are unable to show scalable experiments. A simulated annealing procedure for this problem together with empirical results has been proposed by Kim and Shekhar (2005). They also provide a sketch of the proof that the problem is NP−complete. A tabu-based heuristic was proposed by Tuydes and Ziliaskopoulos (2006) for the problem that significantly reduces the search space to be explored. They focus their study on a specialized version, where they permit lane reversals with partial capacities. Hamza-Lup et al. (2004) proposed a heuristic algorithm for the single source evacuation modeling to tackle the contraflow leading to finding the optimal paths to sinks. However, this approach does not fully consider the overall capacity of the network. A few other studies in the literature that are not analytical in nature were also proposed. Theodoulou and Wolshon (2004) rely on simulation-based methods and decision support tools.

Kim et al., (2008) are the first to give integer programming formulations (macroscopic) of the problem and proposed two heuristics; Greedy heuristic and Bottleneck Relief heuristic. The former determines the condition of congestion and flips highly congested arc in a greedy manner and the latter identifies the bottleneck and increases its capacity by contraflow. However, their solution lacks analyticity. Rebennack et al. (2010) discuss different network contraflow problems with analytical solutions and their complexities. Moreover, the maximum static contraflow ($MSCF$) problem for general graphs and the maximum dynamic contraflow problem ($MDCF$) for networks with a single source and a single sink have been considered and presented the first polynomial time algorithms, based on graph transformation, for both problems. In their papers, it is shown that the quickest contraflow problem can be solved in strongly polynomial time complexity for a single source and a single sink, but the quickest transshipment contraflow problem and fixed switching cost contraflow problems are NP-hard. Dhamala and Pyakurel (2013) formulated a mathematical model of the earliest arrival contraflow problem by flipping the direction of the arcs at time zero and gave strongly polynomial time algorithm to solve it on a two-terminal series-parallel graph. Every cycle in the residual network has nonnegative cycle length in such graph.

Pyakurel et. al (2014), considering the flow loss in the network, introduce the generalized maximum dynamic contraflow (GMDCF) problem and present a pseudo-polynomial time algorithm for this problem for single source and single sink lossy network. Their algorithm is based on the algorithm of Rebennack et al. (2010) and Arulselvan (2009) for computing the MDCF on general graphs and the algorithm of Gross and Skutella (2012) for computing a GMDF on lossy networks with both gain factors and transit times on the arcs. The presented algorithm also gives the generalized earliest arrival contraflow (GEACF) solution on single source and single sink lossy network. Pyakurel and Dhamala (2015) consider the lexicographic contraflow problems that optimize the feasible flow leaving or entering the terminals in the given order after reversing the direction of arcs. They present lexicographically maximum contraflow and lexicographically maximum dynamic contraflow with first polynomial time algorithms to solve them. For the latter problem, they have considered the constant travel time and node capacity but arc reversal capability is assumed at each integer time point, unlike in Dhamala and Pyakurel (2013).
All the works to till date about earliest arrival contraflow problem are of the single sink network as there exists no earliest arrival flow on the multiple sinks network, Gale (1959).

3. Basic Notations and Models

A *graph* is a pair $G = (V, E)$ where V is the set of nodes i (intersection points in the case of transportation problems) and E is the set of arcs $e = (i, j)$ (roads or streets) joining any two nodes i and j. Set E is a pair given by $E = V \times V$. If the orientation is fixed on these arcs then the above graph is a directed graph or simply a *digraph*. We consider the model of network with source nodes set S, sink nodes set D and the intermediate nodes set I. s and d represent the single-source and single-sink, respectively. We assign some capacities $c_e \geq 0$ to each of the arcs $e \in E$ and holding capacities of node $c_i \geq 0$ to each of the nodes $i \in V \backslash S \cup D$. The set $N = (G, S, D, c_e)$ is a *network* structure for flow. For some problems a non-negative costs are assigned for each arc $e \in E$. As we experience in daily life, flows depend both on the structure of the network and the various capacities of its arcs.

Given a network $N = (G, S, D, c_e)$ a function $\mathrm{h}: \mathrm{E} \rightarrow \mathrm{R}^+$ is a *static flow function* on arcs that obeys the capacity constraints $0 \leq \mathrm{h_e} \leq c_e$ for all $\mathrm{e} \in \mathrm{E}$. A static flow h is said to observe flow conservation in node i if h holds

$$\sum_{\mathrm{e} \in \delta^+(\mathrm{i})} \mathrm{h_e} = \sum_{\mathrm{e} \in \delta^-(\mathrm{i})} \mathrm{h_e} \tag{1}$$

where $\delta^+(\mathrm{i}) = \{(\mathrm{i}, \mathrm{j}) \in \mathrm{E}\}$ and $\delta^-(\mathrm{i}) = \{(\mathrm{j}, \mathrm{i}) \in \mathrm{E}\} \, \forall \, \mathrm{j} \in \mathrm{V}$. That is $\delta^+(\mathrm{i})$ and $\delta^-(\mathrm{i})$ respectively denote the set of arcs heading towards node i and the set of arcs leaving node i. A *static circulation* is a static flow h that also satisfies flow conservation constraints at origin and destination nodes. The *residual network* corresponding to a static flow h with respect to capacities c_e is the network redefined with residual capacities $c_e^h = c_e - h$ in each forward arc and $c_e^h = h$ for each backward arc.
An $s - d$ flow satisfying the flow conservation for intermediate nodes i.e. nodes in $V/\{s, d\}$ and capacity constrains for all arcs is said to be a feasible flow.
The flow value v of an $s - d$ flow is given by

$$v = \sum_{\mathrm{e} \in \delta^+(\mathrm{i})} \mathrm{h_e} - \sum_{\mathrm{e} \in \delta^-(\mathrm{i})} \mathrm{h_e}$$

$$= \begin{cases} -\mathrm{h_v}, & \text{for i = s,} \\ 0, & \text{for i} \in \text{I,} \\ \mathrm{h_v}, & \text{for i = d,} \end{cases} \quad \text{where } h_v \geq 0, \forall \, i \in V. \tag{2}$$

An $s - d$ flow with maximum v is said to be a maximum $s - d$ flow.
For transshipment problem with multiple sources and multiple sinks, flow conservation has to hold

only for intermediate nodes. For terminal nodes the flow function h has to satisfy the supplies and demands i.e.

$$\sum_{e\in\delta^-(u)} h_e - \sum_{e\in\delta^+(u)} h_e = d(\omega);\quad \forall\, u \in S \cup D \qquad (3)$$

Such transshipment problems can be reduced to an $s-d$ flow problem by introducing an arbitrary supersource and a supersink in the network.

A sequence of distinct nodes $v_1, v_2, \dots\dots, v_n$ of digraph $G=(V,E)$ is a directed chain or a path if $(v_i, v_{i+1}) \in E, \forall i = 1, \dots, n-1$. We define another flow function $h': P \to R^+$ in terms of the flow along the chains, P, from s to d. A feasible flow h of value v could be decomposed into a set of such chains satisfying

$$v = \sum_{i=1}^{|P|} h'_i\ . \qquad (4)$$

The dynamic flow for a given network$N = (G, S, D, c_e)$ depends not only on capacity but also the transit time, $\tau: E \to R^+$ on its each arc $e = (i,j) \in E$ and such flow units can be sent through it repeatedly reaching the sink within available time horizon T. Therefore, Dynamic flow for discrete time steps over dynamic network is a real valued function $f: E \times \{0,1,2,\dots,T\} \to R^+$. The amount of flow entering the particular arc per unit time, $f(e,\theta)$, can be interpreted as flow rate in the network. $f(e,\theta)$ be the amount of the flow leaving i at time θ along $e=(i,j)$ and reaching at j at time $\theta + \tau_{e=(i,j)}$. For a given time horizon T, this flow rate is bounded by the capacity of corresponding arc; i.e. $0 \le f(e,\theta) \le c_e, \forall\theta \in \{0,1,2,\dots,T-1\}$. Flow conservation in discrete model is satisfied in node $v \in V\backslash\{s,d\}$ if the following holds

$$\sum_{e\in\delta^+(v)} \sum_{\theta=0}^{T} f(e,\theta) \le \sum_{e\in\delta^-(v)} \sum_{\theta=0}^{T} f(e,\theta). \qquad (5)$$

The dynamic $s-d$ flow for discrete time steps can be defined as

$$Value(F) = \begin{cases} \sum_{e\in\delta^-(d)} \sum_{\theta=0}^{T} f(e,\theta) - \sum_{e\in\delta^+(d)} \sum_{\theta=0}^{T} f(e,\theta) \\ \sum_{e\in\delta^+(s)} \sum_{\theta=0}^{T} f(e,\theta) - \sum_{e\in-(s)} \sum_{\theta=0}^{T} f(e,\theta) \end{cases} \qquad (6)$$

If F is the total flow leaving source or reaching sink during time horizon T then the maximal dynamic flow can be formulated as a linear program.

Maximize $value$(F) subjected to the constraints;

$$\sum_{\theta=0}^{T} \sum_{i\in V} [f(s,i;\theta) - f(i,s;\theta - \tau(i,s)] - F = 0; \qquad (7)$$

$$\sum_{i,j\in V} [f(i,j;\theta) - f(j,i;\theta - \tau(j,i)] = 0, i \ne s, d; \qquad (8)$$

$$\sum_{\theta=0}^{T} \sum_{j\in V} [f(d,j;\theta) - f(j,d;\theta - \tau(j,d)] + F = 0 \qquad (9)$$

$$0 \le f(e,\theta) \le c_{e=(i,j)}; \forall\theta \in \{0,1,2,\dots,T-1\}. \qquad (10)$$

If $f(i,j;\theta)$ and F satisfy above constraints we say f is dynamic flow from s to d for time horizon T and say that flow has value F. Moreover, if F is maximal, then f is maximal dynamic flow.

Also, $f(i,i;\theta)$ is the hold-over at node i from time θ to $\theta+1$ and $\tau(i,i) = 1, c_{e=(i,i)} = \infty$ for hold-over at node $\forall i \in V/S \cup D$. However, we can also allow hold-over at the source and sink nodes where we have to add a loop starting in s and ending in s (similar to d) with capacity ∞. Here the role of hold-over is to reduce the congestion in the network instantly by storing the overflow in the arcs that can be sent at later times. Therefore, at each intermediate node of the network, the evacuees can immediately be evacuated or can be hold for later (before T) evacuation.

In the case of multiple-sources-multiple-sinks when fixed supplies at the sources and demands at the sinks are given, the problem of making the sources empty by satisfying the demands at the sinks is called a *transshipment problem*. The problem of minimizing the total time needed to send all the flows of all sources to sink(s) is known as *quickest transshipment problem.* The quickest transshipment problem asks to send a given amount of flow from source to sink in minimum possible time horizon T. The objective here is to determine minimum time horizon T for which there exists a feasible flow satisfying all provided supplies in the sources and demands in the sinks. This problem helps to find a reasonable lower bound of a real evacuation situation. Such problems are useful in the case of evacuation from building. Ford and Fulkerson (1958) suggested an algorithm to compute it in an augmenting path by computing a minimum cost flow problem and proved that there always exists an optimal solution. In the minimum cost flow problem the flow value is fixed and one seeks for minimal cost. Problem of maximizing the dynamic flow f reaching the sink not only for the given time horizon but also for every time step $\theta \in \{0,1,2,\ldots,T-1\}$ is an *earliest arrival transshipment* problem. This problem maximizes the amount of flow sent in to the sinks from the sources simultaneously for each time period $\theta \in \{0,1,2,\ldots,T-1\}$. These transshipment problems are suitable if the number of evacuees is known in advance. If the number of evacuees is unknown in the beginning, dynamic $s-d$ flows are appropriate for evacuation planning. Problem of sending the flows from source to sink as quickly as possible is a quickest s-t flow whereas the earliest arrival $s-d$ flow maximizes the flow from source to sink for every time step $\theta \in \{0,1,2,\ldots,T-1\}$. Earliest arrival flows problem does not necessarily exist in the general network. A latest departure flow is a flow for given time horizon T that maximizes the amount of the flow leaving the source in every interval $[\theta,T]$, $\theta \in \{0,1,2,\ldots,T-1\}$. A dynamic flow with both latest departure and earliest arrival property is called a universally maximum dynamic flow. Gale (1959) answered the question of existence of the earliest arrival flow.

If Supplies and demand at nodes $u \in S \cup D$ (union of source nodes and sink nodes) are given these must be fulfilled by the flow at every step. That is

$$\sum_{e\in\delta^{-}(u)}\sum_{\theta=0}^{T} f(e,\theta) - \sum_{e\in\delta^{+}(u)}\sum_{\theta=0}^{T} f(e,\theta) = d(\omega) \qquad (11)$$

must hold where $d(\omega)$ is the supply-demand function. Moreover, flow in to the sources and flow out of sinks are assumed to be null.

But, in the case of continuous time version dynamic flows, the flow function satisfies the flow conservation at node i if

$$\sum_{e\in\delta^{+}(i)}\int_0^T f(e,\theta)d\theta \le \sum_{e\in\delta^{-}(i)}\int_0^T f(e,\theta)d\theta\,; \quad \forall\theta \in [0,T). \qquad (12)$$

The value of a dynamic $s-d$ flow f with time horizon T is defined by the net flow value that leaves the source over all time steps or enters the sink over all time steps $\theta \in [0,T)$. That is

$$F = value(f) = \begin{cases} \sum_{e\in\delta^{+}(s)}\int_0^T f(e,\theta)d\theta - \sum_{e\in\delta^{-}(s)}\int_0^T f(e,\theta)d\theta \\ \sum_{e\in\delta^{-}(d)}\int_0^T f(e,\theta)d\theta - \sum_{e\in\delta^{+}(d)}\int_0^T f(e,\theta)d\theta \end{cases} \qquad (13)$$

An $s-d$ flow with maximum F is said to be a maximum dynamic $s-d$ flow.

For the case of multiple sources and multiple sinks, the problem becomes the maximum dynamic flow problem.

4. General Solution Strategies

4.1Time Expanded Network

Ford and Fulkerson (1958, 1962) developed time-expanded network $N^T$for each time steps$\theta \in \{0,1,2,\ldots,T-1\}$obtained by the expansion of the dynamic network N. Time expanded networks are the device to solve the dynamic network flow problems since a dynamic flow in N corresponds to a static flow in N^T. Solution methods for static network flow problems can be applied to solve dynamic network flow problems in time-expanded network. Copy each node i of N for T times so that the nodes are of the form$i(\theta)\ \forall\ i \in N$ and $\theta \in \{0,1,2,\ldots,T-1\}$. Next copy the original arcs e between the nodes $i$and j to form the arcs$\left(i(\theta), j\left(\theta+\tau_{i,j}\right)\right), 0 \le \theta \le T-\tau_{i,j}$with capacity $c_{i,j}$. The holdovers at nodes can be modeled with arcs $\left(i(\theta), i(\theta+1)\right), 0 \le \theta \le T$ and capacity ∞ for each node. The major problem associated with time-expanded graphs is its size even for a considerably a few nodes and arcs since it depends on the time horizon T and thus leads to a pseudo-polynomial algorithm.

4.2 Minimum Cost Flow Problem

Another way of solving $s-d$ dynamic flow problem is to work on the underlying static network by interpreting the transit time τ_e as costs$a_e$for each arc and to solve a minimum cost flow problem on it. Ford and Fulkerson (1958, 1962) developed the primal dual algorithm, known to be clever algorithm, to solve the minimum cost flow problem with respect to the time horizon T. Flow obtaining by this algorithm has to be decomposed into chain flows and repeated these over time as many times as possible to get the maximal dynamic flow.

Given a network $N=(v,E)$ with arc costs a_e for each arc $e=(i,j)\in E$, finding the minimum cost of sending a flow of given value v from s to d in N is the minimum cost flow problem.

That is,

$$\text{minimize} \sum_{(i,j)} a_{ij}\, x_{ij} \tag{14}$$

satisfying the following

$$\sum_{j\in V} x_{sj} - v = 0 \tag{15}$$

$$\sum_{j\in V} x_{jd} - v = 0 \tag{16}$$

$$\sum_{j\in V} x_{ji} - x_{ij} = 0 \;\forall i \in V \tag{17}$$

$$0 \le x_{ij} \le c_{ij} \forall i,j \in V \tag{18}$$

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5. Contraflows

While dealing the contraflow network problems one seeks maximum flow in graph while allowing direction reversals of arc to increase the capacity of the arc in the direction of flip. In fact, contraflow is the use of one or more arcs of inbound travel for traffic movement in the outbound direction which increases the operational evacuation capacity, B. Wolshon (2001). In the case of emergency evacuation planning of a part of a city, for example, an evacuation network consists of roads or streets as arcs (edges) and that of intersections of the roads as nodes; unsafe place(s) where accident occurred or going to be occurred soon can be taken as source(s) and the safe place(s) where the people are to be evacuated are assumed as sink(s). Sources and sinks are terminal nodes. Source nodes contain evacuees and sink nodes wait them for shelter. There are capacity constraints (maximum number of evacuees at a unit time) on the nodes and arcs. Moreover, arc travel time for an evacuee is assigned to each arc. In the case of evacuation the flow towards the sources is undesired except for the special surveillances like police vehicles or fire-bridges.

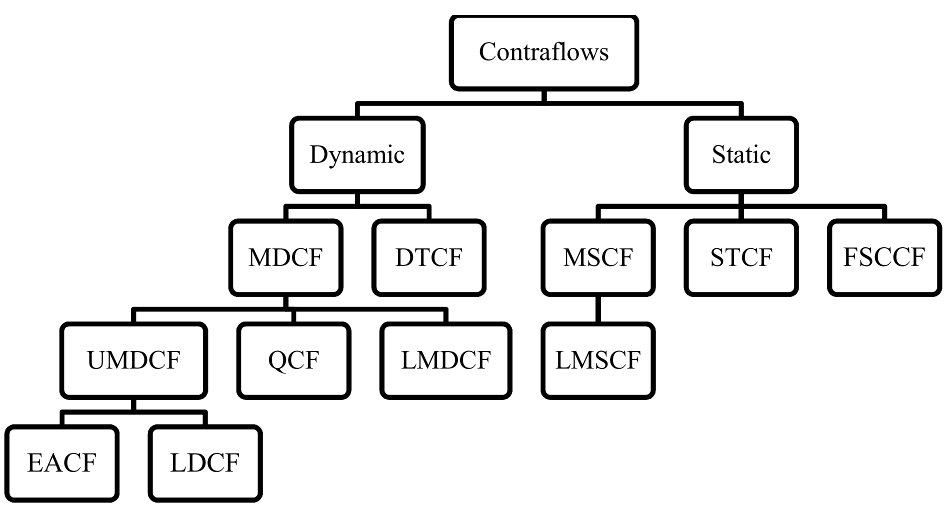

Fig 1 Various Contraflows

Due to that we can reverse direction of some or all arcs towards desired direction to reduce the congestion on the road (arcs) and to increase the total flow rate (number of evacuees) towards the safer zone (sinks). In this paper we study the various contraflow network problems (fig. 1) basically related to evacuation planning.

5.1 Maximum Static Contraflow Problems

$MSCF$is a problem offinding the maximum flow from source node s to sink node d for a given network $N = (G = (V, E), s, d, c_e)$ when the direction of the arcs can be reversed. Rebennack et al. (2010) have studied this problem with their complexity analysis. Algorithm (**P- MSCF)** given by Rebennack et al. (2010) for solving this problem is given below.

Algorithm: P- MSCF

1. *Given network* $N = (G = (V, E),\ s, d, c_e)$ *with integer inputs.*
2. *Solve (by any known algorithm), corresponding MSF problem on* $\tilde{N} = (G = (V, \tilde{E}),\ c_{\tilde{e}})$ *where arc set is defined as* $\tilde{e} = (i, j) \in \tilde{E}$ *if* $\vec{e} = (i, j) \in E$ *or* $\bar{e}$ *= (j, i)∈ E.* *The arc capacity function* $c_{\tilde{e}}$ *is defined as* $c_{\tilde{e}} \coloneqq c_{\vec{e}} + c_{\bar{e}},\ \forall e \in \tilde{E}$.
3. *Perform the flow decomposition into path and cycle flows of the maximum flow obtained from step-2 and remove all cycle flows.*
4. *Arc* $(j, i) \in E$ *is reversed iff the flow along arc* (i, j) *is greater than* $c_{(i,j)}$ *or if there is non-negative flow along arc* (i, j) *not belonging to* E*. Flow now is MSCF with arc reversals for* N.
5. *Get maximum static contraflow solution.*

Theorem: Procedure $P - MSCF$ solves the maximum static contraflow problem for network $N = (G = (V, E),\ s, d, c_e)$ optimally, Rebnnack et al. (2010).

Complexity: It requires $O(h_1(n, m) + h_2(n, m))$ time, where $h_1(n, m) = O(n^2.\sqrt{m})$ and $h_2(n, m) = O(n.m)$ are the times required to solve the MSF problem and the flow decomposition respectively.

5.2Maximum Dynamic Contraflow Problems

MDCF is the problem of finding the maximum dynamic flow from source node s to sink node d for a given digraph $N = (G = (V, E), c_e, \tau_e, T)$ with $\tau_{(i,j)} = \tau_{(j,i)}$ if $(i, j), (j, i) \in E$ to each of the arcs e $= (i, j) \in E$ in given overall time horizon T when the direction of the arcs can be reversed at time 0. Algorithm to solve this problem suggested by Rebennack et al. (2010) is given below.

Algorithm: P- MDCF

1. *Given network* $N = (G = (V, E),\ c_e, \tau_e, T)$*with integer inputs.*
2. *Solve the corresponding MSF problem on* $\tilde{N} = (G = (V, \tilde{E}),\ c_{\tilde{e}}, \tau_{\tilde{e}}, T)$ *where arc set is defined as* $\tilde{e} = (i, j) \in \tilde{E}$, *if*$\vec{e} = (i, j) \in E$ *or* $\bar{e}$ *= (j, i)∈ E.* *The arc capacity function* $c_{\tilde{e}}$, *is defined as* $c_{\tilde{e}} \coloneqq c_{\vec{e}} + c_{\bar{e}},\ \forall e \in \tilde{E}$. *And the travelling time is* $\tau_{\tilde{e}} := \begin{cases} \tau_e \text{ , } if\ e \in E, \\ \tau_{\bar{e}} \text{ , } otherwise, \end{cases} \quad \forall e \in \tilde{E}$
3. *Compute the dynamic, temporally repeated flow on network* N *with capacity* $c_{\tilde{e}}$ *and travelling time* $\tau_{\tilde{e}}$.
4. *Perform the flow decomposition into path and cycle flows of the maximum flow obtained from step-2 and remove all cycle flows.*
5. *Arc* $(j, i) \in E$ *is reversed iff the flow along arc* (i, j) *is greater than* $c_{e=(i,j)}$ *or if there is non-negative flow along arc* (i, j) *not belonging to* E*. Flow now is MDCF with arc reversals for*N.
6. *Get maximum dynamic contraflow solution.*

Theorem: Procedure $P - MDCF$ solves the maximum static contraflow problem for network $N = (G = (V, E),\ c_e, \tau_e, T)$ optimally, Rebnnack et al. (2010).

Complexity: It requires $O(h_2(n, m) + h_3(n, m))$ time, where $h_2(n, m) = O(n.m)$ and $h_3(n, m) = O(n^2.m^3 \log n)$are the times required to solve the flow decomposition and ***MSF*** problem respectively. By reducing from **3-SAT** problem Kim et. al. (2008) proved that $MDCF$ problem is $NP - hard$ in the strong sense even with two sources and one sink or vice versa. The same argument is justified by Rebennack et al. (2010) by using $\boldsymbol{PARTITION}$ problem.

5.3 Generalized Maximum Dynamic Contraflow ($\boldsymbol{GMDCF}$) Problems

Given a network $N = (G = (V, E), s, d, \gamma_e c_e, \tau_e, T)$ with gain factor $\gamma_{e=(i,j)} = \gamma_{e=(j,i))} \in R^+$ the GMDCF problem asks to find the maximum amount of flow that can be sent from s to d within given time horizon T if the direction of the arcs can be reversed at time 0. This problem was first studied by Pyakurel et.al, (2014). The solution procedure they have given is presented here.

Algorithm: GMDCF on Lossy Network

1. *Construct the transformed network* $\tilde{N} = (G = (V, \tilde{E}), s, d, \gamma_{\tilde{e}}, c_{\tilde{e}}, \tau_{\tilde{e}}, T)$ *with arc set defined as* $e = (i,j) \in \tilde{E}$ *if* $(i,j) \in E$ *or* $(j,i) \in E$ *with highest gain factor* γ_e.
2. *The arc capacity function* $c_{\tilde{e}}$ *is defined as* $c_{\tilde{e}} := c_{\vec{e}} + c_{\overleftarrow{e}}$, $\forall e \in \tilde{E}$.
   The travelling time is
   $$\tau_{\tilde{e}} := \begin{cases} \tau_e, & if e \in E, \\ \tau_{\overleftarrow{e}}, & otherwise, \end{cases} \forall e \in \tilde{E}.$$
   And gain factor $\gamma_{\tilde{e}} = 2^{c\tau_{\tilde{e}}}$, $c < 0$.
3. *Generate generalized dynamic temporally repeated flow with unique gain factor in transformed network using algorithm of Gross, M and Skutella, M. (2012).*
4. *Decompose flow into path and cycle flows of resulting network from Step-2. Remove the cycle flows.*
5. *Arc* $(j,i) \in E$ *is reversed iff the flow along arc* (i,j) *is greater than* $c_{e=(i,j)}$ *or if there is non-negative flow along arc* (i,j) *not belonging to E with highest gain and the resulting flow is GMDF with the arc reversal for the original network.*

Complexity: Total complexity of above algorithm is
$$O(|V||\tilde{E}| + O\left(|V||\tilde{E}| log \frac{\tilde{E}}{|V| log |V|} |V|.T\right)).$$

5.4 Quickest Contraflow Problems

Given network $\mathrm{N} = (\mathrm{G} = (\mathrm{V}, \mathrm{E}), \mathrm{S} \cup \mathrm{D}, \mathrm{c_e \tau_e}, \mathrm{T})$, the problem of finding the minimum time horizon T required for a feasible flow from S to D when the direction of the arcs can be reversed at time 0 is a quickest contraflow problem.

The quickest contraflow problem can be solved in a strongly polynomial time by the algorithm obtained from parametric search as suggested by Megiddo (1979), and Burkard et al. (1993). Another way of finding solution to the problem is to first compute an upper bound on the quickest time and perform a binary search by repeatedly solving the minimum dynamic contraflow problem. The former task can be done in polynomial time, the latter; however, leads weakly polynomial time algorithm since it needs to compute a path from source to sink and temporally repeating flow along the path until all commodity at the source is sent to sink.

5.5 Earliest Arrival Contraflow Problems

Given network $N = (G = (V, E), S, d, c_e, \tau_e, T)$, earliest arrival contraflow is a maximum dynamic contraflow that finds a feasible flow from S to d that is maximum for all time periods $0 \leq \tau \leq T$, if the direction of the arcs can be reversed only once at time 0. A strongly polynomial time algorithm developed by Dhamala and Pyakurel (2013) for series parallel graph is presented here.

Algorithm: $\boldsymbol{s - d}$ $\boldsymbol{EACF}$

1. *Given evacuation network* $N = (G = (V, E), s, d, c_e, \tau_e, T)$ *with integer inputs.*
2. *Solve [by Wilkinson(1971) and Minieka(1973)] the corresponding EAF problem on auxiliary network* $\tilde{N} = (G = (V, \tilde{E}), \mathrm{s}, \mathrm{d}, c_{\tilde{e}}, \tau_{\tilde{e}}, T)$ *,where arc set is defined as* $\tilde{e} = (i,j) \in \tilde{E}$, *if* $\vec{e} = (i,j) \in E$ *or* $\overleftarrow{e}$ *= (j, i)*$\in E$.

   The arc capacity function $c_{\tilde{e}}$ *is defined as* $c_{\tilde{e}} := c_{\vec{e}} + c_{\overleftarrow{e}}$, $\forall e \in \tilde{E}$.

   And the travelling time is
   $$\tau_{\tilde{e}} := \begin{cases} \tau_e, & if e \in E, \\ \tau_{\overleftarrow{e}}, & otherwise, \end{cases} \forall e \in \tilde{E}$$

3. *Arc* $(j,i) \in E$ *is reversed iff the flow along arc* (i,j) *is greater than* $c_{e=(i,j)}$ *or if there is non-negative flow along arc* (i,j) *not belonging to* E. *Flow now is in EACF with arc reversals for* N.
4. *Get an earliest arrival contraflow solution.*

Complexity**:** Optimal solution to the $s-d\ EACF$ problem for series parallel graph with arc reversal capability at time 0 can be obtained in $O(nm + m\log m)$, a strongly polynomial time.

5.6 Lexicographically Maximum Contraflow Problems

The problem of finding a feasible flow that lexicographically maximizes amounts entering (leaving) the terminals in the given orders of multi terminals network $N = (G = (V,E), S \cup D, c_e, c_n)$ if the direction of the arcs can be reversed. Pyakurel and Dhamala (2015) studied lexicographically maximum contraflow problem and gave the algorithm to solve it which follows as below.

Algorithm: LMSCF

1. *Given network* $N = (G = (V,E), S \cup D, c_e, c_n)$ *with integer inputs.*
2. *Solve the corresponding LMSF problem on* $\tilde{N} = \left(G = \left(V, \tilde{E}\right), S \cup D, c_{\tilde{e}}, c_n\right)$ *by Minieka, E. (1973)*
3. *Arc* $(j,i) \in E$ *is reversed iff the flow along arc* (i,j) *is greater than* $c_{e=(i,j)}$ *or if there is non-negative flow along arc* (i,j) *not belonging to* E. *Flow now is in LMSF with arc reversals for*N.
4. *Get lexicographically maximum static contraflow solution.*

5.7 Lexicographically Dynamic Maximum Contraflow Problems

For the graph network $N = (G = (V,E), S \cup D, c_e, c_n, \tau_e, T)$ symmetric travel time on each arc, overall time period T, and μ_s, ϑ_d the supply and demand vectors to the ordered set of multi-sources and multi-sinks, the $LMDCF$ is a problem to find a feasible flow that lexicographically maximizes the amount leaving out or entering in the terminals in the priority order, if the direction of the arcs can be reversed at time zero. The algorithm to solve it which is presented below was also given by Pyakurel and Dhamala (2015).

Algorithm: LMDCF

1. *Given network* $N = (G = (V,E), S \cup D, c_e, c_n, \tau_e, \mu_s, \vartheta_d, T)$*with integer inputs.*
2. *Solve the corresponding LMDF problem on* $\tilde{N} = (G = \left(V, \tilde{E}\right), S \cup D, c_{\tilde{e}}, c_n, \tau_{\tilde{e}}, \mu_s, \vartheta_d, T)$, *by Hoppe, B. and Tardos, E. (2000)*
3. *Arc* $(j,i) \in E$ *is reversed iff the flow along arc* (i,j) *is greater than* c_e *or if there is non-negative flow along arc* (i,j) *not belonging to* E. *Flow now is in LMDF with arc reversals for* N.
4. *Get lexicographically maximum dynamic contraflow solution.*

5.8 Dynamic Transhipment Contraflow (DTCF) Problems

For given network $\mathrm{N} = (\mathrm{G} = (\mathrm{V},\mathrm{E}),\ \mathrm{S} \cup \mathrm{D}, \mathrm{c_e}\tau_\mathrm{e}, \mathrm{T}), DTCF$ problem is a decision problem to the maximum dynamic contraflow problem that has to check whether there is a feasible dynamic flow within the given time horizon T, allowing each arc to be reversed once at time 0.

$DTCF$ is NP-complete in the strong sense, Rebennack et al. (2010). They showed that adding only one more terminal node to the $MDCF$ makes the problemNP-complete. In fact, unlike in the case of $MCF$and $MDCF$, in the case of multiple-sources and multiple-sinks, the use of both arcs in the network leads the problem to remember whether an arc is reversed. This memory and the tradeoff of reversing the arc immediate or later time makes the problem NP-complete. A sketch of the proof outline can be found in the work of Kim and Shekhar (2005) also.

5.9 Fixed Switching Cost Contraflow (FSCCF) Problems

Finding a feasible (static) flow f with minimal total cost in given digraph $G = (V,E)$ with a set of sources S, a set of sinks D excess $b \in Z^{|V|}$ capacities $c_e \geq 0$ and arc switching cost $b_e^f$to each of the arcs $\mathrm{e} = (i,j) \in E$ when the direction of the arcs can be reversed is $FSCCF$ problem.

Theorem: Fixed switching cost contraflow is equivalent to $0/_1 -$ minimum improvement flow, Rebennack et al. (2010).

6. Conclusion with Further Directions

The importance and applicability of the idea of contraflow is increasing day by day. The static contraflow problem and single-source-single-sink maximum and quickest contraflow problem with constant transit times are polynomially solvable. However, the general contraflow problem is computationally hard. The contraflow problems which we discussed here consider constant transit times on the arcs. However, some real life situations cannot be dealt with these approaches considering constant transit times on the arcs. Flow may vary on the arc over time and transit time for a unit of flow depends on the amount of flow currently present on the arc. Dynamic flow problems with flow-dependent transit times have been well studied for the past few years. Nevertheless, contraflow problems have not been studying with respect to flow dependent transit times and time dependent transit time yet. Another part of further research can be partial contraflow and total path flip instead of arc only. Continuous time contraflow models are not available in the literature till date. For low mobility population transit based contraflow evacuation planning which still lacks sufficient study must be helpful. Moreover, for the large cities of underdeveloped country, a bi-level (pedestrian based and transit based) integrated contraflow model is suitable because almost no people of such areas have their own car, and they have to walk on foot along narrow local streets.

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