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Linear Transformation and its Properties with Application in Time Series Filtering

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Abstract

The primary objective of this article is to justify the concept of a linear transformation that derives its algebraic properties by means of variation matrices represented so far. The mathematical formulas will be used to elaborate and justify the argument of linear transformation. By displaying the significance of the t-transformation for estimation of latent variables in the time series decomposition, the paper obtains a general expression for smoothing matrices characterized by symmetric and asymmetric weighting system.The article further elaborates the concept of sub-matrix of the symmetric weights that is t-invariant, whereas the sub matrices of the asymmetric weights are the t-transformation of each other. By virtue of this relation, the properties of the t-transformation project useful imperative informationon the smoothing of time series data, which may be required to clarify the concept of the topic raised so far. Eventually, the paper illustrates the role of the t-transformation on the weighting systems of several smoothers often applied for trend cycle estimation such as the locally weighted regression smoother, the cube smoothing spline, the Gaussian Kernel and 13-term trend cycle Henderson filter. By so doing, the article will pose the interrelated facts of linear transformation, from which the prospective researchers can benefit.

Introduction

In the time series analysis, it is often used that the data generating process can be decomposed into various components representing the trend cyclic fluctuation seasonal effects and irregulars these related variables are estimated by Applying liner filter of weights to the observation in the movingmanner. The filter can be arranged in matrix form such that applied to the vector observation produce the corresponding estimated values. If the linerfilters are symmetric say of length 2M+1 with m>0 and applied to a series of length N>2M+1. Then it is evident that the component cannot be estimatedfor the first and last aim observation. It has great importance to estimate thelatent variables up to including

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the most recent observation. Asymmetric filters must be applied to the being and ending M values of the series. In particular, we show that predictor metric is centrosymmetric and it is founda submetrices of symmetric weight that is t-invariant where the t-transformto each other is.

Linear Transformations

The notion of a linear transformation is much older than matrix notation. Indeed, matrix notation was developed for the needs of calculation with linear transformations over finite-dimensional vector spaces. The details of this are in these notes.

The dear reader would be advised to take to heart, when getting to the calculational parts of these notes, the slogan for matrix multiplication \ column-by-column-on-the-right". That is, if A is a matrix with k columns, listed as C1,C2, . . . , Ck (in order) and A0 0 is any matrix over the same field with the column

B@ a1...ak1 CA, then the column of AA0 is a1C1 + ¢ ¢ ¢ + akCk. She would also be well advised to \forget" until further notice that matrix multiplication is associative. An easy consequence of the \column-by-column" rule is the distributive law A (A0 + A00) = AA0 + AA00, whenever it makes sense; just look at the columns of both sides.

A linear transformation is a homomorphism between vector spaces (possibly the same one). The algebraic structure of a vector space over the field F consists of vector addition and multiplication of vectors by the scalars in F.

Suppose that V and W are vector spaces over the same field F. A functionT : V ¡! W is an F-linear transformation if 1. T(~v1 +V ~v2) = T~v1 +W T~v2 for all ~v1; ~v2 2 V (T preserves or respects vector addition), and 2. T(® ¢F;V

~v) = ® ¢F;W T~v for any ~v 2 V and ® 2 F (T preserves or respects scalar multiplication).

(As usual, we will leave o® the subscripts whenever we can, which is just about always. I put them on in the definition just to emphasize where the operations were taking place. We will also leave o® the F if it is understood.Please do not write \T is closed under addition", or \T is closed under scalarmultiplication"| these phrases make no sense.)

In case V = W, a linear transformation from V to itself is usually called a linearoperator on V . In case W = F, a linear transformation from V to F is called a linear functional on V . In case T is not only a linear transformation, but is also a one-to-one and onto function from V to W, it is an isomorphism of vector spaces.

The most basic kind of example of a linear transformation is this: Suppose that V = Fn and W = Fm for some field F and positive integers m and n. Let Abe any m £ n matrix with entries from F and TA : Fn ¡! Fm be given.

Filtering Time Series

A filter is a device that separates entities into their constituting components.The term initially described physical devices such as membranes that remove pollution from liquids or gases. In the twentieth century, electroniccircuits were developed for enhancing communication signals by separating transmission noise from the signal. Such electronic devices and the mathematical models describing their characteristics were also termed filters. As the abstraction level of mathematical filtering models increased, they were adopted in a wide array of scientific disciplines. This thesis describes a selection of filtering procedures and applies them to economic time series. In common with physical filters, they will be used to separate elements from mixtures of components, although these components are entities that are more abstract. An important example is the separation of movements in economic activity into a slowly changing general direction (trend), cyclical variation of medium duration (business cycle), predictably repeating fluctuations throughout each year (season), and unpredictable movements without permanent impact (noise).

The filtering algorithms that form the subject of this thesis are generally applicable to time series models of a specific form. These models aim to represent a sequence of chronologically arranged numerical observations ofsome phenomenon as realizations of stochastic processes in discrete time, which are indirectly observed, and often polluted by random disturbances. This class of models is referred to as a state space model, and enjoyed a surgein popularity in engineering and related disciplines following the publicationof the Kalman filter algorithm (Kalman, 1960). Mathematical filters are used to reconstruct or estimate the indirectly observed stochastic processes fromthe observations by sequentially processing new observations when they become available.

Additional algorithms are available to project the processes to periods in whichobservations are not (yet) available, yielding forecasts and interpolations. Due to the statistical nature of the filters, these projections are accompanied by measures of uncertainty, which are generally not available from non- probabilistic interpolation and extrapolation algorithms.

Early applications of Kalman and related filtering techniques were found primarily in the problems of estimating and tracking the location of physical objects such as rockets and projectiles. Improving the clarity of imperfectly transmitted communication signals was another important utilization. At an abstract level, filtering in state space models addresses the general problem of estimating indirectly observed dynamic systems. As such, it is not surprising that the state space approach has gained acceptance in a widerange of disciplines.

A filter is a device for removing solids or suspended particles from liquids. In the late 17th century, the term began to be used by the natural philosophersin a manner that gave expression to their understanding of the nature of light. It was recognized that white light is a compound of colored lights of differing wavelengths. A colored glass was seen as a device that selectively transmits some of the light, corresponding to a range of wavelengths, whileblocking the remainder. Therefore, it was described as an optical filter.

A direct analogy with light led engineers, in the early 20th century, to talk of electronic filters. Electronic filters are constructed from capacitors, resistors, and inductors. A circuit in which a voltage signal passes through an inductor, or in which a capacitor provides a path to earth, imposes less attenuation on low-frequency signals than on high-frequency signals. Therefore, it constitutes a low-pass filter. If the signal passes through a capacitor, or has apath to earth through an inductor, then the circuit imposes less attenuationon high-frequency signals than on low-frequency signals, and it constitutes high pass filter.

A Kalman filter, or a Hidden Markov Model, starts with some notion of the dynamics of a system and then seeks to match it to observations. As powerfulas these ideas are, what if you are given a signal without a priori insight into the system that produced it? What if your goal is to learn more about the nature of the system, not just what its state is? This is the domain of time series analysis. The field is as broad as time itself; it is defined not by any particular tools, but rather by the intent of their use. Time series problems arise in almost all disciplines, ranging from studying variations in currency exchange rates to variations in heart rates. Wherever they occur, there are three recurring tasks:

Characterize: What kind of system produced the signal? How many degrees of freedom does it have? How random is it? Is it linear? Howdoes noise influence the system?

Forecast: Based on an estimate of the current state, what will the systemdo next?

Model: What are the governing equations for the system? What is their long-term behavior?

These are closely related but not identical. For example, a model with good long-term properties may not be the best way to make short-term forecasts and vice versa. In addition, although it's possible to characterize asystem without explicitly writing down a model, some of the most powerfulcharacterization techniques are based on first building a model.

This chapter will assume that the analyst is an observer, not a participant. Beyond modeling comes manipulation. If it is possible to influence a systemthen these kinds of descriptions can be used to choose informative inputs, and to drive the system to a desired state (by reversing the model to predictinputs based on outputs, the domain of control theory (Doyle et al., 1992; Auerbach et al., 1992).

Time series originally were analyzed, not surprisingly, in the time domain. Characterization consisted of looking at the series, and the only kind of forecasting or modeling was simple extrapolation. A major step was Yule's 1927 analysis of the sunspot cycle [Yule, 1927]. This was perhaps the first time that a model with internal degrees of freedom (what we would now call a linear autoregressive model) was inferred from measurements of an external observable (the sunspot series). This rapidly bloomed into the theory of linear time series, which is mature, successful, ubiquitous, and applicable only to linear systems. It arises in two very different limits: deterministic systems that are so simple they can be described by linear governing equations, or systems, which are so stochastic that their deviationfrom ideal randomness is governed by linear random variable equations.

In between these two extremes lies the rest of the world, for which nonlinearity does matter. The theories of nonlinear differential equationsor stochastic processes in general have no general results, but rather there are many particular tractable cases. However, there is a powerful theory emerging for the characterization and modeling of nonlinear systems withoutmaking any linear assumptions. This chapter starts with the linear canon andcloses with these newer ideas.

The t-transformation and centrosymmetric matrices

Let R^{m+n} denote the set of m×n real matrices and let a belongs to $A\in R^{m+n}$ with the matrices of generic element a_{ij},i=1...m,j=1...n with m,n< ∞. The t-transformation defined as follows.

$t:R^{m\times n}\rightarrow R^{m\times n}, A\rightarrow t(A)$such that

$a_{ij}\rightarrow a_{m+1-i,n+1-j}$ ....(1)for i=1...m,j=1. n

The action of t on A can be described means of two permutation metrices ofequal form but different dimension. $t(A)=E_m A E_n$...(2)

$E_k \in R^{k\times k}$ where the permutation metrices are with once on the crossdiagonal and 0 elsewhere.

Definition

m×n

$A \in R$ and a $a_{ij=\ m+1-i,n+1-j}$

And For all i=1...m,j=1..n then A is rectangular centrosymmetric.

Properties of t-transformation

The t-transformation inherits desirable properties from the properties of the permutation metrices, which is

a) Symmetric

b) Orthogonal

c) Reflection

I. t is linearProof:

$t(A+B)=E_m(A+B)E_n=E_mAE_n+E_mBE_n=t(A)+t(B)$

$t(\lambda A)=E_m(\lambda A)E_n=\lambda E_mAE_n=\lambda t(A)$Therefore t is linear proved.

II. $A \in R^{m\times n}, t(t(A))=A$ Proof :

$t(t(A))=E_m(E_mAE_n)E_n=(E_mE_m)a(E_nE_n)=A$

III. $A \in R^{m\times n}, t^{-1}(A)=t(A)$Proof:

It follows by 2 given

$t^{-1}:R^{m\times n}\rightarrow R^{m\times n}, t(A)\rightarrow A$

proved

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