

GSM Mobile Positioning based on Path Loss using Hyperbola and Interpolation

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Abstract—The Location of a mobile user is an utmost importance because of its seamless mobility feature. The technology for finding the such user in mobile networks is now one of the most promising and research-needed fields. Classification of potential location based system (LBS) applications can be done as for public safety, consumer needs and enterprise business. There are various methods that have been studied but many of them have good accuracy in outdoor RF environment only. Such environment consists of good Line-of-sight (LOS) and thus multipath propagation and other penetration loss have a very less effect. Some of the method studied here are Receive signal strength (RSS), Time of arrival (TOA), Difference in time of arrival (DTOA), Global Positioning system (GPS) etc. These methods have their own pros and cons. The proposed method is developed to improve the accuracy of location of MS using hyperbolic technique based on path loss of three Base station transmitter (BTS) and polynomial interpolation of appropriate degree for error forecast. Such path loss is used to estimate the reference distance between BTS and MS by using one of the empirical model. During the calculation of such distance, the ground altitude is also taken into consideration to determine the value of tower height. There are three different error sources which effects the estimated reference distance. First is the error in RSS measured by MS, second is error in the network parameter and third is error in empirical model. These errors are interpolate so that combine error can be forecasted using appropriate degree of polynomial interpolation which then get adjusted with estimated distance thus produce the more accurate enhanced reference distance. Thus location of the Mobile device can be calculated more accurately.

I. INTRODUCTION

According to the strategy analytics, around 80% of time people kept them indoor. Either for their work or for domestic purpose, they enclosed themselves inside the building. Hence 70% of cellular and 80% of data originates from indoor [1]. It means the location technique should be able to perform for indoor as compare to outdoor.

The most accurate technique for positioning is GPS and hence nowadays all most all smart phone are equipped with GPS. It works by receiving signals from satellite. Whereas satellite is power limited system, it transmits a low power to MS and hence GPS signal become weaker inside the building as due to penetration loss and multipath loss. Accurate indoor location is critical in order to assist specially for public safety such as locating emergency call tracking, location base service, requires seamless and ubiquitous positioning technology in order to enable continuity of service inside building [1].

As GPS is power limited system thus it has low accuracy for indoor purpose, there is another way to calculate positioning of MS which is network based system. In network based system,

signal transmitted between mobile phone and the network is used to determine the mobile position [2]. Cellular Network is design to work for indoor as well. Thus it uses a high power transmitting system which includes high power transmitter and high gain antenna. In urban area, the type of building and its location greatly varies.

The structure of those different types of building is different to each other and hence penetration loss is also different. Multipath signal are also greatly affected due to high number of building between transmitter and receiver. As there is very less chance of Line of sight (LOS) signal indoor environment hence these two factors should be well addressed in order to more accurately locate mobile.

This research has the objective to find the location of Mobile users using differential path loss between three receive signal strengths by plotting hyperbola and polynomial interpolation of appropriate degree in a wireless communication network.

II. LITERATURE STUDY

A. Angle of Arrival (AOA)

This method utilizes the Triangulation principle as it uses two known reference points and one unknown point which location is need to find. In BTS there are antenna array deployed. These antennas give the angle at which they are receiving signal from MS. There should be at least two angles being determined from two different BTS. The Crossing between these two angles is the location of MS. For finding precise AOA needs a clear LOS signal since AOA is calculated using phase difference between signals from different array antenna otherwise the accuracy is decreased [2]. It has accuracy of 50 meter to 150 meter [3].

B. Time of arrival (TOA)

This method also utilizes the Multilateration principle as it uses three known reference points. Time taken to reach the signal from BTS to MS is calculated known as time travel duration (TTD). Such duration is then converted into distance using known velocity of radio wave propagation (i.e. approx 300m per Microsecond). A circle is made with a radius equal to the distance measured from above model centered at that particular BTS. Thus three circles are made at three BTS which common point is location of MS. It has accuracy of 100 meter to 400 meter [3].

C. Time difference of arrival (TDOA)

This method also utilizes the hyperbolic principle. The TTD difference between two BTS to MS is plotted using hyperbola as shown in fig 1.. Similarly another TTD difference between two BTS to MS is plotted among which one of the BTS is same with previous plotting. The common point between these two hyperbolic curves is location of MS. It has accuracy of 50 meter to 150 meter [3]. It's accuracy further optimized by Maximum Likelihood Estimation using fuzzy logic upto 15 meter [11].

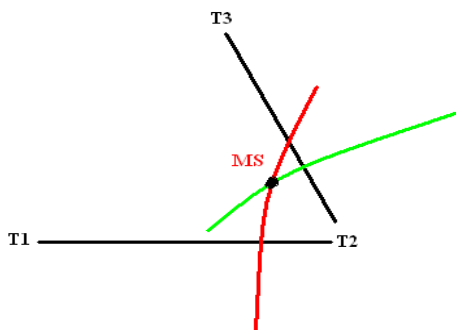


Fig. 1. TDOA Technique

D. Finger printing/database correlation

Location estimation using database of receive signal strength has been widely used nowadays. It contains the large database of RSS of coverage area already measured called as fingerprints. The required location of MS is analysis by comparing the RSS of MS from different BTS with fingerprints of RSS. The Main drawback is the scalability and availability of such database. It has showed that it has 44m accuracy in GSM network [2].

E. Receive signal strength (RSS) using circular positioning.

In this method, signal comes in downlink direction from (BTS) to user MS is analyzed. A circle is made with a radius equal to the distance measured from knowledge of the propagation characteristics model centered at that particular BTS [4]. Thus three circle is made at three BTS which common point is location of user MS as shown as fig 2. It has accuracy mean error 393 meter for 3 BTS and 246 meter for 6 BTS [12].

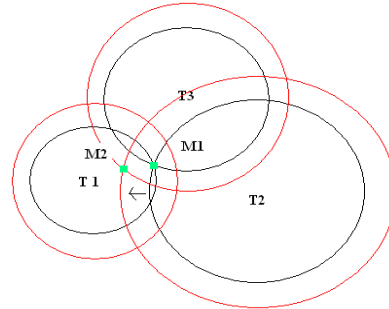


Fig. 2. RSS Technique using circulation

F. Receive signal strength (RSS) using Circular positioning and Pearson's Correlation Coefficient.

Here when the location of MS is calculated, normally there are maximum 6 BTS RSS values is obtained. Instead of using only 3 BTS, the data from the remaining 3 BTS also can be taken into considering. With the RSS value from remaining BTS, the relation between Loss and distance is made more robustness thus provide more accurate estimated distance and hence provide more accurate location. This scheme provides mean error below than 150 meter and maximum error less than 550 meter [10].

G. Receive signal strength (RSS) using hyperbolic positioning.

This method utilizes the Multilateration principle as it uses three known reference points. In this method, the signal comes in downlink direction from (BTS) to user MS is analyzed. This method also utilizes the hyperbolic principle. The distance between MS and BTS is estimated using Okumara-hata model. Such distances are from two pairs of BTS. After estimating the distance from two pair of BTS to MS, such distance is use to plot two hyperbolas. The common point between these two hyperbolic curves is location of MS. The distance estimation between MS and BTS here also plays an important role to locate MS more accurately. Using Okumara-hata model for distance estimation, the locations mean error is found to be 345 meter with 3BTS and 282 meter with 6 BTS [12].

H. Stationary signal strength difference using hyperbolic positioning.

SSSD is the new technique that uses the difference in receives signal from two BTS and estimated distance from one of the BTS to estimate the distance from MS to another remaining second BTS. The Estimated distance from first BTS is done by using Okumara-hata model. Since the second BTS distance is calculated with the difference in receive signal, the computational burden to calculate second BTS distance become less. After estimating the distance from two pair of BTS to MS,

such distance is use to plot two hyperbolas. The common point between them is location of MS. The accuracy thus obtain is mean error of 343 meter with 3 BTS and 298 meter with 6 BTS [12].

I. Weighted Least Squares Techniques for Improved Received Signal Strength using hyperbola and circular.

As any channel model is not perfect on estimating the RF environment hence it affect heavily on location of MS. Such RF model give the rough estimation of distance from BTS to MS thus give a rough estimation of MS. Either use of any Multilateration technique, due to rough estimation of distance would result more than one location of MS. In the below figure 2.4, the small circle is the actual distance of MS to BTS thus it give the accurate location which is the intersection of three circle. But due to error in estimated distance the location of MS is then found to be at three common locations which is also intersection of three bigger circles. Hence in this situation, a best location of MS can be estimated by using weighted least square estimation [13].

III. METHODOLOGY

In this section, the methodology that will be used in this research is listed. Following are the steps involves.

A. Study of mathematical principles

Here we studied circular and, hyperbolic.

B. Select one of the position techniques and enhance its accuracy

Here we have used the hyperbolic technique along with polynomial interpolation to enhance the accuracy.

C. Select performance metric

Location accuracy means different in location. Such accuracy here we measured in meter. Here we select RMSE.

D. Select Validating technique

It will be validated by Implementing the concept in real network and compare the result with a real value. The drive test data will give the RSS value then it will be computed with enhanced distance and plotted using hyperbola.

E. Compare the result with one of the existing technique.

Here it will be compared with RSS with hyperbola. Same data se will be used for plotting location using hyperbola with and without enhanced distance.

IV. EXPERIMENTATION

The experimental setup includes the following.

A. Tools Used:

The tools used for the experimentation are – (i) Ericsson Toms Investigation 9.1.4; (ii) Visual basic script; (iii) Google Earth; (iv) Mapinfo; (v) Graph

B. Steps Performed

1) Propagation model Selection

The First step of this experiment is to select some of the propagation model to compare with the measured path loss data and thus later select one of them which is most accurate to such measured data. The models selected to compare in terms of their use in GSM RF model prediction are – (i) Hata model; (ii) Cost 231 Hata; (iii) Walfisch-ikegami.

2) Data collection via RF drive test

Data collection has been performed in 16 locations at Sanepa area which is covered by 16 BTS. In each location on average 500 samples has been taken for the study thus total 6000 Samples has been recorded. Out of 500 samples, average RSS is taken for each BTS signal received which have significant samples i.e. more than 100. BTS Data collection includes following parameters given in table 1

TABLE I. RF DRIVE TEST DATA COLLECTION.

MS location	BTS ID [ID_Name_Sector]	RSS [dbm]	Long. of BTS [deg]	Lat. of BTS [deg]
CG	KTM383_SachalSanepa-C	-55	85.307	27.685
CG	KTM059_Balkhu-B	-66	85.301	27.681
CG	KTM327_Bakundol-C	-73	85.312	27.683

3) Datasheet preparation

The second step of experiment is to collect the parameter for the calculating RF Model which is as follows according to BTS location and Mobile location given in table 2 and 3. The tower height is taken by considering the altitude of the ground as well.

TABLE II. PARAMETER 1 FOR THE CALCULATING RF MODEL

MS location	BTS ID [ID_Name_Sector]	Tx-Rx Actual Distance [Km]	Other building Height [m]	Tower Height [m]
CG	KTM383_SachalSanepa-C	0.330	21	33
CG	KTM059_Balkhu-B	0.488	17	29
CG	KTM327_Bakundol-C	0.607	31	43

TABLE III. PARAMETER 2 FOR THE CALCULATING RF MODEL

MS location	Road width [m]	Angle incident [deg]	Freq. [MHz]	Height of mobile [m]
CG	11	1	950	16
CG	11	1	950	7
CG	11	1	950	18

4) Calculate Path loss from three different path loss models

With the given table 3, the path loss is calculated for three propagation models defined in step 1 as given in table 4.

TABLE IV. CALCULATED PATH LOSS

<i>BTS ID [ID_Name_Sector]</i>	<i>Hata Model PL [db]</i>	<i>Cost231 Hata PL [db]</i>	<i>Walfisch-Ikigami PL [db]</i>
KTM383_SachalSanepa-C	72.28	71.14	99.95
KTM059_Balkhu-B	102.04	100.64	112.42
KTM327_Bakundol-C	74.93	74.25	118.29

5) Compare path loss and select one of the model

After calculating Path loss (PL) at different place and at different distance, it is compare with the measured path loss in terms of its RMSE. Measured path loss is calculated by using RSS and Tx power in table 5 and 6.

TABLE V. COMPARISON OF PATH LOSS MODEL

<i>MS location</i>	<i>BTS ID [ID_Name_Sector]</i>	<i>Tx-Rx Actual Distance [Km]</i>	<i>RSS [dbm]</i>	<i>Transmit power [dbm]</i>
sports	KTM362_Jhamshikhel-B	0.040	-39	42
farmarmart	KTM362_Jhamshikhel-C	0.077	-41	42
ilfc	KTM383_SachalSanepa-C	0.111	-40	42
chakupat_patan	KTM334_Chakupat-B	0.160	-55	42

TABLE VI. COMPARISON OF PATH LOSS MODEL

<i>MS location</i>	<i>Tx-Rx Actual Distance [Km]</i>	<i>Measured PL [db]</i>	<i>Hata PL [db]</i>	<i>Cost231-Hata PL [db]</i>	<i>Walfisch-Ikegami PL [db]</i>
sports	0.040	81.00	16.19	13.14	77.44
farmarmart	0.077	83.00	25.65	23.18	85.40
ilfc	0.111	82.00	55.76	53.65	84.70
chakupat_patan	0.160	97.00	50.25	48.54	96.16

TABLE VII. ERROR OF DIFFERENT MODEL PATH LOSS WITH RESPECT TO MEASURED PATH LOSS

<i>BTS ID [ID_Name_Sector]</i>	<i>Error Hata [db]</i>	<i>Error Cost231 [db]</i>	<i>Error Walfisch-Ikegami [db]</i>
KTM383_SachalSanepa-C	24.72	25.86	-2.95
KTM059_Balkhu-B	5.96	7.36	-4.42
KTM327_Bakundol-C	44.07	44.75	0.71

As it can be seen from the table 8 comparing RMSE error between measured path loss and calculated path loss, it is clearly observed that Walfisch-ikegami has less error 4.05 among remaining path loss models.

TABLE VIII. RMSE OF DIFFERENT MODEL USING GROUND ALTITUDE

<i>Propagation model</i>	<i>RMSE value</i>
Hata	55.89675
Cost 231 Hata	56.72973
Walfisch-Ikegami	4.052552

During calculation of path loss, the ground altitude has been considered to determine the height of tower. In the case ground altitude is not considered, the RMSE of Walfisch-Ikegami is still the least among three which is 4.39 as given in below table 9

TABLE IX. RMSE OF DIFFERENT MODEL WITHOUT GROUND ALTITUDE

<i>Propagation model</i>	<i>RMSE value</i>
Hata	6.878266
Cost 231 Hata	6.909241
Walfisch-Ikegami	4.395694

Thus from above three comparisons, Walfisch-Ikegami becomes the most suitable propagation model to predict path loss in present conditions.

6) Estimate the distance from BTS to MS

With the selected Walfisch-Ikegami Path loss model, the estimated distance from the BTS which signal has been received by MS is calculated as given in table 10 keeping all other parameter as same as given in table 2. Such estimated distance is then used to determine the location of MS by applying hyperbola.

TABLE X. DISTANCE ESTIMATED BY WALFISCH-IKEGAMI MODEL

<i>MS location</i>	<i>BTS ID [ID_Name_Sector]</i>	<i>Measu red PL [db]</i>	<i>Distance estimate using Walfisch-Ikegami [m]</i>
CG	KTM383_SachalSanepa-C	97	0.276
CG	KTM059_Balkhu-B	108	0.373
CG	KTM327_Bakundol-C	119	0.633

Note: Remaining data is listed under Appendix H

7) Predict the error in distance estimation using polynomial interpolation

Such estimated distance in above table 10 has error as compare to actual distance as given in table 12 for location CG.

TABLE XI. ERROR BETWEEN ACTUAL AND ESTIMATED DISTANCE

<i>MS location</i>	<i>BTS ID [ID_Name_Sector]</i>	<i>Distance estimate Walfisch-Ikegami [m]</i>	<i>Error [m]</i>
CG	KTM383_SachalSanepa-C	0.276	0.054
CG	KTM059_Balkhu-B	0.373	0.115
CG	KTM327_Bakundol-C	0.633	-0.027

Such error along with the estimated distance is then subjected to polynomial interpolation for prediction of error in required estimated distance. There are n-1 degree of polynomial interpolation which give forecasted error. To select the most accurate degree, predict the error in every degree then computed with estimated distance to give new enhanced distance. Again new error is calculated between actual distances and enhanced estimated distance as given in table 13.

TABLE XII. ERROR DISTANCE WITH 12 DEGREE OF INTERPOLATION

S.N	Error between actual and estimated distance [m]	Error predicted by 12 degree interpolation [m]	New enhanced distance estimated [m]	New Error between actual and enhanced distance [m]
1	-0.010	-0.018	0.031	0.009
2	0.010	0.030	0.096	-0.019
3	0.017	-0.005	0.089	0.022
4	0.056	0.042	0.200	0.014
5	-0.008	0.047	0.216	-0.056
6	-0.011	0.030	0.245	-0.042

Now compute the RMSE value for such new error. Thus comparing the Minimum RMSE value, required degree of interpolation is selected as given table 13.

TABLE XIII. RMSE FOR DISTANCE ERROR USING 3 TO 13 DEGREE OF INTERPOLATION

N	Error between actual and estimated distance [m]	Error between actual and new enhanced distance using N degree of polynomial interpolation [m]				
		9	10	11	12	13
RMSE	115	101	99	99	98	98

Here, it has found that Minimum RMSE is 98 computed from 12 degree of interpolation among all other. Thus 12 degree of polynomial interpolation is used to predict error which was computed by tabular data using table 11 given by equation 1.

$$Y = -1285907.62 * X_{12} + 7487962.29 * X_{11} - 19199843 * X_{10} + 28537359.2 * X_9 - 27241356 * X_8 + 17486699.1 * X_7 - 7684499.02 * X_6 + 2309945.99 * X_5 - 466912.631 * X_4 + 61235.8445 * X_3 - 4891.36120311 * X_2 + 211.65115869 * X - 3.72197105 \quad (1)$$

8) Compute the new enhanced estimate distance.

The error now is predicted using 12 degree of interpolation for any distance estimation. Such error is now added/subtracted with the estimated distance to get new enhanced estimated distance as given table 14.

TABLE XIV. NEW ENHANCED DISTANCE ESTIMATED USING 12 DEGREE OF POLYNOMIAL INTERPOLATION

MS location	BTS ID [ID_Name_Sector]	Distance estimate using Walfish-Ikigami [m]	New enhanced distance estimate d [m]
CG	KTM383_SachalSanepa-C	0.276	0.298

CG	KTM059_Balkhu-B	0.373	0.440
CG	KTM327_Bakundol-C	0.633	0.634

V. COMPUTE LOCATION USING HYPERBOLA

The equation of Hyperbola is given as below

$$\frac{X^2}{a^2} - \frac{Y^2}{b^2} = 1 \quad (2)$$

Where, x,y are the coordinate of locus points. a is one of the points on axis referred to x,y. in which

$$a = (f_2 - f_1) / 2 \quad (3)$$

f1 and f2 are the distance from two focal points (B1 & B2) to one of the locus point P.

$$b^2 = c^2 - a^2 \quad (4)$$

c is the distance from the center point of hyperbola (C) to its focal point (B1) or 2c is the distance between two BTS.

Where

$$c = \text{distance between two BTS} / 2 \quad (5)$$

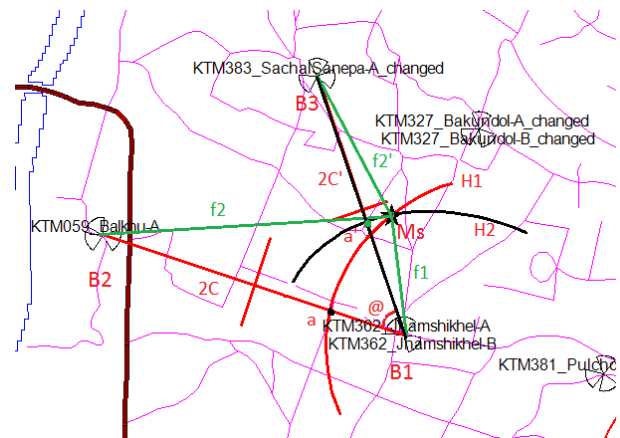


Fig. 3. Intersection of two Hyperbolas with respect to BTS and MS location

The following steps have been performed in order to calculate the hyperbola and its use to locate the position of MS.

1) Mark the coordinate of BTS.

The Coordinate of 3 BTS is computed using the distance between them.

- Suppose the 3 BTS are marked as B1, B2 and B3.



- Let their distance between B1 and B2 as “ 2C “ and between B1 and B3 as “ 2C’ “

For the First hyperbola,

- Let assume first hyperbola is between BTS B1 and B2
- Let the mid of BTS B1 and B2 is origin (0,0).
- Thus, coordinate of BTS B1 become ((2C)/2,0) and of BTS B2 become ((-2C)/2,0).

Similarly for the second Hyperbola,

- Let assume second hyperbola is between BTS B1 and B3
- Let the mid of BTS B1 and BTS B3 is origin(0,0).
- Thus, coordinate of BTS B1 become ((2C’)/2,0) and of BTS B3 become ((-2C’)/2,0).

2) 5.2 Compute the additional value of parameter using enhanced Estimate distance.

Compute below values for both hyperbola after calculating distance f1, f2 and f2`.

- Compute the value of “ a “ using equation 3
- Compute the value of “ b “ using equation 4
- Compute the value of “ c “ using equation 5
- Compute the angle @ between line connecting BTS B1 & B2 and line connecting BTS B1&B3.

3) 5.3 Calculate the List of locus of points

Calculate the List of locus of points given by two hyperbolas by using eq. 2.

Since the locus of points given by second hyperbola between BTS B1 and B3 is in the same direction as with first hyperbola till now, it should be rotated to its original location which is having some angle given by angle @. The rotation is done by using rotation Matrix rotated by angle @ centered at common BTS B1. Thus the common points between these locus of points between two hyperbola is location of MS.

VI. RESULT

The final output of this research is given below. There are total 16 locations of MS has been computed as given below table 15. Such location has been computed with both estimated distance and enhanced estimated distance.

TABLE XV. LOCATION ERROR WITH ESTIMATED DISTANCE AND ENHANCED ESTIMATED DISTANCE

S. N	MS location	Location Error Using Estimated distance [m]	Location Error Using enhanced distance [m]	Result [m]
1	gyanodaya	191	90	101
2	CG	116	75	41

3	ilfc	85	45	40
4	spid4	137	103	35
5	ThadoDunga	120	88	32
6	kishnagali	80	50	30
7	farmarmart	54	25	28
8	Summit	79	55	24
9	chakupat_patan	98	78	20
10	Mart	237	248	-11
11	Kandevasthan	75	105	-29
12	StMaryschool	40	78	-38
13	FriendColony	82	120	-38
14	patan3	73	126	-52
15	un	99	161	-62
16	chakupat2	133	206	-72

VII. CONCLUSION AND FUTURE WORK

The model chose the hyperbola technique which can mitigate any error which is common while measuring the RSS value from different BTS. Similarly it use the Walfish-Ikigami model which shows the superior performance among other RF model to estimate distance between BTS and MS more accurately which ultimately affect the location accuracy of MS. While using such RF model, the ground altitude of BTS location also has been considered while calculating the total tower height. Lastly the error from estimated distance given by Walfish-Ikigami model is modeled to polynomial interpolation which is used to predict the error in required distance. Such predicted error then computed along with the estimated distance and provide the new enhanced distance.

The outcome of the model shows improve minimum location error as 25 meter among 16 locations better than alone hyperbola used which was 54 meter. This is 53% better in the minimum range. Similarly it has slightly degraded maximum error as 248 meter compare to 237 meter. This is 4.6 % worse in the maximum range. The reason for worsen result is nature of non-linear error that cannot be forecasted perfectly by polynomial interpolation. Out of 16 locations, the accuracy is better in 9 (56 %) location and worse in 7 (44 %) location. Among the 9 better locations, the minimum error reach is 25 meter compare to 54 meter and maximum error reach is 103 meter compare to 191 meter where hyperbola is alone used. Hence it is concluded that mobile location using hyperbolic technique based on pathloss and appropriate degree of interpolation can improve the location of MS.

The accuracy of location of MS can be further improved by enhancing the estimated distance. Here same degree of interpolation is used to forecast the error. In fact various different degree of interpolation can be used to forecast error depending upon the positioning of required distance. This approach will give more accurate location of MS.

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