Journal of Engineering Issues and Solutions

Evaluation of weibull parameter estimators for wind speed of Jumla, Nepal

Ayush Parajuli[1,*]

[1] *Department of Mechanical Engineering and Science, Kyoto University, Kyoto, Japan*

**Corresponding email: parajuliyush@gmail.com*

Received: January 10, 2021; Revised: February 22, 2021; Accepted: February 26, 2021

Abstract

Weibull Probability Distribution Function (PDF) is widely used across world for estimation of wind power. Weibull function is a two parameter probability distribution function. The methods employed for the evaluation of these two parameters are critical for the efficient use of Weibull PDF. In the present study, three different Weibull PDF parameter estimators have been evaluated. For this purpose, the daily averaged wind speed data of Jumla Station, Nepal for period of 10 year (2004 – 2014: 2012 excluded) is studied. The parameter estimator evaluated in this study are Method of Moments (MoM), Least Square Error Method (LSEM) and Power Density Method (PDM). It has been found that Method of Moments (MoM) is the best estimator for evaluating Weibull Parameters.

Keywords: *Least square error method; method of moments; power density method; weibull distribution*

1. Introduction

To address the demand of the clean and renewable energy, wind energy is getting broader attention even in remote terrain. Wind farm design analysis has a key objective to reduce the Levelised Cost of Energy (LCoE) while addressing the pressing environmental and social concerns. LCoE is used in various studies which gives the cost to generate 1 MWh energy incorporating all lifetime cost. Primarily, it is function of initial investment, revenue generated through farm operation, routine operational and maintenance cost and salvage value. To reduce LCoE, wind farm modeling/design analysis is done. The fidelity of such model will hinge on selection of objective function, optimization method, and definition of wind characteristics, turbine modeling and wind farm interaction.

This study is limited to improving the definition of wind characteristics. The wind characteristics is defined using large data sets of available wind speeds and direction in a certain period of time. So, the probability distribution of different wind speeds is used as a representation of these data sets. Further, the probability distribution will assist:

a) To retrospectively characterize past conditions.

b) To predict future power generation at one location.

c) To predict power generation within a grid of turbines.

d) To calibrate meteorological data.

Figure 1: Site location of synoptic station

As seen from the literature, much concentration has been given to Weibull function because it is found to give best fit to the observed wind speed data at flat terrains, mountains and off shore (Abbas et al., 2012; Ahmed, 2013; Baseer et al., 2017; Deep et al., 2020; Hulio et al., 2019; Katinas et al., 2017; Odo et al., 2012; Oner et al., 2013; Oyedepo et al., 2012; Pishgar-Komleh et al., 2015; Rehman et al., 1994; Shoaib et al., 2017). Further, the author have also shown Weibull function to perform better than Rayleigh distribution in earlier research (Parajuli, 2016).

Weibull distribution is a two parameter function. The two parameters are shape parameter (k) and scale parameter (c). Shape parameter is a dimensionless parameter and scale parameter is a dimensional parameter (Wais, 2017). Several methods can be deployed to estimate the parameters and the efficiency of these estimators determine the accuracy of the PDE (Chaurasiya et al., 2018). Therefore, this study is done with a single agenda i.e. to find the best Weibull parameter estimator for the given site. The reader should refer to Section **2.1** for details of the site.

2. Materials and Methods

2.1 Site Location and Data Collection

Jumla, Nepal is a potential site for wind energy generation and has been shown to be a class III site in the previous study done by the author (Parajuli, 2016). Department of Hydrology and Meteorology (DHM) has a synoptic station in Chandannath Municipality of Jumla. The site is located at at 2300m above sea level. Wind speed was measured at height of 10m from ground level and average daily wind speed was available. Wind speed from 2004 to 2014 (2012 excluded) was used for analysis. The location of site is as shown in Figure 1. Previous studies to find best Weibull parameter estimators were done on sites at lower altitudes and flat terrains (Ahmed, 2013; Azad et al., 2014; Guarienti et al., 2020; Saeed et al., 2020; Tiam Kapen et al., 2020). We know that the effect of shear and mountain wakes are critical in higher altitude as atmosphere is a density stratified medium. Therefore, the application of findings at lower altitude shall be extended to higher altitude with caution. Therefore this study, which has been done with wind data of a site at 2300m altitude, will provide insights whether the findings at lower altitude stay valid at higher altitude.

2.2 Vertical Extrapolation of Wind Speed

To calculate the total wind energy potential, the measured wind speed must be modified for turbine hub height above ground level for small and moderate sized turbine. With the assumption of use of moderately sized turbine for difficult terrain, the single point extrapolation to mean hub height is done using the following equation (Abbas et al., 2012):

$$v = u\left(\frac{z}{y}\right)^{a} \tag{1}$$

Where, u is the wind speed at measured wind speed (m/s), y is the measuring equipment height above ground (m) and z is the turbine hub height (m). The exponent *a* can be termed as shear parameter and depends on several factors such as atmospheric stability and surface roughness. According to the literature, for neutral stable condition, *a* is approximately 0.143, which is commonly assumed to be constant in wind resource assessments (Pishgar-Komleh et al., 2015). The author notes that an extensive study is also required for value of shear parameter in mountainous terrain at high altitude. Because of lack of available data of shear parameter for this region, the author cautiously uses the value estimated in previous study for different terrain.

2.3 Wind Speed Probability Distribution and Parameter Estimators

Researches have shown that Weibull function fits the wind probability distribution more accurately compared to others. Weibull distribution is a two parameter function characterized by scale parameter c (m/s) and shape parameter k (dimensionless) and is given by (Ahmed, 2013; Weibull, 1951):

$$f_w(v) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^{k}\right] \tag{2}$$

Shape parameter k and scale parameter c can be calculated using several methods. This study will evaluate following estimators:

2.3.1. Method of Moments (MOM)

This technique is commonly used in field of parameter estimation. The method is used as an alternative to the Maximum Likelihood Method which has its associated computational cost. The shape and scale factor can be calculated as (Azad et al., 2014):

$$k = \left(\frac{0.9874}{\frac{\sigma}{\bar{v}}}\right)^{1.0983} \tag{3}$$

$$c = \frac{\bar{v}}{\Gamma(1 + 1/k)} \tag{4}$$

where Γ is the gamma function.

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dx \quad (5)$$

2.3.2 Least Square Error Method (LSEM)

This method tends to minimize the square of the error in probability plotting. The equation of CDF is linearized with natural log and least square method is employed to estimate the value of k and c:

$$k = \frac{n\sum_{i=1}^{n} \ln v_i \ln[-\ln\{1-F(v_i)\}] - \sum_{i=1}^{n} \ln v_i \sum_{i=1}^{n} \ln[-\ln\{1-F(v_i)\}]}{n\sum_{i=1}^{n} (\ln v_i)^2 - \left\{\sum_{i=1}^{n} \ln v_i\right\}^2} \quad (6)$$

$$c = \exp\left\{\frac{k\sum_{i=1}^{n} \ln v_i - \sum_{i=1}^{n} \ln[-\ln\{1-F(v_i)\}]}{nk}\right\} \quad (7)$$

2.3.3 Power Density Method (PDM)

This method was suggested by Akdag et al (Akda & Dinler, 2009). The energy pattern factor is ratio of average of cube of velocity to cube of average velocity and *k* can be estimated using energy pattern factor. Value of *c* can be estimated using relationship of *k* and *c* in Equation 4.

$$E_{pf} = \frac{\overline{v^3}}{\overline{v}^3} \quad (8)$$

$$k = 1 + \frac{3.69}{(E_{pf})^2} \quad (9)$$

2.4 Evaluation of Estimators

In order to check how accurately the estimator predicted theoretical probability density fits measured data, in this paper, we employ root mean square error (RMSE) parameter (Azad et al., 2014; Chang, 2010; Ouarda et al., 2015).

$$RMSE = \left[\frac{1}{N}\sum_{1}^{N} (f_m - f_w)^2\right]^{1/2} \quad (10)$$

Where N is number of observations, f_m is frequency of observation or ith calculated value from measured data, f_w is frequency of weibull or ith calculated value from the weibull distribution.

3. Results and Discussion

The Weibull parameters *k* and *c* were estimated using the three methods employed. Instead of evaluating the parameters of every year and evaluating a large data set, the data for all ten years were considered as a single data set and the parameters were calculated. The same is tabulated in Table 1. The readers interested in parameter trend and seasonal fluctuations of the wind speed at the location can refer to previous article by the author (Parajuli, 2016). The probability densities for the actual wind speed data and the fits are presented in Figure 2. It can be noted that all three methods underestimated the peak frequency. However, the underestimation of the peak by Power Density Method (PDM) is larger compared to other two. Further, it can be noted that, for high and low wind speeds, the performance of all three methods are similar. To evaluate the overall error, root mean square error method has been employed in this research and the same is tabulated in last column of Table 1. From the error analysis, we can see that Method of Moment (MoM) is the best estimator for evaluating Weibull Parameter for this site. Similarly, Power Density Method has been noted to have poor performance out of three estimators.

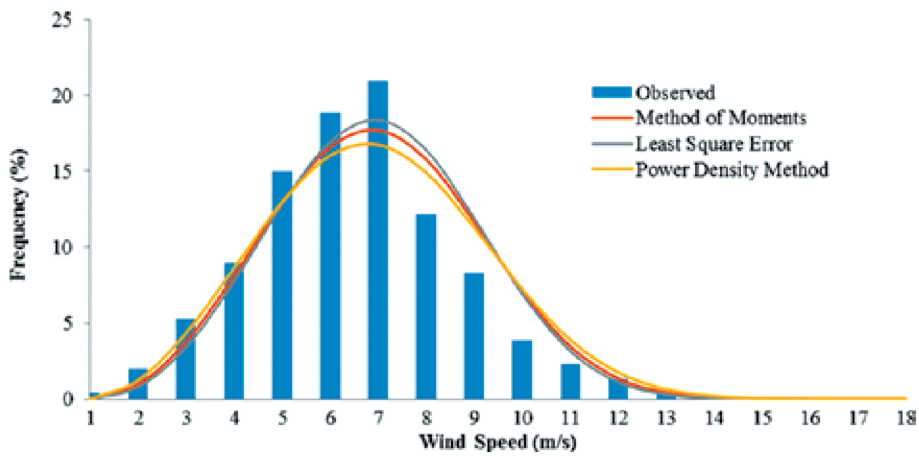

Figure 2: Observed histogram and weibull fits of various method

Table 1: Parameters Estimation and RMSE

Method	Shape Factor (k)	Scale Factor (c)	RMSE
Method of Moments (MoM)	3.03	6.69	0.391
Least Square Error Method (LSEM)	3.16	6.68	1.036
Power Density Method (PDM)	2.86	6.70	1.457

In the research done by Ahmed et al. (Ahmed, 2013) for Halabja City, Iraq, Power Density Method has been shown to under predict the peak frequency and not favourable method for prediction of Weibull parameters. It is to be noted that the altitude level of Halabja City is 692m. Similarly, Azad et al. (Azad et al., 2014) showed for three sites of Bangladesh, which are below 10m altitude level, that Method of Moments (MOM) is the best estimator for Weibull Function. The previous researches corroborate with the finding of this analysis despite the differences in the altitude and terrain. In the previous study done by the author at the same site, Method of Moments (MoM) was used to estimate the Weibull parameters. Therefore, the reader interested in power estimation and wind speed characteristics of the site can refer to the article (Parajuli, 2016).

4. Conclusion

Based on the wind speed data from Jumla, Nepal which is located at around 2300m above sea level, it has been found that Method of Moments (MOM) is the best estimator for prediction of Weibull Parameters. The finding is similar to studies done by other researchers at low altitude. To further strengthen the claim, the future studies shall be done on multiple locations at similar altitudes. Similarly, the variation of shear parameter shall also be duly noted in future studies.

Acknowledgements

The authors would like to thank the Department of Hydrology and Meteorology for providing wind speed data of the site.

Conflict of Interests

Not declared by authors.

References

Abbas, K., Alamgir, K., Ali, A., Khan, D., & Khalil, U. (2012). Statistical analysis of wind speed data in Pakistan. *World Applied Sciences Journal.*

Ahmed, S. A. (2013). Comparative study of four methods for estimating Weibull parameters for Halabja , Iraq. *International Journal of Physical Sciences*, *8*(5), 186–192. https://doi.org/10.5897/IJPS12.697

Akda, S. a., & Dinler, A. (2009). A new method to estimate Weibull parameters for wind energy applications. *Energy Conversion and Management*, *50*(7), 1761–1766. https://doi.org/10.1016/j.enconman.2009.03.020

Azad, A., Rasul, M., & Yusaf, T. (2014). Statistical Diagnosis of the Best Weibull Methods for Wind Power Assessment for Agricultural Applications. *Energies*, *7*(5), 3056–3085. https://doi.org/10.3390/en7053056

Baseer, M. A., Meyer, J. P., Rehman, S., & Alam, M. M. (2017). Wind power characteristics of seven data collection sites in Jubail, Saudi Arabia using Weibull parameters. In *Renewable Energy* (Vol. 102). https://doi.org/10.1016/j.renene.2016.10.040

Chang, T. P. (2010). Wind Speed and Power Density Analyses Based on Mixture Weibull and Maximum Entropy Distributions. In *International Journal of Applied Science and Engineering* (Vol. 8, Issue 1, pp. 39–46).

Chaurasiya, P. K., Ahmed, S., & Warudkar, V. (2018). Study of different parameters estimation methods of Weibull distribution to determine wind power density using ground based Doppler SODAR instrument. *Alexandria Engineering Journal*, *57*(4), 2299–2311. https://doi.org/10.1016/j.aej.2017.08.008

Deep, S., Sarkar, A., Ghawat, M., & Rajak, M. K. (2020). Estimation of the wind energy potential for coastal locations in India using the Weibull model. *Renewable Energy*, *161*, 319–339. https://doi.org/10.1016/j.renene.2020.07.054

Guarienti, J. A., Kaufmann Almeida, A., Menegati Neto, A., de Oliveira Ferreira, A. R., Ottonelli, J. P., & Kaufmann de Almeida, I. (2020). Performance analysis of numerical methods for determining Weibull distribution parameters applied to wind speed in Mato Grosso do Sul, Brazil. *Sustainable Energy Technologies and Assessments*, *42*. https://doi.org/10.1016/j.seta.2020.100854

Hulio, Z. H., Jiang, W., & Rehman, S. (2019). Techno - Economic assessment of wind power potential of Hawke's Bay using Weibull parameter: A review. *Energy Strategy Reviews*, *26*. https://doi.org/10.1016/j.esr.2019.100375

Katinas, V., Marčiukaitis, M., Gecevičius, G., & Markevičius, A. (2017). Statistical analysis of wind characteristics based on Weibull methods for estimation of power generation in Lithuania. *Renewable Energy*, *113*, 190–201. https://doi.org/10.1016/j.renene.2017.05.071

Odo, F. C., Offiah, S. U., & Ugwuoke, P. E. (2012). Weibull distribution-based model for prediction of wind potential in Enugu , Nigeria. *Advances in Applied Science Research*, *3*(2), 1202–1208.

Oner, Y., Ozcira, S., Bekiroglu, N., & Senol, I. (2013). A comparative analysis of wind power density prediction methods for Canakkale, Intepe region, Turkey. *Renewable and Sustainable Energy Reviews*, *23*, 491–502. https://doi.org/10.1016/j.rser.2013.01.052

Ouarda, T. B. M. J., Charron, C., Shin, J.-Y., Marpu, P. R., Al-Mandoos, A. H., Al-Tamimi, M. H., Ghedira, H., & Al Hosary, T. N. (2015). Probability distributions of wind speed in the UAE. *Energy Conversion and Management*, *93*, 414–434. https://doi.org/10.1016/j.enconman.2015.01.036

Oyedepo, S. O., Adaramola, M. S., & Paul, S. S. (2012). Analysis of wind speed data and wind energy potential in three selected locations in south-east Nigeria. *Int. J. Energy Environ. Eng.*, *3*(1), 7. https://doi.org/10.1186/2251-6832-3-7

Parajuli, A. (2016). A Statistical Analysis of Wind Speed and Power Density Based on Weibull and Rayleigh Models of Jumla, Nepal. *Energy and Power Engineering*, *08*(07), 271–282. https://doi.org/10.4236/epe.2016.87026

Pishgar-Komleh, S. H., Keyhani, A., & Sefeedpari, P. (2015). Wind speed and power density analysis based on Weibull and Rayleigh distributions (a case study: Firouzkooh county of Iran). *Renewable and Sustainable Energy Reviews*, *42*(0), 313–322. https://doi.org/http://dx.doi.org/10.1016/j.rser.2014.10.028

Rehman, S., Halawani, T. O., & Husain, T. (1994). Weibull parameters for wind speed distribution in Saudi Arabia. *Solar Energy*, *53*(6), 473–479. https://doi.org/10.1016/0038-092X(94)90126-M

Saeed, M. A., Ahmed, Z., Yang, J., & Zhang, W. (2020). An optimal approach of wind power assessment using Chebyshev metric for determining the Weibull distribution parameters. *Sustainable Energy Technologies and Assessments*, *37*. https://doi.org/10.1016/j.seta.2019.100612

Shoaib, M., Siddiqui, I., Amir, Y. M., & Rehman, S. U. (2017). Evaluation of wind power potential in Baburband (Pakistan) using Weibull distribution function. *Renewable and Sustainable Energy Reviews*, *70*, 1343–1351. https://doi.org/10.1016/j.rser.2016.12.037

Tiam Kapen, P., Jeutho Gouajio, M., & Yemélé, D. (2020). Analysis and efficient comparison of ten numerical methods in estimating Weibull parameters for wind energy potential: Application to the city of Bafoussam, Cameroon. *Renewable Energy*, *159*, 1188–1198. https://doi.org/10.1016/j.renene.2020.05.185

Wais, P. (2017). Two and three-parameter Weibull distribution in available wind power analysis. *Renewable Energy*, *103*, 15–29. https://doi.org/10.1016/j.renene.2016.10.041

Weibull, W. (1951). A Statistical Distribution Function of Wide Applicability. In *Journal of Applied Mechanics* (Vol. 103).