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Compatible Mappings and Its Various Variants in Metric Space

Uday Kumar Karn[1,2], Chet Raj Bhatta[2], Ajay Kumar Chaudhary[3,*]

[1]Department of Mathematics, Patan Multiple Campus, Tribhuvan University, Nepal
[2]Central Department of Mathematics, Kirtipur, Tribhuvan University, Kathmandu, Nepal
[3]Department of Mathematics, Tri-Chandra Multiple Campus, Tribhuvan University, Kathmandu, Nepal

*Correspondence to: Ajay Kumar Chaudhary, Email: akcsaurya81@gmail.com

Abstract: *A study on compatible mappings becomes necessary to do research on multiple self-mappings to find the fixed points, called common fixed points in fixed point theory. To obtain the weaker forms of commuting mappings [16], G. Jungck introduced the notion of compatible mappings [18] in metric space in 1986.*

This article presents a study on compatible mappings and their various variants of non-commuting mappings in metric space and verifies its relationship through examples. It facilitates comparative analysis and strengthens the relationship between them.

Keywords: Commuting mappings, Weakly commuting mapping, Compatible mappings, Reciprocal continuous

1 Introduction

Stephen Banach Contraction mapping [4] works in single self-mapping and obtain fixed point theorem. A common fixed point of f with identity mapping on X can be thought of as the study of fixed points of self-mappings. However, K. Goebel [14] introduced the notion in 1968 of substituting another self-mapping g on X for the identity mapping and established the coincidence theorem in order to derive the common fixed point theorem. Despite the fact that Machuca [21] initially examined this issue in 1967, under some strict topological restrictions.

In order to derive common fixed point theorems for contractive type mappings, we always require a commutativity condition, a restriction on the ranges of mappings, continuity of one or more mappings, and a contractive condition. Furthermore, obtaining a necessary version of one or more of these conditions or weakening them is the objective of all significant fixed point theorems and common fixed point theorems. In 1976, Jungck [16] introduced the commuting mappings and established common fixed point theorems by using constructive procedures of sequence of iterations in metric space. This condition is too strong. So, naturally searches weaker forms. In 1982, Sessa [31] gave weakly commuting mapping and extended a variety of fixed point theorems by substituting weakly commutativity for commutativity. Thereafter less restrictive contractive mapping, compatible mapping introduced by Jungck [18] in 1986 in metric space. Elegancy of this result many authors have introduced various other contractive conditions on more than one self-mappings like compatible type conditions, readers may see references [[6, 7, 8, 9, 10],[12, 13, 15], [25]].

This article discusses on only various variants of compatible mappings in metric space introduced by different mathematics researchers and established common fixed theorems in metric space by using these weaker forms of commuting mappings. Here, we show commuting mappings weaker forms interrelationship through verified examples. This paper boost up the comparative study of compatible mappings with its different types which helps to develop their interrelationship, and also helps readers for their research study.

2 Preliminaries

Definition 2.1. [2] Let A and B are two self-mappings on X. Then, for some $z \in X$ is called coincidence point of A and B if $z = Ax = Bx$. Here, z is called point of coincidence of A and B.

Definition 2.2. [6] Let A and B are two self-mappings on X. Then, a point $x \in X$ is said to be common fixed point of A and B if $x = Ax = Bx$.

Definition 2.3. [16] Let A and B are two self-mappings on X in metric space (X, d). Then, a pair (A, B) is said to be commuting mapping if and only if $ABx = BAx$ for all $x \in X$.

Definition 2.4. [31] Let A and B are two self-mappings on X in metric space (X, d). Then, a pair (A, B) is said to be weakly commuting mapping if and only if $d(ABx, BAx) \leq d(Ax, Bx)$ for all $x \in X$.

Definition 2.5. [1] Let A and B are two self-mappings on X in metric space (X, d). Then, a pair (A, B) is said to satisfy E. A. property if there exists a sequence $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some z in X.

Note 1 It is observed that the (E.A.) property is equivalent to the previously known notion of tangential mappings introduced by Sastry et al. [30].

Definition 2.6. [18] Let A and B are two self-mappings on X in metric space (X, d). Then, a pair (A, B) is said to be compatible mapping if and only if $\lim_{n\to\infty} d(ABx_n, BAx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some z in X.

Definition 2.7. [11] Two self-mappings A and B of a metric space (X, d) are said to be semi-compatible if

(i) $Ax = Bx \implies ABx = BAx$;

(ii) $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some z in $X \implies \lim_{n\to\infty} d(ABx_n, Bx) = 0$.

Definition 2.8. [16] Two self-mappings A and B of a metric space (X, d) are said to be weakly compatible if they commute at their coincidence points, that is, if $ABx = BAx$, whenever $Ax = Bx$, x in X.

Note 2 It is also called coincidentally commuting by Dhage [12], Partially commuting by Sastry and Murthy [30], compatible type (N) by Shrivastava, Bawa, and Singh [32].

Definition 2.9. [22] Two self-mappings A and B of a metric space (X, d) are said to be non-compatible mapping if there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some z in X but $\lim_{n\to\infty} d(ABx_n, BAx_n)$ is either non-zero or non-existent.

Remark 2.10. Weak commutativity is essentially a point property, while the notion of compatibility uses the machinery of sequences. Compatibility or weak commutativity of a pair of self-mappings on a metric space depends on the choice of the metric. [2]

Definition 2.11. [34] Two self-mappings A and B of a metric space (X, d) are said to be common limit in the range (CLR) of B property if there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = Bx$ for some x in X

Remark 2.12. E. A. property and (CLR) property both are for notion of non-compatibility. These properties are well suited for studying common fixed points of strict contractive conditions, non-expansive type mapping pairs or Lipschitzian- type mapping pairs in ordinary metric spaces, which are not even complete. [2]

Definition 2.13. [22] A pair (A, B) of self-mappings of a metric space (X, d) is said to be R-weakly commuting if there exists some real number $R > 0$ such that $d(ABx, BAx) \leq Rd(Ax, Bx)$, for all $x \in X$.

Remark 2.14. [22]

(i) Every R-weakly commuting pair is weakly commuting if $R = 1$.

(ii) Weak commutativity $\implies$ R-weak commutativity. But R-weak commutativity $\implies$ weak commutativity only when $R \leq 1$.

Definition 2.15. [22] A pair (A, B) of self-mappings of a metric space (X, d) is said to be point wise R-weakly commuting on X, if and only if given $x \in X$, there exist $R > 0$ such that
$d(ABx, BAx) \leq Rd(Ax, Bx)$.

Remark 2.16. [2] It is observed from above definition that A and B can fail to be point-wise R-weakly commuting only if there exists some $x \in X$ such that $Ax = Bx$ but $ABx \neq BAx$, i.e. only if they possess a coincidence point at which they do not commute.

Definition 2.17. [24] A pair (A, B) of self-mappings of a metric space (X, d) is said to be reciprocal continuous if $\lim_{n\to\infty} ABx_n = Az$, and $BAx_n = Bz$, whenever $\{x_n\}$ is a sequence in X such that
$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some z in X.

Remark 2.18. [24] If A and B both mappings are continuous then they are obviously reciprocal continuous but converse is not true.

Definition 2.19. [35] A pair (A, B) of self-mappings of a metric space (X, d) is said to be weakly reciprocal continuous if $\lim_{n\to\infty} ABx_n = Az$, or $BAx_n = Bz$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = z$ for some z in X.

Remark 2.20. [35] If A and B both mappings are reciprocal continuous then they are obviously weakly reciprocal continuous but converse is not true.

Definition 2.21. [26] Two self-mappings A and B of a metric space (X, d) are said to be weakly uniformly contraction mappings if and only if $d(ABx, BBx) \leq d(Ax, Bx)$, and $d(AAx, BAx) \leq d(Ax, Bx)$, for all $x \in X$.

Definition 2.22. [19] A pair (S, T) of self-mappings of a metric space (X, d) is said to be compatible mappings of type (A) if and only if $\lim_{n\to\infty} d(SSx_n, TSx_n) = 0$, and $\lim_{n\to\infty} d(STx_n, TTx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some z in X.

Definition 2.23. [27] A pair (S, T) of self-mappings of a metric space (X, d) is said to be compatible mappings of type (B) if and only if $\lim_{n\to\infty} d(STx_n, TTx_n) \leq \frac{1}{2} [\lim_{n\to\infty} d(STx_n, Sz) + \lim_{n\to\infty} d(Sz, SSx_n)]$, and $\lim_{n\to\infty} d(TSx_n, SSx_n) \leq \frac{1}{2} [\lim_{n\to\infty} d(TSx_n, Tz) + \lim_{n\to\infty} d(Tz, TTx_n)]$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some z in X.

Definition 2.24. [28] A pair (S, T) of self-mappings of a metric space (X, d) is said to be compatible mappings of type (C) if and only if
$\lim_{n\to\infty} d(STx_n, TTx_n) \leq \frac{1}{3} [\lim_{n\to\infty} d(STx_n, Sz) + \lim_{n\to\infty} d(Sz, SSx_n) + \lim_{n\to\infty} d(Sz, TTx_n)]$, and
$\lim_{n\to\infty} d(TSx_n, SSx_n) \leq \frac{1}{3} [\lim_{n\to\infty} d(TSx_n, Tz) + \lim_{n\to\infty} d(Tz, TTx_n) + \lim_{n\to\infty} d(Tz, SSx_n)]$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some z in X.

Remark 2.25. [2] If $S, T : (X, d) \to (X, d)$ be continuous mappings. Then the following are equivalent:

(i) S and T are compatible of type (A);

(ii) S and T are compatible of type (B);

(iii) S and T are compatible of type (C); and

(iv) S and T are compatible mappings.

Definition 2.26. [29] A pair (S, T) of self-mappings of a metric space (X, d) is said to be compatible mappings of type (P) if and only if $\lim_{n\to\infty} d(SSx_n, TTx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some z in X.

Definition 2.27. [36] A pair (S, T) of self-mappings of a metric space (X, d) is said to be compatible mappings of type (C) if and only if $\lim_{n\to\infty} d(SSx_n, TTx_n) = 0$, and $\lim_{n\to\infty} d(STx_n, TSx_n) = 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some z in X.

Definition 2.28. [37] A pair (S, T) of self-mappings of a metric space (X, d) is said to be compatible mappings of type (E) if and only if $\lim_{n\to\infty} TTx_n = \lim_{n\to\infty} TSx_n = Sz$, and $\lim_{n\to\infty} SSx_n = \lim_{n\to\infty} STx_n = Tz$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some z in X.

Splitting definition of compatible mapping of type (E) in two forms as:

Definition 2.29. [37] A pair (S, T) of self-mappings of a metric space (X, d) is said to be T-compatible mappings of type (E) if and only if $\lim_{n\to\infty} TTx_n = \lim_{n\to\infty} TSx_n = Sz$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some z in X.

Definition 2.30. [37] A pair (S, T) of self-mappings of a metric space (X, d) is said to be S-compatible mappings of type (E) if and only if $\lim_{n\to\infty} SSx_n = \lim_{n\to\infty} STx_n = Tz$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some z in X.

Definition 2.31. [3] A pair (S, T) of self-mappings of a metric space (X, d) is said to be occasionally weakly compatible (OWC) if $STx = TSx$ for some $x \in C(S, T)$.

In the sense of Jungck and Rhodes, A pair (S, T) of self-mappings of a metric space (X, d) is said to be occasionally weakly compatible (OWC) if there exists at least one coincidence point at which S and T commute, i.e., if $ST = TS$ for some x in X, then $STx = TSx$.

The generalization of compatible mappings is called biased mappings. It is introduced by Jungck and Pathak in 1995.

Definition 2.32. [20] A pair (S, T) of self-mappings of a metric space (X, d) is said to be S-biased if and only if $\alpha \lim_{n\to\infty} d(Sx_n, STx_n) \leq \alpha \lim_{n\to\infty} d(Tx_n, TSx_n)$, where $\alpha = liminf. or\, limsup.$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some z in X.

Similarly, the definition of T-biased can be obtained from the definition of S-biased by interchanging the role of S and T. [20]

3 Main Results

3.1 Interrelationship I

Here, we verify compatible mappings variants interrelationship through examples.

Interrelationship 3.1. *[31] Weakly commuting mappings need not to be commuting.*

Example 3.2. Let $A, B : [0, 2] \to [0, 2]$ defined by $A(x) = \frac{x}{x+2}, B(x) = \frac{x}{2}$, for all x. Then, (A, B) is weakly commuting mapping but not commuting.

Interrelationship 3.3. *[11] Semi compatible mappings need not be compatible mappings.*

Example 3.4. Let $X = [2, 6]$, d be the usual metric on X. Defining $A, B : X \to X$ by
$Ax = 2$, if $x < 3$, $Ax = 4$, if $x = 3$, $Ax = \frac{x+21}{12}$, if $3 \leq$ x ≤ 6
$Bx = 2$, if $x = 2, Bx = 2x$, if $2 < x \leq 3$, $Bx = \frac{2x}{3}$. Then, A and B are semi-compatible mapping.

Interrelationship 3.5. *[18] Weakly commuting mappings are compatible but converse does not hold.*

Example 3.6. Let $A, B : X \to X$ defined by $Ax = x^3, Bx = 2x^3$, for all x, where $X = [0, \infty)$, d be usual metric. Then, $d(ABx, BAx) > d(Ax, Bx)$ which shows that A, and B are compatible but not weakly commuting, and also not commuting mapping.

Interrelationship 3.7. *[22] Commuting mappings are R- weakly commuting.*

Example 3.8. Let $X = [1, \infty)$ d be the usual metric on X. Defining $A, B : X \to X$ by $Ax = 2x - 1$, $Bx = x^2$, for all $x \in X$.

Here, $d(ABx, BAx) = 2(x-1)^2$, and $d(Ax, Bx) = (x-1)^2$. So, A, and B are R- weakly commuting with $R = 2$. But since $d(ABx, BAx) = 2(x-1)^2 \neq 0$ for all $x \neq 1 \in X \implies A$ and B are not commuting.

Interrelationship 3.9. *[28] Compatible mappings of type $(A) \implies$ compatible mappings of type $(B) \implies$ compatible mappings of type (C), but the converse is not true in general.*

Example 3.10. Let $X = [1, 20]$, d be the usual metric on X. Defining $S, T : X \to X$ as below:
$Sx = 1$, if $x = 1$, $Sx = 3$, if $1 \leq x \leq 7$
$Tx = x - 6$, if $7 < x \leq 20$, $Tx = 1$, if $1 \cup (7, 20], Tx = 2$, if $1 < x \leq 7$.
Taking sequence $\{x_n\}$ as $x_n = 7 + \frac{1}{n}$, $n > 0$. Then, S and T are compatible of type (C), but neither compatible nor compatible of type (A) nor compatible of type (B).

Interrelationship 3.11. *[19] Compatible mapping and compatible mappings of type (A) are independent to each other.*

Example 3.12. Let $X = [2, 12]$, d be the usual metric on X. Defining $S, T : X \to X$ as below:
$Sx = 2, ifx = 2$ or $x > 5$, $Sx = 12$, if $2 \leq x \leq 5$
$Tx = 2$, if $7 < x \leq 20, Tx = 12$, if $2 \leq x \leq 5$, $Tx = x + 13$, if $x > 5$.
Taking sequence $\{x_n\}$ as $x_n = 5 + \frac{1}{n}, n > 0$. Then, S and T are compatible of type (A), but neither commuting nor compatible mappings.

Example 3.13. Let $X = \mathbb{R}$ equipped with the usual metric d. Defining self-mappings S and T as below:

$$Sx = x,$$

and

$$T(x) = \begin{cases} 0, & \text{if } x \quad is \quad an \quad integer, \\ 1, & \text{if } x \quad is \quad not \quad an \quad integer. \end{cases}$$

Taking sequence $\{x_n\}$ as $x_n = 1 + \frac{1}{n+1}, n > 0$. Then, S and T are compatible mapping, but not compatible mapping of type (A).

Interrelationship 3.14. *[37] If $Sz = Tz$, then compatible of type (E) implies compatible (compatible of type (A), compatible of type (B), compatible of type (C), compatible of type (P)); however, the converse may not be true. Further, if $Sz \neq Tz$ then compatible of type (E) is neither compatible nor compatible of type (A), (compatible of type (C), compatible of type (P)).*

Example 3.15. Let $X = [0, 1]$, $d(x, y) = |x - y|$. We define self-maps S and T as $Sx = 1, Tx = 0$, for $x \in [0, \frac{1}{2}] - \{\frac{1}{4}\}, Sx = 0, Tx = 1$ for $x = \frac{1}{4}$, and $Sx = \frac{1-x}{2}$, $Tx = \frac{x}{2}$, for $x \in (\frac{1}{2}, 1]$. Clearly S, and T are not continuous at $x = \frac{1}{2}, \frac{1}{4}$.
Suppose that $x_n \to \frac{1}{2}$, $x_n > \frac{1}{2}$ for all n. Then, we have $Sx_n = \frac{1-x_n}{2} \to \frac{1}{4} = z$, and $Tx_n = \frac{x_n}{2} \to \frac{1}{4} = z$. Also, we have $SSx_n = S(\frac{1-x_n}{2}) = 1 \to 1$, $STx_n = S(\frac{x_n}{2}) = 1 \to 1$, $T(z) = 1$, and $TTx_n = T(\frac{x_n}{2}) = 0 \to 0$, $TSx_n = T(\frac{1-x_n}{2}) = 0 \to 0$, $S(z) = 0$. Therefore, (S, T) is compatible of type (E) but the pair (S, T) is neither compatible nor compatible of type (A) (compatible of type (C), compatible of type (P)).

Example 3.16. Let $X = [0, 1]$, with usual metric $d(x, y) = |x - y|$. We define self-maps S and T as $Sx = Tx = \frac{1}{2}$, for $x \in [0, \frac{1}{2})Sx = Tx = \frac{2}{3}$ for $x = \frac{1}{2}$, and $Sx = 1 - x, Tx = x$ for $x \in (\frac{1}{2}, 1]$.

Consider a sequence $\{x_n\}$ in X such that $x_n \to \frac{1}{2}$, $x_n > \frac{1}{2}$ for all n. Then, we have $Sx_n = 1 - x_n \to \frac{1}{2} = z$, and $Tx_n = x_n \to \frac{1}{2} = z$. Since, $(1 - x_n) < \frac{1}{2}$ for all n. We have $SSx_n = S(1 - x_n) = \frac{1}{2} \to \frac{1}{2}$, $STx_n = S(x_n) = 1 - x_n \to \frac{1}{2}$, and $SSx_n = S(x_n) = x_n \to \frac{1}{2}$, $TSx_n = T(1 - x_n) = \frac{1}{2} \to \frac{1}{2}$.
Also, we have $S(z) = \frac{2}{3} = T(z)$, but $ST(z) = ST(\frac{1}{2}) = S(\frac{2}{3}) = \frac{1}{3}$,$TS(z) = TS(\frac{1}{2}) = T(\frac{2}{3}) = \frac{2}{3}$.
However, $\frac{1}{3} = ST(z) = 6 = TS(z) = \frac{2}{3}$, where $z = \frac{1}{2}$. Therefore, (S, T) is compatible (compatible of type (A), compatible of type (B), compatible of type (C), compatible of type (P); but the maps are not compatible of type (E). Moreover, it has to be noted that the maps are not commuting at the coincidence point.

Interrelationship 3.17. *[37] Compatible of type* $(E) \implies$ *both* $T-$ *and* $S-$ *compatible of type* (E)*, however* $S-$ *or* $T-$*compatible of type* (E) *do not imply compatible of type* (E)*.*

Example 3.18. Let $X = [0, 1]$, with usual metric $d(x, y) = |x - y|$. We define self-maps S and T as $S(x) = 1, T(x) = \frac{1}{5}$, for $x \in [0, \frac{1}{2}] - \{\frac{1}{4}\} Sx = 0, T(x) = 1$ for $x = \frac{1}{4}$, and $Sx = \frac{1-x}{2}, Tx = \frac{x}{2}$ for $x \in (\frac{1}{2}, 1]$. Since S ans T are not continuous at $x = \frac{1}{2}, \frac{1}{4}$, suppose that $x_n \to \frac{1}{2}, x_n > \frac{1}{2}$, for all n. Then, we have $Sx_n = \frac{1-x_n}{2} \to \frac{1}{4} = z$, $Tx_n = \frac{x_n}{2} \to \frac{1}{4} = z$. Consequently, we have $SSx_n = S(\frac{1-x_n}{2}) = 1 \to 1$, $STx_n = S(\frac{x_n}{2}) = 1 \to 1, T(z) = 1$, and $TTx_n = T(\frac{x_n}{2}) = \frac{1}{5} \to \frac{1}{5}$, $TSx_n = T(\frac{1-x_n}{2}) = \frac{1}{5} \to \frac{1}{5}, S(z) = 0..$ Therefore, (S, T) is $S-$compatible of type (E) but not compatible of type (E).

Interrelationship 3.19. *[8] Weakly compatible is occasionally weakly compatible but the converse is not true.*

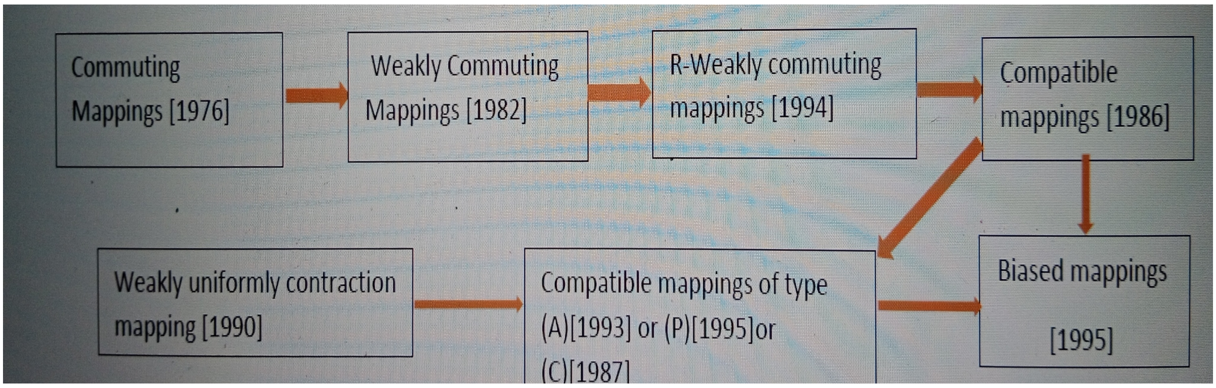
Example 3.20. Let $\mathbb{R}$ be a usual metric space and define two self-mappings S and T by $S(x) = 3x$, and $T(x) = X^2$ for all $x \in \mathbb{R}$. We see here that $S(x) = T(x)$ for $0, 3$, and $ST0 = TS0$ but $ST3 \neq TS3$. So, S and T are are not weakly compatible but occasionally weakly compatible.

Interrelationship 3.21. *[20] If* S *and* T *are compatible, then they are both* S*-biased and* T*-biased but converse is false.*

Example 3.22. Let $X = [0, 1]$, with usual metric $d(x, y) = |x - y|$. We define self-maps S and T as $S(x) = 1 - 2x, T(x) = 2x$ for $x \in [0, \frac{1}{2}]$, and $Sx = 0, Tx = 1$ for $x \in (\frac{1}{2}, 1]$. Then, S and T are both S-biased and T-biased but not compatible.

3.2 Interrelation II

Here, we show interrelationship through arrow how non-commuting mappings are connected to each other with or without continuity of mappings:



4 Conclusion

This article discusses on comparative study of compatible mappings in metric space, and describes various variant topological properties. Here, explaining their interrelation through examples and also show it through a chart. It helps researchers for comparative studies and to solve many related open problems in this domain.

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