Extended Kumaraswamy Exponential Distribution with Application to COVID-19 Data set

Arun Kumar Chaudhary[1,*], Lal Babu Sah Telee[1], Vijay Kumar[2]

[1]Department of Management Science, Nepal Commerce Campus, Kathmandu, Nepal
[2]Department of Mathematics and Statistics, Deen Dayal Upadhyaya Gorakhpur University, Gorakhpur, India

*Correspondence to: Arun Kumar Chaudhary, Email: akchaudhary1@yahoo.com

Abstract: *There are many probability models describing the time related events data. In this study, the exponential distribution is modified by adding one more parameter to get more flexible probability model called Extended Kumaraswamy Exponential (EKwE) distribution using the New Kw-G family (NKwG) of distributions. We have studied some of the statistical characteristics of the model, such as its reliability function, hazard rate function, and quantile function. For testing the applicability of the model, a real data set based on COVID-19 data is taken. The Cramer-von Mises (CVM) approach, Least Square Estimation (LSE), and Maximum Likelihood Estimation (MLE) are used to estimate the model's parameters. Validity of the model is checked by using P-P plot and Q-Q plot. Akaike Information Criterion (AIC), Corrected Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC) and Hannan-Quinn Information Criterion (HQIC) are also used for model comparison. Goodness of fit of the proposed model is tested using Kolmogrov-Smirnov (KS), Cramer-Von Mises (CVM) and Anderson-Darling (An) test statistics along with respective p-values. All the analysis of the study is performed by using R programming.*

Keywords: Family of distribution, Information criteria, Maximum likelihood, R programming, Test statistics

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1 Introduction

There are different probability models in existing literature that describes the time related data. In recent time, we can find some data that can be explained and analyzed adequately using the classical probability models. One of the important aspects of the research is to explore the existing knowledge as well as getting some new potentiality of the data. New probability models become essential for analyzing the recent data more precisely. In literature, we can get various methods of formulating new probability model that explains data with more precise results. Addition of some extra parameters to the existing probability model, modifying the existing probability models, inversing the variables used in models and using some family of probability models are the main techniques of defining new probability models. Modification of exponential distribution was done by Kus [12] to get a new lifetime model having decreasing hazard function. According to Barreto-Souza and Cribari-Neto [5], a new lifetime distribution is generalized by adding a power parameter creating a new distribution known as "A generalization of the Exponential-Poisson distribution". There are various exponentiated models such as the exponentiated generalized class of distributions by Cordeiro and Ortega [9], exponentiated Weibull distribution by Nadarajah et al. [15] and exponentiated distributions by Al-Hussaini and Ahsanullah [2] etc. Telee et al. [23] introduced exponentiated generalized exponential geometric distribution using Beta Exponential family by Alzaatreh et al. [3]. Chaudhary and Kumar [7] introduced the logistic NHE distribution using the extension of exponential distribution by Nadarajah and Haghighi [16]. Chaudhary and Kumar [8] have also developed new model called half-Cauchy modified exponential distribution using half-Cauchy family of distribution by Ristić and Balakrishnan [19]. There are many modified distributions. Weibull distribution [10] was modified to introduce modified Weibull distribution by Sarhan and Zaindin [20]. Modified inverse Rayleigh distribution by Khan [11] is the modified form of the Weibull distribution [10].

In this study, we have used new Kumaraswamy generalized family of distributions by Tahir et al. [21] to introduce the new probability model called extended Kumaraswamy exponential (EKwE) distribution. Proposed model has two parameters defined on single continuous variable. The cumulative distribution function of the new Kw-G family (NKwG) is given by equation (1)

$$F(x;\eta,\beta,\theta) = 1 - \left(1 - \left(1 - \bar{G}(x;\eta)^{G(x;\eta)}\right)^{\beta}\right)^{\theta}; x \geq 0, (\eta,\beta,\theta) \geq 0. \quad (1)$$

We have set a parameter $\theta = 1$ to get the special case of new Kw-G family (NKwG) as equation(2)

$$F(x;\eta,\beta) = \left(1 - \bar{G}(x;\eta)^{G(x;\eta)}\right)^{\beta}; x \geq 0, (\eta,\beta) \geq 0. \quad (2)$$

The cumulative distribution function of the exponential distribution which is taken as the base function is given by(3)

$$G(x;\lambda) = 1 - e^{-\lambda x}; x \geq 0, \lambda > 0, \quad (3)$$

and

$$\bar{G}(x;\lambda) = e^{-\lambda x}; x \geq 0, \lambda > 0.$$

Substituting the $G(x;\lambda)$ and $\bar{G}(x,\lambda)$ in eq. (2), we get the CDF of the proposed model EKwE given by expression(4)

$$F(x;\lambda,\beta) = \left(1 - \left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)}\right)^{\beta}; x \geq 0, (\lambda,\beta) > 0. \quad (4)$$

2 Model Analysis

The probability density function (pdf) of the proposed model EKwE is given by expression(5) as

$$f(x;\lambda,\beta) = \lambda\beta\left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)}\left(1 - \left(e^{-\lambda x}\right) + \lambda x\left(e^{-\lambda x}\right)\right)\left(1 - \left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)}\right)^{\beta-1}; x \geq 0, (\lambda,\beta) > 0. \quad (5)$$

The probability density curves for some values of the parameters are displayed here. It is clear that the probability curve is flexible depending upon the values of the parameter indicating that the distribution will fit different set of data adequately.

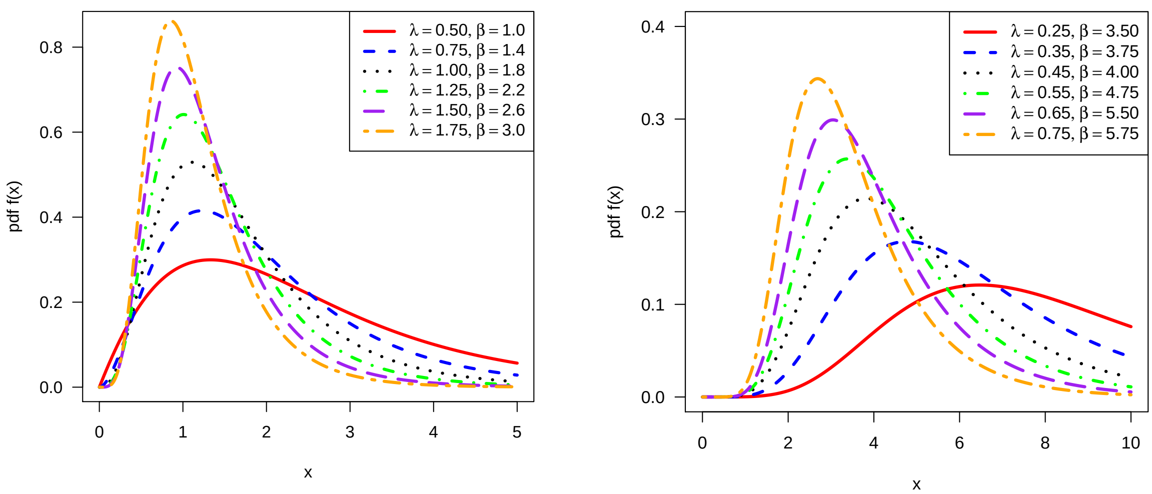

Figure 1: Probability density function

The Figure 1 represent probability density curves of EKwE.
Some of the properties of the model is mentioned below

2.1 Survival function (S(x))

The survival function(6) of the probability model is complementary of the cdf and is given by,

$$S(x) = 1 - F(x; \lambda, \beta) = 1 - \left(1 - \left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)}\right)^{\beta}; x \geq 0, (\lambda, \beta) > 0. \tag{6}$$

2.2 Hazard rate function (h(x))

Hazard rate function (7)of the proposed model is defined as

$$h(x) = \lambda\beta\left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)} \left(1 - \left(e^{-\lambda x}\right) + \lambda x \left(e^{-\lambda x}\right)\right) \left(1 - \left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)}\right)^{\beta-1} \left(1 - \left(1 - \left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)}\right)^{\beta}\right)^{-1} \tag{7}$$

The failure rate curves of the proposed model for numerous values of parameters are exhibited in figure [2]. It is found that the hazard curve is of different shape depending upon the values of parameters. The curve is increasing-decreasing, and inverted bathtub shaped.

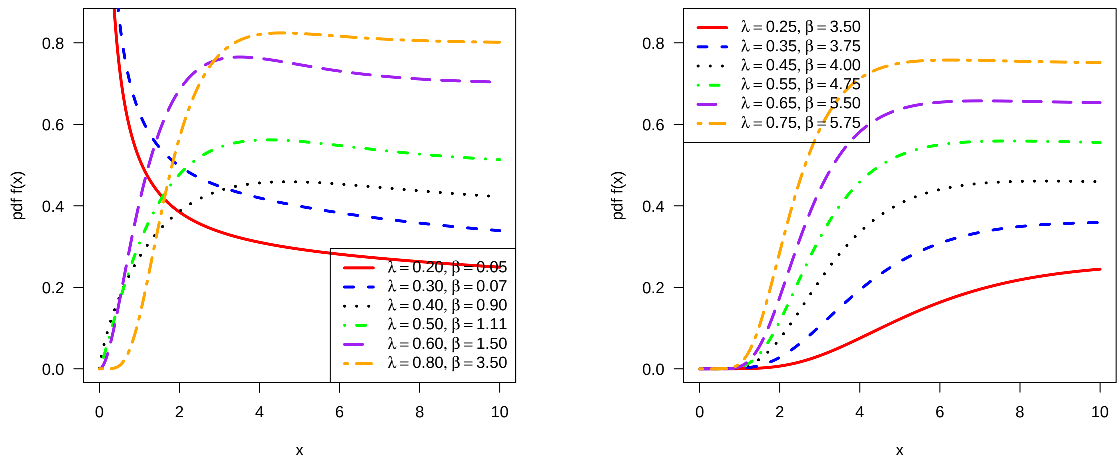

Figure 2: Hazard rate function

The Figure 2 represent hazard rate curves of EKwE.

2.3 Reverse hazard function

We can define reverse hazard function as(8)

$$h_{rev}(x) = \beta\left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)} \left(1 - \left(e^{-\lambda x}\right) + \lambda x \left(e^{-\lambda x}\right)\right) \left(1 - \left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)}\right)^{-1} ;x > 0, (\lambda, \beta) > 0. \tag{8}$$

2.4 Cumulative hazard rate function (H(x))

The cumulative hazard rate function H(x) is given by expression(9) as

$$H(x) = -\ln S(x) = -\ln\left[1 - \left(1 - \left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)}\right)^{\beta}\right]. \tag{9}$$

2.5 Quantile function

Quantile function of the model is an alternative of the distribution function that helps more study different characteristic such as central tendency, dispersion and moments etc. Quantile function of the model is given by equation(10)

$$\log(1-p^{1/\beta})+\lambda x(1-e^{-\lambda x})=0;\ 0\le p\le 1. \tag{10}$$

2.6 Asymptotic properties of the Model

Asymptotic properties of the density function can be found by verifying that $\lim_{x\to 0} f(x)=\lim_{x\to\infty} f(x)$. If model satisfies the asymptotic properties, then mode of the model will exist. Taking limiting at end points

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\lambda\beta\left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)}\left(1-\left(e^{-\lambda x}\right)+\lambda x\left(e^{-\lambda x}\right)\right)\left(1-\left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)}\right)^{\beta-1}=0; \tag{11}$$

$$\lim_{x\to \infty} f(x)=\lim_{x\to \infty}\lambda\beta\left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)}\left(1-\left(e^{-\lambda x}\right)+\lambda x\left(e^{-\lambda x}\right)\right)\left(1-\left(e^{-\lambda x}\right)^{\left(1-e^{-\lambda x}\right)}\right)^{\beta-1}=0. \tag{12}$$

Here, $\lim_{x\to 0} f(x)=\lim_{x\to\infty} f(x)$ so modal value of the proposed model will exist.

2.7 Skewness and kurtosis

Skewness describes about the consistency of the data. Here we have used Bowley's coefficient of skewness by Al-saiary et al. [1] based on quantiles as

$$\text{SK (B)} = \frac{\text{Q (0.75) +Q (0.25) -2*Q (0.50)}}{\text{Q(0.75)-Q(0.25)}}.$$

Coefficient of Octiles Kurtosis by Moors [14] and Al-saiary et al. [1] can be calculated using relation

$$\text{K}_\text{u}=\frac{\text{Q (0.375)-Q(0.625)-Q (0.125)+Q (0.875)}}{\text{Q (0.75)-Q(0.25)}}.$$

3 Methods of Parameters Estimation

Parameters can be estimated applying different methods. We have applied following methods.

3.1 Methods of Maximum Likelihood Estimation (MLE)

We define the log likelihood function for the proposed model in equation (13). Let $\underset{-}{x} = (x_1,\ldots,x_n)$ be a random sample of size n from EKwE then the log likelihood function can be written as

$$\ell(\lambda,\beta|\underline{x})=n\log(\lambda\beta)+(\beta-1)\sum_{i=1}^{n}\log\left[1-\left(e^{-\lambda x_i}\right)^{\left(1-e^{-\lambda x_i}\right)}\right]$$

$$-\lambda\sum_{i=1}^{n}x_i\left(1-e^{-\lambda x_i}\right)+\sum_{i=1}^{n}\log\left(1-e^{-\lambda x_i}+\lambda x_i e^{-\lambda x_i}\right). \tag{13}$$

After differentiating (13) with respect to λ and β, we can get the first order and second order partial derivatives of log likelihood function as

$$\frac{\partial\ell}{\partial\lambda}=(\beta-1)\sum_{i=1}^{n}x_i e^{-\lambda x_i}\left(1-e^{-\lambda x_i}\right)\left(1-e^{-\lambda x_i}+x_i e^{-\lambda x_i}\right)\left[1-\left(e^{-\lambda x_i}\right)^{\left(1-e^{-\lambda x_i}\right)}\right]^{-1}$$

$$+\frac{n}{\lambda}-\sum_{i=1}^{n}x_i\left(1-e^{-\lambda x_i}+\lambda x_i e^{-\lambda x_i}\right)+x_i e^{-\lambda x_i}\left(2-x_i\right)\left(1-e^{-\lambda x_i}+\lambda x_i e^{-\lambda x_i}\right)^{-1}; \tag{14}$$

$$\frac{\partial \ell}{\partial \beta}=\frac{n}{\beta}+\sum_{i=1}^{n}\log\left[1-\left(e^{-\lambda x_i}\right)^{\left(1-e^{-\lambda x_i}\right)}\right]. \tag{15}$$

Solving above first order derivatives setting to zero, parameters of the proposed model can be estimated. Solution of above equation is not possible so computer programming can be used. Let $\hat{\Theta}=(\hat{\lambda},\hat{\beta})$ and $\Theta=(\lambda,\beta)$, are estimated constants and parameter vector respectively then resulting asymptotic normality will be $\left(\hat{\Theta}-\Theta\right)\rightarrow N_3\left[0,(I(\Theta))^{-1}\right]$.
The Fisher's information matrix $I(\Theta)$ can be given by,

$$I(\Theta)=-\begin{bmatrix}E\left(\frac{\partial^2 l}{\partial\lambda^2}\right) & E\left(\frac{\partial^2 l}{\partial\lambda\partial\beta}\right)\\ E\left(\frac{\partial^2 l}{\partial\beta.\partial\lambda}\right) & E\left(\frac{\partial^2 l}{\partial\beta^2}\right)\end{bmatrix}.$$

Asymptotic variance $(I(\Theta))^{-1}$of MLE is worthless because Θ cannot be obtained. Let $O\widehat{(\Theta)}$ be the observed fisher information matrix. Estimate $O(\hat{\Theta})$ of $I(\Theta)$ Hessian matrix H can be obtained as

$$O\widehat{(\hat{\Theta})}=-\begin{bmatrix}\left(\frac{\partial^2 l}{\partial\lambda^2}\right) & \left(\frac{\partial^2 l}{\partial\lambda\partial\beta}\right)\\ \left(\frac{\partial^2 l}{\partial\beta.\partial\lambda}\right) & \left(\frac{\partial^2 l}{\partial\beta^2}\right)\end{bmatrix}=-H(\Theta)_{|_{(\Theta=\hat{\Theta})}}. \tag{16}$$

Variance covariance matrix is

$$\left[-H(\Theta)_{|_{(\Theta=\hat{\Theta})}}\right]^{-1}=\begin{bmatrix}Var(\hat{\lambda}) & Cov.(\hat{\lambda},\hat{\beta})\\ Cov(\hat{\beta},\hat{\lambda}) & Var(\hat{\beta})\end{bmatrix}. \tag{17}$$

Here, 100(1-γ)% C. I. for λ and β are

$$\hat{\lambda}\pm Z_{\gamma/2}\sqrt{Var(\hat{\lambda})}\&\hat{\beta}\pm Z_{\gamma/2}\sqrt{Var(\hat{\beta})}$$

3.2 Method of Least Square (LSE)

Let $X_{(1)}<X_{(2)}<\ldots<X_{(n)}$ is ordered random variables and a random sample $\{X_1,X_2,\ldots,X_n\}$ of size n is taken from a distribution function $F(\cdot)$. We define a function A using $F(X_{(i)})$ as CDF of ordered statistics by equation (18)

$$A(x;\lambda,\beta)=\sum_{i=1}^{n}\left[F(X_{(i)})-\frac{i}{n+1}\right]^2=\sum_{i=1}^{n}\left[\left\{1-\left(e^{-\lambda x_{(i)}}\right)^{\left(1-e^{-\lambda x_{(i)}}\right)}\right\}^{\beta}-\frac{i}{n+1}\right]^2. \tag{18}$$

Minimizing the function (18), the parameters of proposed model EKwE can be obtained. For minimization of (18), getting partial derivatives of A with respect to parameters as

$$\frac{\partial A}{\partial\lambda}=2\beta\sum_{i=1}^{n}x_{(i)}\left[1-\left(e^{-\lambda x_{(i)}}\right)^{\left(1-e^{-\lambda x_{(i)}}\right)}\right]^{\beta-1}\left(e^{-\lambda x_{(i)}}\right)^{\left(1-e^{-\lambda x_{(i)}}\right)}$$
$$\left(1-e^{-\lambda x_{(i)}}+x_{(i)}e^{-\lambda x_{(i)}}\right)\left[F(X_{(i)})-\frac{i}{n+1}\right];$$

$$\frac{\partial A}{\partial\beta}=2\sum_{i=1}^{n}\left[1-\left(e^{-\lambda x_{(i)}}\right)^{\left(1-e^{-\lambda x_{(i)}}\right)}\right]^{\beta}\log\left[1-\left(e^{-\lambda x_{(i)}}\right)^{\left(1-e^{-\lambda x_{(i)}}\right)}\right]\left[F(X_{(i)})-\frac{i}{n+1}\right].$$

Parameters can be also obtained by weighted LSE minimizing the function D in (19)

$$D(X;\lambda,\beta)=\sum_{i=1}^{n}w_i\left[F(X_{(i)})-\frac{i}{n+1}\right]^2=\sum_{i=1}^{n}w_i\left[\left\{1-\left(e^{-\lambda x_{(i)}}\right)^{\left(1-e^{-\lambda x_{(i)}}\right)}\right\}^{\beta}-\frac{i}{n+1}\right]^2, \tag{19}$$

where

$$w_i=\frac{1}{Var(X_{(i)})}=\frac{(n+1)^2(n+2)}{i(n-i+1)}.$$

Using the CDF of the order statistics and weight w_i in above expression and by differentiating (20) with respect to λ and β, we can get weighted least square estimates

$$D\left(X;\lambda,\beta\right)=\sum_{i=1}^{n}\frac{\left(n+1\right)^{2}\left(n+2\right)}{i\left(n-i+1\right)}\left[\left\{1-\left(e^{-\lambda x_{(i)}}\right)^{\left(1-e^{-\lambda x_{(i)}}\right)}\right\}^{\beta}-\frac{i}{n+1}\right]^{2}. \quad (20)$$

3.3 Cramers-Von Mises method of estimation

Using this method, parameters λ and β can be estimated by minimizing the function (21)

$$Z\left(X;\lambda,\beta\right)=\frac{1}{12n}+\sum_{i=1}^{n}\left[F\left(x_{i:n}|\lambda,\beta\right)-\frac{2i-1}{2n}\right]^{2}=\frac{1}{12n}+\sum_{i=1}^{n}\left[\left\{1-\left(e^{-\lambda x_{(i)}}\right)^{\left(1-e^{-\lambda x_{(i)}}\right)}\right\}^{\beta}-\frac{2i-1}{2n}\right]^{2}. \quad (21)$$

Differentiating (21) with respect to λ and β, we can get the first and second order partial derivatives of function Z as

$$\frac{\partial Z}{\partial\lambda}=2\beta\sum_{i=1}^{n}x_{(i)}\left[1-\left(e^{-\lambda x_{(i)}}\right)^{\left(1-e^{-\lambda x_{(i)}}\right)}\right]^{\beta-1}\left(e^{-\lambda x_{(i)}}\right)^{\left(1-e^{-\lambda x_{(i)}}\right)}$$
$$\left(1-e^{-\lambda x_{(i)}}+x_{(i)}e^{-\lambda x_{(i)}}\right)\left[F(X_{(i)})-\frac{2i-1}{2n}\right]; \quad (22)$$

$$\frac{\partial Z}{\partial\beta}=2\sum_{i=1}^{n}\left[1-\left(e^{-\lambda x_{(i)}}\right)^{\left(1-e^{-\lambda x_{(i)}}\right)}\right]^{\beta}\log\left[1-\left(e^{-\lambda x_{(i)}}\right)^{\left(1-e^{-\lambda x_{(i)}}\right)}\right]\left[F(X_{(i)})-\frac{2i-1}{2n}\right]. \quad (23)$$

Solving $\frac{\partial Z}{\partial\lambda}=0$ and $\frac{\partial Z}{\partial\beta}=0$, CVM estimates can be obtained.

4 Estimation and Analysis

For testing the applicability of the proposed model, we have applied the model on a real data set. The data set consist of mortality rate of 106 patients during COVID-19 pandemic in Mexico during the period between March 4, 2020 to July 20, 2020 by Bantan et al. [4]. For simplicity, the rate is divided by five. The set is as follows:

1.7652, 1.2210, 1.8782, 2.9942, 2.0766, 1.4534, 2.6440, 3.2996, 2.3330, 1.2030, 2.1710, 1.2244, 1.3312, 0.6880, 1.1708, 2.1370, 2.0070, 1.0484, 0.8668, 1.0286, 1.5260, 2.9208, 1.5806, 1.2740, 0.7074, 1.2654, 0.9460, 0.6430, 1.8568, 2.5756, 1.7626, 2.0086, 1.4520, 1.1970, 1.2824, 0.6790, 0.8848, 1.9870, 1.5680, 1.9100, 0.6998, 0.7502, 1.3936, 0.6572, 2.0316, 1.6216, 1.3394, 1.4302, 1.3120, 0.4154, 0.7556, 0.5976, 0.6672, 1.3628, 1.5708, 1.6650, 1.7120, 0.6456, 1.4972, 1.3250, 1.2280, 0.9818, 0.9322, 1.0784, 2.4084, 1.7392, 0.3630, 0.6654,1.0812, 1.2364, 0.2082, 0.3600, 0.9898, 0.8178, 0.6718, 0.4140, 0.6596, 1.0634, 1.0884, 0.9114, 0.8584, 0.5000, 1.3070, 0.9296, 0.9394, 1.0918, 0.8240, 0.7884, 0.6438, 0.2804, 0.4876, 0.6514, 0.7264, 0.6466, 0.6054, 0.4704, 0.2410, 0.6436, 0.5852, 0.5202, 0.4130, 0.6058, 0.4116, 0.4652, 0.5052, 0.3846.

Figure[3] displays the boxplot and the TTT plot of the data taken in consideration. Boxplot shows that the data is positively skewed and non normal in nature. Similarly the nature of the TTT curve is concave indicating that there is increasing failure rate.

Table [1] shows the descriptive measures of the data illustrated. It is found from measure that the data set is positively skewed with non-normality.

Table 1: Descriptive statistics of the data.

Min	Q1	Median	Mean	Q3	Sd	Skewness	Kurtosis	Max
0.2082	0.6578	1.0559	1.1646	1.5188	0.6499868	0.9736882	3.667506	3.2996

Parameters of the model are estimated using MLE, LSE and CVME methods using *optim*() function of R software R Core Team 2022 [18] is used. In table [2], the estimated parameters and standard error of estimates (SE) are listed

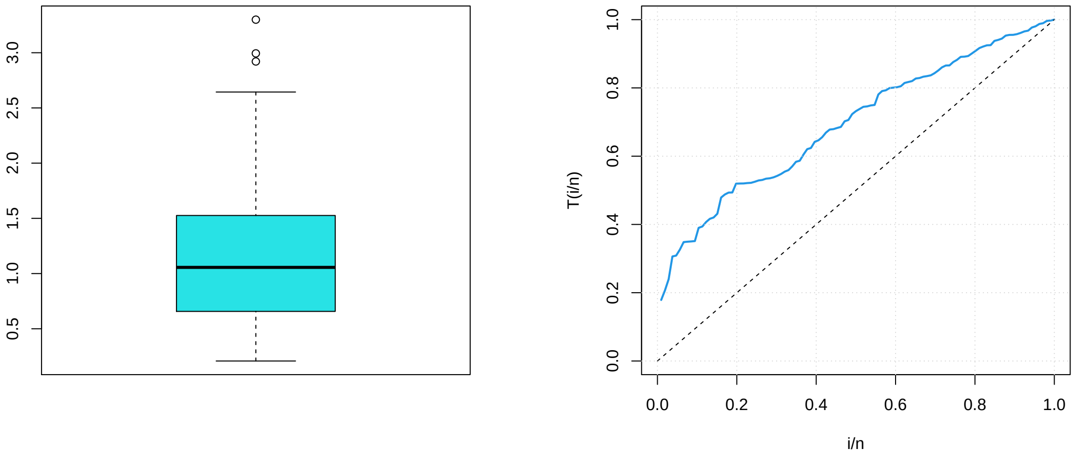

Figure 3: Box plot (Left) and TTT plot (Right).

Table 2: Estimated Parameters and SE.

Methods	Lambda	Beta
MLE	1.4987(0.1576)	1.9135(0.3502)
LSE	1.3266(0.6827)	1.5349(1.4204)
CVME	1.3503(0.6918)	1.5861(1.4801)

Figure [4] displays the histogram of the illustrated data versus the fitted density curve using MLE. It also contains the empirical cumulative distribution function (ecdf) versus the fitted distribution curve.
In addition, we have determined the Log likelihood values and various information criterion values, including the AIC, BIC, CAIC, and HQIC values, for parameters estimated using each of the three methods of estimation displayed in table[3].

Table 3: Log likelihood (LL), AIC, BIC, CAIC and HQIC.

Model	LL	AIC	BIC	CAIC	HQIC
MLE	-91.4946	186.9892	192.3161	187.1057	189.1482
LSE	-92.2444	188.4888	193.8157	188.6053	190.6478
CVM	-92.0404	188.0809	193.4078	188.1974	190.2399

For testing the validity of the model, we have also plotted the P-P plot and Q-Q plots of the proposed model and are displayed in figure [5].

Table[4] represents the test statistics values of goodness of fit using Kolmgorove-Smirnov (KS), Cramer–von Mises (W) and Anderson-Darling (A^2) along with respective p-values for different methods of estimations.

Table 4: KS, W, and (A^2) statistics with corresponding p-values.

Methods	KS(p-value)	W(p-value)	A^2(p-value)	CAIC	HQIC
MLE	0.0648(0.7648)	0.0627(0.7982)	0.3428(0.9027)	187.1057	189.1482
LSE	0.0513(0.9429)	0.0422(0.9221)	0.3359(0.9088)	188.6053	190.6478
CVME	0.0539(0.9178)	0.0416(0.9256)	0.3088(0.9313)	188.1974	190.2399

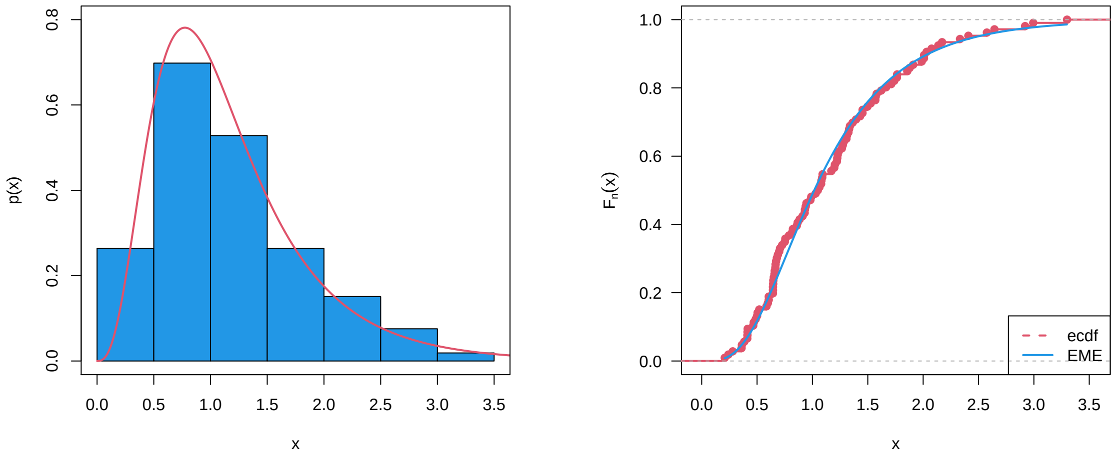

Figure 4: Histogram vs Pdf (Left) and Emperical CDF vs ECDF (Right).

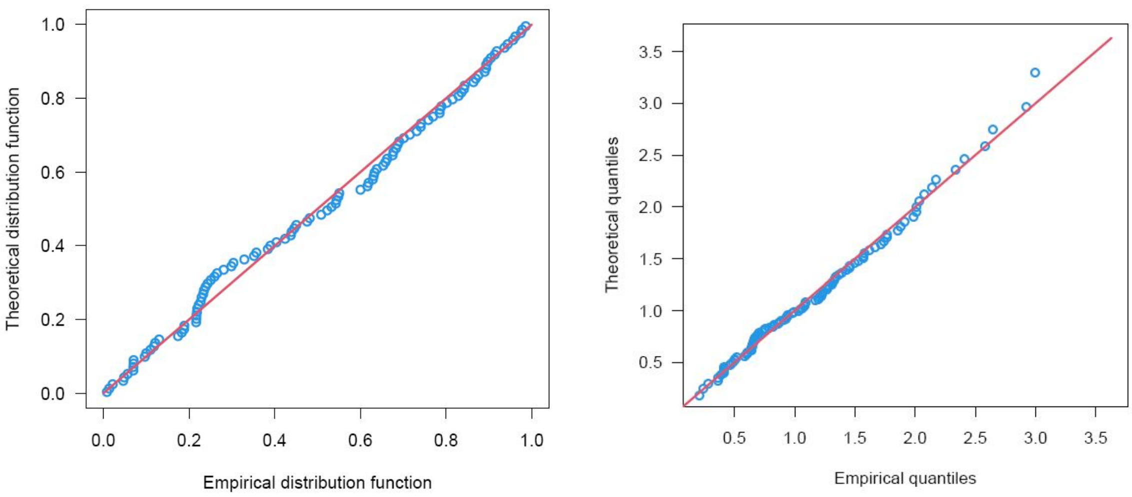

Figure 5: P-P plot (left) and Q-Q plot (Right) of EKwE.

For model comparison, we have considered five already published probability models. The models considered are Odd Lomax Exponential (OLE) distribution by Ogunsanya et al. [17], Logistic Inverse Exponential (LIE) Distribution by Chaudhary and Kumar [6], Lindely Generalized Inverted Exponential(LGIE) distribution by Telee and Kumar [24], Weibull Extension (WE) distribution by Tang et al. [22] and Modified Weibull (MW) distribution by Lai et al. [13]. Parameters of the considered models are estimated using MLE and are tabulated in table [5]. This table also represents the SE for all the distributions considered. Table [6] contains the LL, AIC, BIC, CAIC and HQIC for EKwE along with considered models. Since the information criteria values for proposed model is less than the considered model indicating that the data fits proposed model better compared to the considered models.

Table 5: Estimated parameters and their SE for EKwE along with considered models.

Model	Alpha	Beta	Theta	Lambda	HQIC
EKwE	-	1.9135(0.3502)	-	1.4987(0.1576)	189.1482
OLE	0.1479(0.0603)	0.0119(0.0126)	-	0.1059(0.0355)	190.6478
LGIE	7.7120(7.1960)	-	0.6487(0.6326)	1.4727(0.2671)	190.2399
WE	20.2560(54.2696)	1.9589(0.2467)	-	10.3291(32.3227)	
MW	0.5718(0.1698)	1.8937(0.3440)	-	0.0225(0.2355)	
LIE	2.0429(2.0429)	-	-	0.6717(0.6717)	

Table 6: Log likelihood (LL), AIC, BIC, CAIC, and HQIC.

Model	LL	AIC	BIC	CAIC	HQIC
EKwE	-91.4946	186.9892	192.3161	187.1057	189.1482
OLE	-92.4894	190.9788	198.9691	191.2141	194.2173
LGIE	-93.1517	192.3033	200.2937	192.5386	195.5419
WE	-93.7893	193.5786	201.5689	193.8139	196.8171
MW	-93.8558	193.7115	201.7018	193.9468	196.9500
LIE	-96.3881	196.7763	202.1032	196.8928	198.9353

5 Conclusion

In this study, we have presented a two-parameter continuous probability model called extended Kumaraswamy exponential (EKwE) distribution using new Kumaraswamy generalized family of distribution. Parameters of the model are estimated using Maximum Likelihood, Cramer–von Mises and Least Square Methods. Different statistical properties like survival, hazard, and quantile functions etc. of the novel model are analyzed. For testing the applicability testing of the model, a real data set based on COVID-19 data is considered. To find the nature of the data, boxplot and TTT plot are plotted. We have also mentioned the exploratory measure of the data. For model validation, different curve like *P*-*P* plot and *Q*-*Q* plots are displayed. We have also analyzed Log Likelihood values, Akaike information criterion, Bayesian information criterion, Corrected Akaike information and Hannan-Quinn information values. For testing the goodness of fit of the model, Kolmogorov-Smirnov, Cramer-Von Mises and Anderson Darling test are used. For comparison of the proposed model, we have considered five other probability models. The suggested model performs better than other existing models when compared to various validation criteria.

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