Advection-Dispersion Equation for Concentrations of Pollutant and Dissolved Oxygen

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Abstract: *A coupled system of advection-dispersion equations based water pollution model is presented that incorporates different parameters. The major concerns of the research are to observe the concentrations of pollutant and dissolved oxygen in the river. One dimensional model is used to observe the concentrations by taking the dimension along the length of the river. Since the processes of pollution and aeration are ongoing, the investigation of steady states is made possible by taking into account the elimination of pollutants by aeration. In this model coupled advection-dispersion equations are solved by taking dispersion coefficient as zero and non-zero, respectively.*

Keywords: Concentration, Dispersion, Pollutant, Dissolved oxygen

DOI: https://doi.org/10.3126/jnms.v5i1.47375

Nomenclature

A	River's cross-sectional area (m^2).
X	Dissolved oxygen concentration ($kg.m^{-3}$).
D	Dispersion coefficient of pollutant in the x-direction ($m^2.day^{-1}$).
D_x	Dispersion coefficient of dissolved oxygen in the x-direction ($m^2.day^{-1}$).
S	Saturated oxygen concentration ($kg.m^{-3}$).
C	Pollutant concentration ($kg.m^{-3}$).
k	Concentration of half-saturated oxygen demand for pollutant degradation ($kg.m^{-3}$).
k_1	Pollutant degradation rate coefficient (day^{-1}).
k_2	Degradation rate coefficient for dissolved oxygen (day^{-1}).
L	Polluted river length (m).
q	Added pollutant rate along the river ($kg.m^{-1}$.day^{-1}).
α	Mass transfer of oxygen from air to water (m^2.day^{-1}).
t	Time (day).
u	Water velocity in the direction of x (m.day^{-1}).
x	Position (m).

1 Introduction

Water pollution is one of the main environmental issues that we are facing, as more than 70% of the earth's surface is covered by water. Water pollution takes place due to unfavourable substances come into water that replaces quality of water and which is dangerous to human health [1]. Drinking water is required to be safe for public health. Being a common solvent, water is a main source of many communicable diseases. The data of the World Health Organization (WHO) shows that 80% of the diseases and 3.1% of the deaths happen because of low water quality [2]. This study was inspired by the poor water quality of the Bagmati River in Kathmandu 1, Nepal. The main causes of pollution in the Bagmati River in Kathmandu, Nepal, include rapid urban growth, low levels of awareness, industrial waste, continual dumping of solid wastes, insufficient waste-water treatment facilities, and domestic sewage.

A mathematical model for river pollutiom was proposed by Pimpunchat et al. [13]. To study the impact

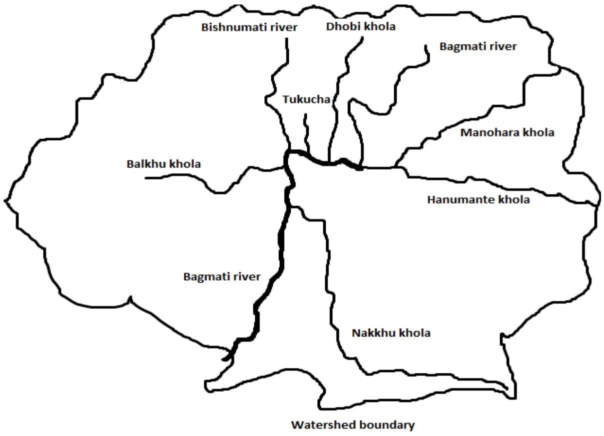

Figure 1: Map of Bagmati River and its tributaries (**Left**), and Polluted Bagmati River [22] (**Right**).

of aeration, they had employed a coupled advection-dispersion equation.

The analytical solution to the advection-dispersion equation of pollutant concentration was presented by Paudel et al. [11]. To solve this equation, they used the Laplace transformation method. Water pollution models in one and two dimensional formats were presented by Pochai et al. [14] and Tabuenca et al. [19], respectively. Van Genuchten and Alves [20] provided analytical solutions for a physical system in a semi-infinite domain with zero initial concentration. The advection-diffusion equation in semi-infinite media has a numerical solution that Savovic and Djordjevich [18] developed. Analytical solutions to the one-dimensional advection-dispersion equation have been provided by Kumar et al. [6].

Solutions to unsteady advection-dispersion equations were stated analytically by Wadi et al. [21]. These equations give a one-dimensional description of pollutant concentration. The Laplace transformation technique was used to arrive at the solutions. A model was put up by Marusic [9] to describe the dispersion of pollutants in river type systems. He was successful in describing how pollutant concentrations change over time. According to the study of Johari et al. [3], advection-dispersion equation was used to forecast concentration of pollutant transport. Salkuyeh [17] presented convection-diffusion equation with exact solution.

Manitcharoen and Pimpunchat [8] suggested unsteady state solutions for the advection-dispersion equations governing pollutant concentration. Both analytical and numerical solutions have been achieved using the Laplace transformation technique. Many methods have been developed in recent years to solve partial differential equations [4, 5, 15, 16].

A coupled system of advection-dispersion equations based water pollution model is presented that incorporates different parameters. One dimensional model is used to observe the concentrations by taking dimension along the length of river. Then, analytical solutions are constructed. Preliminary versions of these results were included in Paudel [10].

2 Mathematical Model

We consider coupled equations for pollutant $C(x,t)$ and dissolved oxygen $X(x,t)$ concentrations. When oxygen reacts with pollutant, coupling situation appears. To observe concentrations, one dimensional model is used. We take dimension along the lengh of river. So the concentrations $C(x,t)$ and $X(x,t)$ satisfy advection-dispersion equations. The coupled equations [8, 12] are expressed in one dimension as

$$\frac{\partial(AC)}{\partial t} = D\frac{\partial^2(AC)}{\partial x^2} - \frac{\partial(uAC)}{\partial x} - k_1\frac{X}{X+k}AC + qH(x); \quad 0 \le x < L, t > 0 \tag{1}$$

$$\frac{\partial(AX)}{\partial t} = D_x \frac{\partial^2(AX)}{\partial x^2} - \frac{\partial(uAX)}{\partial x} - k_2 \frac{X}{X+k} AC + \alpha(S - X); \quad 0 \le x < L, t > 0. \tag{2}$$

The Heaviside function $H(x)$ is represented in equation (1) by

$$H(x) = \begin{cases} 1, & \text{if} \quad 0 < x < L, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

In equation (1) and (2), u denotes water velocity in the direction of x, D denotes pollutant's dispersion coefficient in the same direction, D_x is the dissolved oxygen's dispersion coefficient in the same direction, S is the saturation oxygen concentration, k_1 is the pollutant's degradation rate coefficient, k_2 is the dissolved oxygen's degradation rate coefficient, the mass transfer of oxygen from air to water is represented by α, the additional pollutant rate along the river is represented by q, the half-saturated oxygen demand concentration for pollutant decay is represented by k, and the cross-section of the river is represented by A.

The parameters A, u, q, α and S are taken as constants [7, 21]. We take into account a river where pollutants are dumped along with contaminants. For dissolved oxygen, it is considered that the saturation concentration S less the concentration X, i.e., $\alpha(S - X)$, determines the rate of growth of concentration by movement from air into the river. Both the pollutant concentration C and the dissolved oxygen concentration X interact. We consider cases with and without dispersion, k negligible ($k \approx 0$) and k non-zero.

3 Steady State Analysis

Here, we consider various special cases given in figure 2. The model we used is steady-state model.

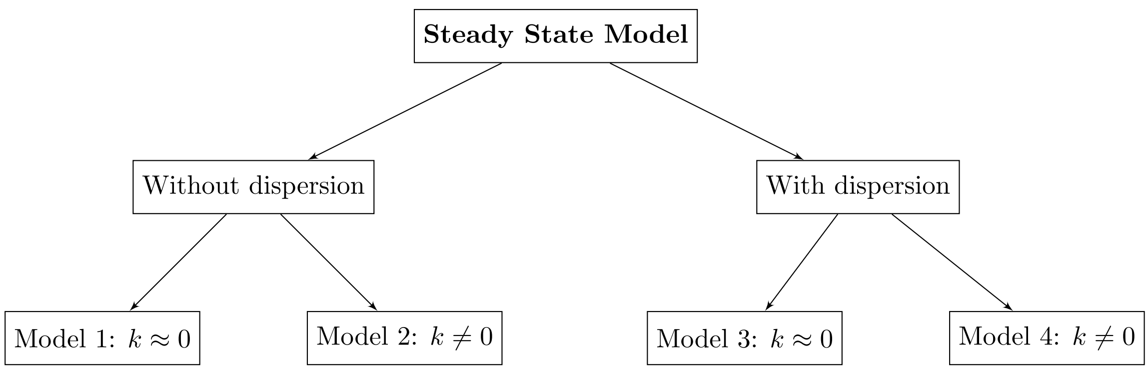

Figure 2: Special cases of model [12].

Model 1. We have no dispersion ($D = 0, D_x = 0$) in this model and k is negligible (*i.e.*, $k \approx 0$)[11].

$$\frac{d(uAC(x))}{dx} = -k_1 AC(x) + q; \quad (x > 0, t > 0) \tag{4}$$

$$\frac{d(uAX(x))}{dx} = -k_2 AC(x) + \alpha(S - X(x)); \quad (x > 0, t > 0) \tag{5}$$

We consider k negligible ($k \approx 0$). The boundary conditions are $C(0) = 0$ and $X(0) = S$.

From equation (4)

$$uA\frac{dC(x)}{dx} + k_1 AC(x) = q$$

$$\Rightarrow \frac{dC}{dx} + \left(\frac{k_1}{u}\right) C = \frac{q}{uA}$$

As the last differential equation is linear in C, we take the integrating factor

$$I.F. = \exp\left(\int \frac{k_1}{u} dx\right) = \exp\left(\frac{k_1 x}{u}\right).$$

The solution is

$$C \exp\left(\frac{k_1 x}{u}\right) = \frac{q}{uA}\int \exp\left(\frac{k_1 x}{u}\right) dx + c_1$$

$$\Rightarrow C \exp\left(\frac{k_1 x}{u}\right) = \frac{qu}{uk_1 A}\exp\left(\frac{k_1 x}{u}\right) + c_1$$

$$\Rightarrow C \exp\left(\frac{k_1 x}{u}\right) = \frac{q}{k_1 A}\exp\left(\frac{k_1 x}{u}\right) + c_1.$$

Using $C(0) = 0$,

$$c_1 = \frac{-q}{k_1 A}.$$

So, the pollutant concentration becomes

$$C(x) = \frac{q}{k_1 A}\left\{1 - \exp\left(\frac{-k_1 x}{u}\right)\right\}, \tag{6}$$

and the limit is given by

$$\lim_{x\to\infty} C(x) = \frac{q}{k_1 A}. \tag{7}$$

Figure 3 illustrates how C varies in the range $0 \leq x \leq 4$ described by equation (6). We consider parameters A, u, q, k_1 to be 1 [21] to test the model. Figure 3 shows that C increases as x increases. It reaches to maximum as $x \to \infty$. Generally concentration of pollutant increases as x increases.

From equation (5),

$$uA\frac{dX}{dx} = -k_2 AC + \alpha(S - X)$$

$$\Rightarrow \frac{dX}{dx} + \left(\frac{\alpha}{uA}\right) X = \left(\frac{\alpha S}{uA} - \frac{k_2 C}{u}\right)$$

Integrating factor

$$I.F. = \exp\left(\int \frac{\alpha}{uA} dx\right) = \exp\left(\frac{\alpha x}{uA}\right).$$

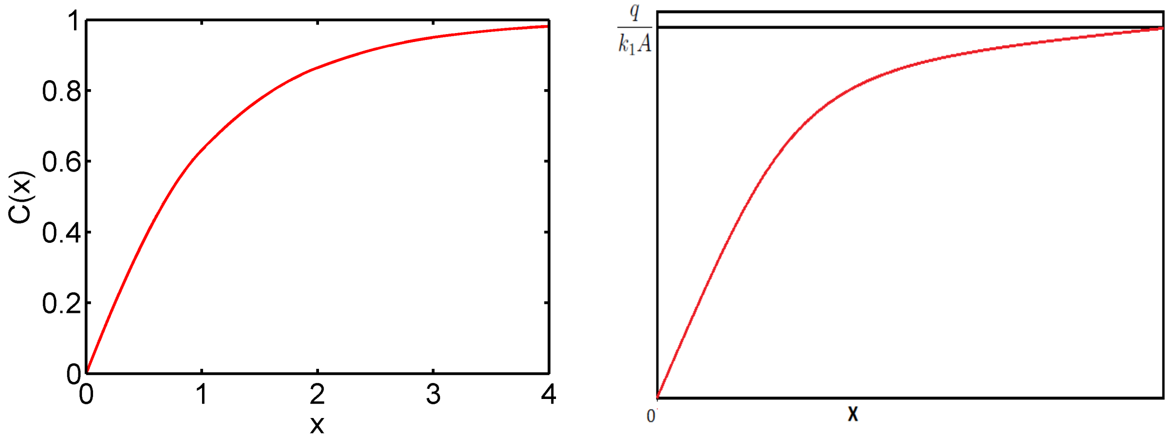

Figure 3: Solution for C without dispersion and $k \approx 0$.

The solution is

$$X \exp\left(\frac{\alpha x}{uA}\right) = \int \left(\frac{\alpha S}{uA} - \frac{k_2 C}{u}\right) \exp\left(\frac{\alpha x}{uA}\right) dx + c_2$$

$$\Rightarrow X \exp\left(\frac{\alpha x}{uA}\right) = \int \left[\frac{\alpha S}{uA} - \frac{k_2}{u}\frac{q}{k_1 A}\left\{1 - \exp\left(\frac{-k_1 x}{u}\right)\right\}\right] \exp\left(\frac{\alpha x}{uA}\right) dx + c_2$$

$$\Rightarrow X \exp\left(\frac{\alpha x}{uA}\right) = \int \left(\frac{\alpha S}{uA} - \frac{k_2 q}{uk_1 A}\right) \exp\left(\frac{\alpha x}{uA}\right) dx + \int \frac{k_2 q}{uk_1 A} \exp\left(\frac{\alpha - k_1 A}{uA} x\right) dx + c_2$$

$$\Rightarrow X \exp\left(\frac{\alpha x}{uA}\right) = \left(\frac{\alpha S}{uA} - \frac{k_2 q}{uk_1 A}\right) \frac{uA}{\alpha} \exp\left(\frac{\alpha x}{uA}\right) + \frac{k_2 q}{uk_1 A}\left(\frac{uA}{\alpha - k_1 A}\right) \exp\left(\frac{\alpha - k_1 A}{uA} x\right) + c_2$$

$$\Rightarrow X = S - \frac{k_2 q}{k_1 \alpha} + \frac{k_2 q}{k_1(\alpha - k_1 A)} \exp\left(\frac{-k_1 x}{u}\right) + c_2 \exp\left(\frac{-\alpha x}{uA}\right).$$

Using $X(0) = S$,

$$c_2 = \frac{k_2 q}{k_1 \alpha} - \frac{k_2 q}{k_1(\alpha - k_1 A)} = -\frac{k_2 q A}{\alpha(\alpha - k_1 A)}$$

$$\Rightarrow X(x) = S - \frac{k_2 q}{\alpha k_1} + \frac{k_2 q}{k_1(\alpha - k_1 A)} \exp\left(\frac{-k_1 x}{u}\right) - \frac{k_2 q A}{\alpha(\alpha - k_1 A)} \exp\left(\frac{-\alpha x}{uA}\right). \tag{8}$$

Equation (8) gives the dissolved oxygen concentration. And at far downstream

$$\lim_{x \to \infty} X(x) = S - \frac{k_2 q}{\alpha k_1}. \tag{9}$$

This is shown in figure 4 which shows that oxygen level decreases due to reaction with pollutants. Now the solutions for concentrations of pollutant and dissolved oxygen are

$$\lim_{x \to \infty} (C(x), X(x)) = \left(\frac{q}{k_1 A}, S - \frac{k_2 q}{\alpha k_1}\right). \tag{10}$$

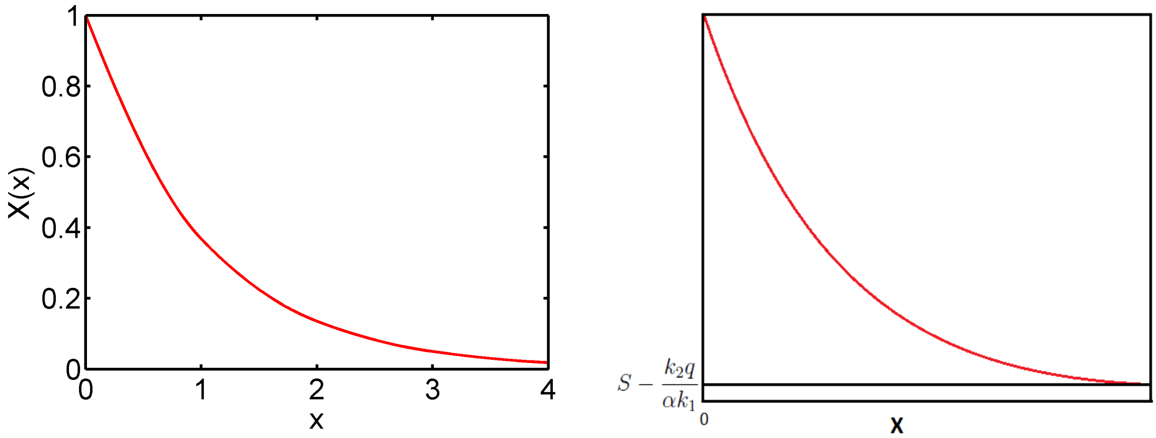

Figure 4: Solution for X without dispersion and $k \approx 0$

Model 2. For no dispersion ($D = 0, D_x = 0$) and k is non-zero ($i.e.,\ \ k \neq 0$), the model [12] is

$$\frac{d(uAC(x))}{dx} = -k_1 \frac{X(x)}{X(x)+k} AC(x) + q; \quad (x > 0, t > 0) \tag{11}$$

$$\frac{d(uAX(x))}{dx} = -k_2 \frac{X(x)}{X(x)+k} AC(x) + \alpha(S - X(x)); \quad (x > 0, t > 0). \tag{12}$$

Boundary conditions are $C(0) = 0$ and $X(0) = S$, which are same. The solutions are

$$\lim_{x \to \infty} (C(x), X(x)) = \left(\frac{q}{k_1 A} + \frac{\alpha k q}{k_2(\frac{\alpha k_1 S}{k_2} - q)}, S - \frac{k_2 q}{\alpha k_1} \right). \tag{13}$$

The solutions depend upon k and q whereas in the model 1, these solutions depend on q only. If $q \geq \alpha k_1 S/k_2$, the solution does not exist. Figure 5 shows the variation of C and X in the range $0 \leq x \leq 20$ as k varies. To test the model we suppose $A, S, u, k_1, k_2, \alpha$ to be 1 and q to be 1/4.

Model 3. We have dispersion terms $D \neq 0, D_x \neq 0$ and $k \approx 0$. Then, the model [12] is

$$D \frac{d^2(AC)}{dx^2} - \frac{d(uAC)}{dx} - k_1 \frac{X}{X+k} AC + qH(x) = 0; \quad (x > L, t > 0) \tag{14}$$

$$D_x \frac{d^2(AX)}{dx^2} - \frac{d(uAX)}{dx} - k_2 \frac{X}{X+k} AC + \alpha(S - X) = 0; \quad (x > L, t > 0). \tag{15}$$

In this model ($k \approx 0$), the equations (14) and (15) become

$$D \frac{d^2(AC)}{dx^2} - \frac{d(uAC)}{dx} - k_1 AC + q = 0 \tag{16}$$

and

$$D_x \frac{d^2(AX)}{dx^2} - \frac{d(uAX)}{dx} - k_2 AC + \alpha(S - X) = 0. \tag{17}$$

From equation (16)

$$DA \frac{d^2 C}{dx^2} - uA \frac{dC}{dx} - k_1 AC + q = 0$$

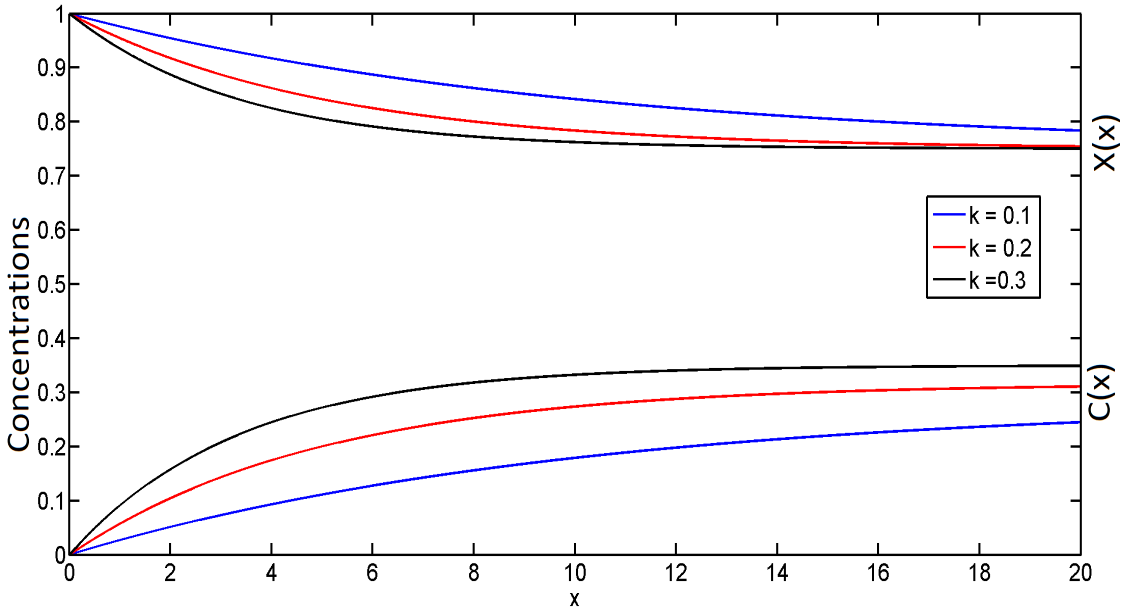

Figure 5: Solution for C and X with dispersion and $k \neq 0$.

$$\Rightarrow \frac{d^2C}{dx^2} - \frac{u}{D}\frac{dC}{dx} - \frac{k_1}{D}C = -\frac{q}{DA}$$

$$\Rightarrow \left(\tilde{D}^2 - \frac{u}{D}\tilde{D} - \frac{k_1}{D}\right)C = -\frac{q}{DA},$$

where $\tilde{D} = \frac{d}{dx}$, which is the second order differential equation.
Its auxiliary equation is

$$m^2 - \frac{u}{D}m - \frac{k_1}{D} = 0$$

$$\Rightarrow m = \frac{\frac{u}{D} \pm \sqrt{\frac{u^2}{D^2} - 4\left(-\frac{k_1}{D}\right)}}{2}$$

$$\Rightarrow m = \frac{u}{2D} \pm \frac{\sqrt{u^2 + 4Dk_1}}{2D}$$

$$\Rightarrow m = \delta \pm \beta,$$

where

$$\delta = \frac{u}{2D}$$

and

$$\beta = \frac{\sqrt{u^2 + 4Dk_1}}{2D}.$$

Therefore, the complementary function (C.F.) is

$$C.F. = c_1 e^{(\delta-\beta)x} + c_2 e^{(\delta+\beta)x}$$

and the particular integral (P.I.) is

$$P.I. = \frac{1}{\tilde{D}^2 - \frac{u}{D}\tilde{D} - \frac{k_1}{D}}\left(-\frac{q}{DA}\right)$$

$$\Rightarrow P.I. = \frac{q}{k_1 A}.$$

The general solution is

$$C(x) = C.F. + P.I.$$

$$\Rightarrow C(x) = c_1 e^{(\delta-\beta)x} + c_2 e^{(\delta+\beta)x} + \frac{q}{k_1 A}.$$

So, the pollutant concentration is

$$C(x) = \begin{cases} \frac{q}{k_1 A}\left[1 - (\frac{\delta+\beta}{2\beta})e^{(\delta-\beta)x}\right], & \text{if } x \geq 0, \\ \frac{q}{k_1 A}\left(\frac{\beta-\delta}{2\beta}\right)e^{(\delta+\beta)x}, & \text{if } x < 0. \end{cases} \tag{18}$$

We use the conditions $C(\infty) < \infty$ and $C(-\infty) < \infty$ [13]. Additionally, we demand $C'(x)$ and $C(x)$ to be continuous at $x = 0$. Since there are only scattered sources of pollution rather than point sources, $C(x)$ is continuous. It follows that $C'(x)$ is continuous because the dispersive flux $DC'(x) - uC(x)$ is continuous as well. From equation (17),

$$AD_x \frac{d^2(X)}{dx^2} - uA\frac{d(X)}{dx} - k_2 AC + (\alpha S - \alpha X) = 0$$

$$\Rightarrow \frac{d^2X}{dx^2} - \frac{u}{D_x}\frac{dX}{dx} - \frac{k_2}{D_x}C - \frac{\alpha}{AD_x}X + \frac{\alpha S}{AD_x} = 0$$

$$\Rightarrow \frac{d^2X}{dx^2} - \frac{u}{D_x}\frac{dX}{dx} - \frac{\alpha}{AD_x}X = \frac{k_2 C}{D_x} - \frac{\alpha S}{AD_x}$$

$$\Rightarrow \left(\tilde{D}^2 - \frac{u}{D_x}\tilde{D} - \frac{\alpha}{AD_x}\right)X = \left(\frac{k_2 AC - \alpha S}{AD_x}\right),$$

which is the second order differential equation.
Its auxiliary equation is

$$m^2 - \frac{u}{D_x}m - \frac{\alpha}{AD_x} = 0$$

$$\Rightarrow m = \frac{\frac{u}{D_x} \pm \sqrt{\frac{u^2}{D_x^2} - 4\left(-\frac{\alpha}{AD_x}\right)}}{2}$$

$$\Rightarrow m = \frac{u}{2D_x} \pm \frac{\sqrt{u^2 + \frac{4\alpha D_x}{A}}}{2D_x}$$

$$\Rightarrow m = \gamma \pm \eta,$$

where

$$\gamma = \frac{u}{2D_x}$$

and

$$\eta = \frac{\sqrt{u^2 + \frac{4\alpha D_x}{A}}}{2D_x}.$$

Therefore, the complementary function (C.F.) is

$$C.F. = c_3 e^{(\gamma-\eta)x} + c_4 e^{(\gamma+\eta)x}$$

and the particular integral (P.I.) is

$$P.I. = \frac{1}{\tilde{D}^2 - \frac{u}{D_x}\tilde{D} - \frac{\alpha}{AD_x}} \left(\frac{k_2 AC - \alpha S}{AD_x} \right),$$

where

$$C(x) = \begin{cases} \frac{q}{k_1 A}\left[1 - (\frac{\delta+\beta}{2\beta})e^{(\delta-\beta)x}\right], & \text{if } x \geq 0, \\ \frac{q}{k_1 A}\left(\frac{\beta-\delta}{2\beta}\right)e^{(\delta+\beta)x}, & \text{if } x < 0. \end{cases}$$

The general solution is

$$X(x) = C.F. + P.I.$$

$$X(x) = \begin{cases} S - \frac{k_2 q}{k_1 \alpha} + \frac{k_2 q}{k_1}\left[\left(\frac{\delta+\eta}{2\eta\alpha} - \frac{\delta+\beta}{4\beta\eta A^*} + \frac{\delta-\beta}{4\beta\eta B^*}\right)e^{(\gamma-\eta)x} - \frac{\delta+\beta}{2\beta A^*}xe^{(\delta-\beta)x}\right], & \text{if } x \geq 0, \\ S + \frac{k_2 q}{k_1}\left[\left(\frac{\delta-\eta}{2\eta\alpha} - \frac{\delta+\beta}{4\beta\eta A^*} + \frac{\delta-\beta}{4\beta\eta B^*}\right)e^{(\gamma+\eta)x} - \frac{\delta-\beta}{2\beta B^*}xe^{(\delta+\beta)x}\right], & \text{if } x < 0, \end{cases} \tag{19}$$

where

$$A^* = 2AD_x(\delta - \beta) - uA,$$

$$B^* = 2AD_x(\delta + \beta) - uA.$$

Equation (19) gives the dissolved oxygen concentration. Here, we used initial conditions $X(\infty) < \infty$ and $X(-\infty) = S$ [12].

Model 4. We have dispersion terms $D \neq 0, D_x \neq 0$ and $k \neq 0$ [13].

$$D\frac{d^2(AC)}{dx^2} - \frac{d(uAC)}{\partial x} - k_1\frac{X}{X+k}AC + qH(x) = 0; \quad 0 \leq x < L, t > 0 \tag{20}$$

$$D_x\frac{d^2(AX)}{dx^2} - \frac{d(uAX)}{dx} - k_2\frac{X}{X+k}AX + \alpha(S - X) = 0; \quad 0 \leq x < L, t > 0. \tag{21}$$

Boundary conditions are $C(-\infty) = 0$ and $X(-\infty) = S$.

Far downstream solution for this model is

$$\lim_{x\to\infty}(C(x), X(x)) = \left[\frac{q}{k_1 A}\left(1 + \frac{k}{X(\infty)}\right), S - \frac{k_2 q}{k_1 \alpha}\right]. \tag{22}$$

4 Conclusions

A coupled system of advection-dispersion equations based water pollution model is presented that incorporates different parameters. We have constructed analytical solutions for mathematical models. One dimensional model is used to observe the concentrations by taking dimension along the length of river. By considering the removal of pollutant by aeration, event of steady states is investigated. In this model, coupled advection-dispersion equations are solved by taking dispersion coefficient as zero and non-zero, respectively.

Acknowledgement

As a recipient of an M.Phil. fellowship (M.Phil. - $75/76 - S\&T - 9$), authors thank the University Grants Commission (UGC) of Nepal for its financial assistance.

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