Three Classical Methods to find the Cube Roots: A Connective Perspective on Lilavati, Vedic and Pande's Procedures

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Abstract: *South Asian region has made a glorious history of mathematics. This area is considered as fertile land for the birth of pioneer mathematicians who developed various mathematical ideas and creations. Among them, three innovative personalities are Bhaskaracarya, Gopal Pande and Bharati Krishna Tirthaji and their specific methods to find cube root are mainly focused on this study. The article is trying to explore the comparative study among the procedures they adopt.*

Gopal Pande disagrees with the Bhaskaracarya's verse. He used the unitary method against that method mentioned in Bhaskaracarya's famous book Lilavati to prove his procedures. However, the Vedic method by Tirthaji was not influenced by the other two except for minor cases. In the case of practicality and simplicity, the Vedic method is more practical and simpler to understand for all mathematical learners and teachers in comparison to the other two methods.

Keywords: Cube roots, Lilavati, Nepali method, Vedic mathematics

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1 Introduction

The history of the calculation of cube roots can be traced back to Babylonian civilization [1]. The calculations of cube roots also appeared in the Chinese mathematical system around the second century BCE [2]. Methods for extracting cube root was also devised by Greek mathematician Hero of Alexandria (10 AD – 70 AD) in the first century [3]. In the classical period of the South Asian region, important contributions were made by scholars like Pingala (around the first century), Aryabhata (476 – 550), Varahamihira (505 – 587), Brahmagupta (598 – 668), Bhaskara-I (around the seventh century), Lalla (720 – 790), Sridhara (870 – 930), Shreepati (1019 – 1066), Bhaskara-II (1114 – 1193) [4, 5, 10]. Among them, the method for determining the cube root of the number having many digits was firstly provided by the great Mathematician or Astronomer Aryabhata in Aryabhatiya [5, 6]. Aryabhata has mentioned in his book Ganitpaad a method to extract the cube root of any number. In the fifth sloka of Aryabhatiya which was mentioned in Sanskrit as [5].

अघनाद भजेद द्वितीयात त्रिगुणेन घनस्य मलवर्गेण ।

वर्ग स्त्रिपूर्व गुणितः शोध्यः प्रथमाद घनस्य घनात् ।।

(आर्यभटीयको गणितपाद)

(Aghanaada Bhajeda dwitiyata trigunena ghanasya malawargena

Warga stripurva gunitah shoddhyah prathamaad ghanasya ghanaat.)

(Aaryabhatiyako ganitpaad)

(The meaning of Aryabhata's and Brahmagupta's stanza (Sloka) are more or less similar to the Bhaskaracarya's stanza [5]. Therefore, common meaning is mentioned after Bhaskara's sloka)

After his time many mathematicians felt that it was complex to understand. Many great mathematicians followed the ideas in Mathematics, even though most of the ideas were likely similar but simplified than Aryabhata's method to find the cube roots. Their methods are closely related to, i.e., $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ (where x and y are variables that they used) Brahmhagupta expressed his method in this regard in his book Brahmsphutsiddhanta Ganitadhyaya in Sanskrit as [5].

छेदोडघनाद् द्वीतियाद् घनमूलकृतिस्त्रिसङ गुणाप्तकृतिः ।

शोध्या त्रिपूर्वगुणिता प्रथमाद् घनतो घनो मूलम् ।। ७ ।।

(Chhedodaghanaad dwitiyaad ghanamulakritistrisang gunaaptakriti

Shodhhya tripurvagunita prathamaad ghanato ghano mulam ||7||)

In the twelfth century, Bhaskara-II was popularly known as Bhaskaracarya. He wrote his Siddhantasiromani which contained four parts: Lilavati, Algebra, Planetary Motion and Astronomy. Before the time of Bhaskaracarya, many contributions were made by mathematicians in the field of Mathematics including finding the cube roots of the numbers. After the period of Bhaskaracarya , mathematician Narayan (1340-1457) slightly revised in his book Ganitkaumudi to the method of Bhaskaracarya [5, 7]. Other mathematicians Samanta Chandrasekhar (1835–1904) and Bapudev Shastri (1821–1900) differed somewhat from those in the Aryabhata's and Bhaskaracarya's method for extracting cube roots [8].

The Sanskrit stanzas specified by Brahmagupta, Aryabhata and Bhaskaracarya-II are more alike similar to each other. Many other dignified mathematicians also accepted these methods and with some minor twists and turns, they have slightly revised these methods. However, their slight revision has not been mentioned in this study for it being not relevant to the topic. Many classical mathematicians prior and of a much later period than Bhaskaracarya-II followed the same method as Bhaskaracarya with some minor modifications in his method to find the cube root of a number. Above mentioned mathematicians who opted for classical procedures has made remarkable contributions. Among them, this article mainly emphasizes on three mathematicians who significantly differed or provided the elementary structure for finding the cube root. Furthermore, only the comparison of methods of Bhaskaracarya, Gopal Pande and Bharati Krishna Tirthaji has been designated for this study and a neutral analysis with connections, comparison and contrast has been conducted for their respective methods to find the cube roots.

Pandit Gopal Pande (1847 - 1920) was the Nepalese student of Bapudev Shastri. He was the first Mathematics teacher of the first school of Nepal, Sanskrit Pathshala (currently Durbar high school) [5]. Another renowned mathematician Bharati Krishna Tirthaji (1884 – 1960) reinstated a new form of mathematics in the South Asian region namely Vedic Mathematics. Tirthaji is known as the father of Vedic Mathematics [9]. The contributions of Pande's and Tirthaji's to find cube roots are remarkable.

The cube roots of a real number x is a number y such that $y^3 = x$. In Mathematics, each real number (except 0) has exactly one real cube root and other two imaginary cube roots. But for non-zero complex numbers have three distinct imaginary cube roots. Finding the cube roots of a number by the conventional method is time consuming and tedious job with more complexity. However, using a proper technique it becomes interesting and fast too. We can find the cube root of any number within a very short period by just looking at the number with perfect practice. This article only focuses on extracting the real part of the cube roots not for imaginary parts of the numbers.

2 A Glance of Three Mathematicians and Their Methods

Many mathematicians contributed a significant role in making the glorious history of Mathematics. Among them, three mathematicians Bhascaracharya-II, Gopal Pande and Bharati Krishna Tirthaji, and their mathematics are more relevant for this article.

2.1 Bhaskaracarya and his Lilavati

Bhaskaracarya or Bhaskara-II was the famous leading Mathematician, Poet, Astronomer of ancient Indian during the twelfth century. Bhaskaracarya wrote his Siddhantasiromani when he was 36 [10]. The first part of Siddhantasiromani, Lilavati mainly deals with Arithmetic but also contains portions of Geometry, Trigonometry and Algebra. Bhaskaracarya's Lilavati was considered as the popular book in mathematics at that time. It has been used as a text-book for the last eight hundred years in many south Asian countries and is still in practice in many Indian provinces [10].

2.1.1 Bhaskaracarya's method for finding cube roots [5, 10, 11]

Bhaskaracharya also found a method to find the cube root of a number. However, the method that on finding the cube root, Bhaskaracarya had his words as follows:

आधं घनस्थानमथाघने द्वे पुनस्तथाऽन्त्याद् घनतो बिशोध्य ।

घनं पृथक्स्थं पदमस्य कृत्या त्रिघ्न्या तदाधं विभजेत् फलं तु ।।१४।।

पङ्क्त्यां न्यसेत् तत्कृतिमन्त्यनिघनीं त्रिघ्नीं त्यजेत् तत्प्रथमात् फलस्य ।

घनं तदाधाद् घनमूलमेवं पंक्तिभर्वेदेवमतस् पुनश्च ।।१५।। [10]

(Aadhyam ghanasthanmathaghane dwe punastathaantyaad ghanatbishodhya

Ghanam prithksthyam padamassya kritya trighnya tadhaadham wibhajet phalam tu ||14||

Panktyam nayset tatkritimantyanighanim trighinim tyajet tatprathamaat phalassya

Ghanam tadadhaad ghanamulmewam panktibharwedewamatassya punascha ||15||)

Meaning: Draw a vertical bar above the digit in the unit's place of the number whose cube root is wanted. Then put horizontal bars on the two digits to its left, vertical bar on the next and repeat until the extreme left-hand digit is reached. From the extreme left-hand section, deduct the highest cube possible and write to the left side the number 'a' whose cube was subtracted. Write to the right of the remainder, the first digit of the next section to get a new sub-dividend. Now divide by $3a^2$ and write the quotient b next to a. Then write the next digit from the section to the right of the remainder obtained above. The next divisor is $3ab^2$. In the next step take b^3 as the divisor. Continue this procedure until the digits in the given number are exhausted [10].

For illustration, consider the perfect cube number 50653.

Table No.1

Root	Description	Given Number	Remarks
a=3	a^3	$\bar{5}\ \dot{0}\ \bar{6}\ \bar{5}\ \dot{3}$ -2 7	The largest perfect cube below 50 is 27
	$3a^2b = 3 \times 3^2 \times 7$	2 3 6 -1 8 9	When $b = 8, 236 - 3a^2b$ is positive but $236 - 3ab^2$ is negative. Therefore, b=8 is not acceptable. So, we proceed to the next small number b=7.
b=7	$3ab^2 = 3 \times 3 \times 7^2$	4 7 5 4 4 1	
	$b^3 = 7^3$	3 4 3 3 4 3	
		0 0 0	

Therefore, $\sqrt[3]{50653} = 37$.

Bhaskaracharya's method of finding the cube root was similar to the method used by Aryabhata, Sridhara, Brahmagupta and Sripati [5].

2.2 Gopal Pande and his method

Gopal Pande (1847–1920) was the first Nepalese mathematician to write Mathematics in the Nepali language. He wrote Wyaktachandrika with consecutively four editions within the period 1883 to 1914 in the Nepali language [5]. Among them his third edition was written in Hindi [5, 12]. He expressed his mathematical techniques in his book from a simple understandable primary level to a higher-level realization [5]. He wrote the book Wyaktachandrika by restructuring the concepts of Lilavati, the idea from his teacher Bapudev Shastri and idea from other books.

Even though, Pande's most of the work was based on Lilavati of Bhaskaracarya [5, 7]. He has published his book and made it the text-book of mathematics. His book was considered as the easy book for teaching mathematics at that time [13]. In 1884, the first Nepali calendar (Panchanga) was published in Nepal. In this calendar, Pande found an error in the forecasting of the Lunar eclipse of that year. After a long discussion, it was found that Gopal Pande's result was true. Due to this achievement, he was honored with Royal Astrologer [12, 13].

Being a neighboring country with the same cultural and social-behavioral, there was a direct influence on the teaching-learning process between Nepal and India. Most of the Nepalese scholars were joined with the Indian scholars and shared the ideas in every field of education. Every mathematical text-books of Nepal was purchased from India, which were written either in Sanskrit or in Hindi. Moreover, India was the main source of Nepalese scholars in every aspect of the subjects at that time [5]. For finding the cube roots of the number, Nepalese mathematician Gopal Pande has disagreed with the Bhaskaracarya's verse. This new approach is considered as a remarkable contribution of Gopal Pande in Nepali mathematics.

Bhaskaracharya expressed in his book Siddhantasiromani that the unitary method or rule of three cannot be applied to find the square root and cube roots. In this regard, he expressed his idea in Sanskrit as [5]:

वर्गं वर्गपदं घनं घनपदं संत्यज्य यद् गण्यते

तत् त्रैराशिकमेव भेदबहुलं नान्यत् ततो विद्यते ।

(सिद्धान्तशिरोमणि प्रश्नाध्याय)

(*Varga vargapadam ghana ghanapadam satyajya yadha ganyate*

Tat trairashikmewa bhedabahulam nanyat tatwo vidhyate.)

- *Siddhanta Siromani Prasnadhyaya*

Meaning: The meaning of the above Sanskrit verse is that the whole mathematical problem can be solved by the rule of three or by unitary method except for finding squares, square roots, cubes and cube roots [5].

Pande's mathematical contribution with innovative techniques was more illustrated by Professor Naya Raj Pant (1913 – 2002) in 1980 on his book Pandit Gopal Pande and his method of getting Cube Root. Gopal Pande claimed that the rule of three can be used for finding squares, square roots, cubes and cube roots of a positive integer [5].

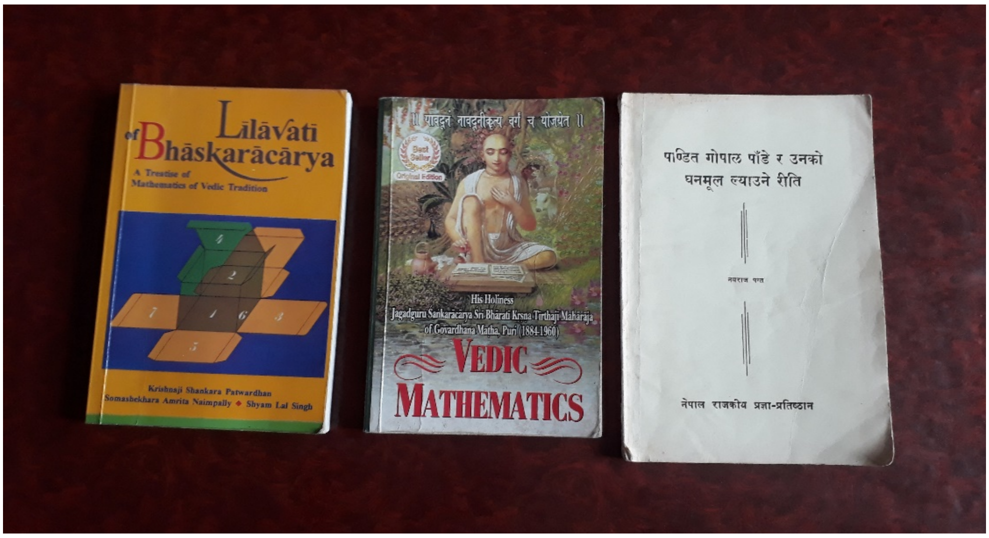

Figure 1: Images of three books: Lilavati of Bhaskaracarya (English Translation), Vedic Mathematics of Bharati Krishna Tirthaji and Pandit Gopal Pande by Nayaraj Pant (Pant's book on Gopal Pande is rarely available in the market. It can be obtained in the central library, Kathmandu.)

2.2.1 Pande's method to find cube roots [5, 14, 15]

Most of the basic mathematical problems can be solved by using a unitary method, which is an alternative technique of solving problems that can be solved by direct and inverse proportion method. In the unitary method, we first find the value of unit quantity by division in the case of direct proportion, and by multiplication in the case of inverse proportion. Then, we find the value of any other quantity by multiplication in the case of direct proportion, and by division in the case of inverse proportion.

For calculation of the cost of 90 books when it was given that the cost of 25 books was \$20, we use the unitary method. If the cost of 25 books was \$20 then the cost of 1 book is $\frac{\$20}{25}$, so the cost of 90 books

$= \frac{\$20}{25} \times 90 = \$72$. According to Bhaskaracarya, the process is not useful for finding squares, square roots, cubes and cube roots. For example, If the square root of 49 is 7 then the square root of 5776 is $\frac{7}{49} \times 5776 = 825.1428$. Here, the result obtained by the unitary method is wrong.

Gopal Pande was not agreed with the Bhaskaracarya's verse about to find the squares, square roots, cubes and cube roots. Pande expressed his techniques differently to find these by taking an example of cube root.

Pande wrote his method in his book Wyktachandrika about cube roots. A piece of method of Pande's in his own language in Nepali for finding cube roots looks like:

रीति . "जुन संख्याको घनमूल लिनु छ तेसका आदिकोअंक माथि बिन्दु दिनु . फेरी दुई २ अङ्क छाडदै बिन्दु दिँदैजानु . तेसमा जति बिन्दु हुन्छन् तेती अङ्क मूलमा आउदछन्।

फेरि अन्त्यका बिन्दुदेखि जति अंक हुन्छन् तेतिको एक संख्या मानी तेसमा जतिको घन घट्न सक्दछ, घटाएर शेषमा आर्को बिन्दु सम्मका अंक उतारी दसले गुण्नु। तेसलाई मूलको र एक जोडियाको मूलको घनको अन्तर , हजारले गुनेकाले भाग लिनु। जो लाब्धि आउँदछ त्यो मूलको अर्को अंक हुन्छ ।फेरी ति दुइ अंक भयाको संख्याको घन गरि आर्को बिन्दुदेखी घटाउनु"…. [5], [16].

Illustration of Pande's method: To perceive the ideas of Pande's method for finding cube roots, it is convenient by taking an example to find the cube root of the number 13824. For this purpose, he found the correct result by expressed his ideas by using the methods stepwise as follows: [5, 16]

- Place dots above the first digit (Unit digit) then above every fourth digit (Thousandth place) then above the seventh digit and so on from the right of the given number which has to be found the cube root, i.e., making the group of three from the right side of the number.
- The number of dots is equivalent to the digits of required cube roots.
- For the number 13824, it will have a 2-digits cube root being two dots $1\dot{3}82\dot{4}$.
- From the left of the number, first, it will be taken cube root of 13 which is greater than 2 but less than 3. Then it will be confirmed that the required cube root will be greater than 20 and less than 30. Therefore, for the difference in cube root by 30–20 = 10, the difference of their cubes is $30^3 – 20^3 = 19000$. That is, when the difference between two cubes is 19000 then the difference between two cube roots would be 10.
- Now the number $13 – 2^3 = 5$ with the number 824 (last or next three numbers) becomes 5824.
- Hence, in this case, Gopal Pande uses the unitary method or rule of three as:

Table No. 2

Difference of cubes	The difference between cube roots	Cube root (By unitary method)	Remarks
19000	10	$\frac{10}{19000} \times 5824 = 3.06$	The next number of required cube root would be 3 + 1=4

Therefore, the cube root of 13824 = 24.

2.3 Bharati Krishna Tirthaji and his vedic methods

A very renowned cultured personality of India, Jagadguru Sankaracharya Sri Bharti Krishna Tirthaji (1884-1960) was a glorious and divine person. He is well known for his formulation and reconstruction of Vedic Mathematics from certain Sanskrit text Veda. According to him, the Vedic system is based on his Sixteen sutras and an equivalent number of sub-sutras cover all branches of mathematics, pure and applied. Vedic Mathematics was formulated and reconstructed by Tirthaji in the time of 1911-1918, was only published in 1965 after his death in 1960 [9, 17, 18]. Vedic Mathematics also relates to mental calculations.

2.3.1 Vedic method to find cube roots [9, 17, 18]

There is a very quick method in Vedic Mathematics to find cube roots of perfect cube numbers, for this we need to remember that:
$1^3 = 1; 2^3 = 8; 3^3 = 27; 4^3 = 64; 5^3 = 125; 6^3 = 216; 7^3 = 343; 8^3 = 512; 9^3 = 729; 10^3 = 1000$
From the above cubes of 1 to 10, we need to remember the following facts:

- **i.** If the last digit of perfect cubes is 1, 4, 5, 6, 9, or 0 then the last digit of cube roots is similar.
- **ii.** If the last digit of perfect cubes is 2, 3, 7, or 8 then the last digit of cube roots is 8, 7, 3, or 2 respectively.

To be a perfect cube, the digital sum of the cubes is necessary to found 0 or 1 or 8. Observe the table:

Table No.3

Numbers	1	2	3	4	5	6	7	8	9
Cubes	1	8	27	64	125	216	343	512	729
Digital Sum	1	8	0	1	8	0	1	8	0

The left digit of a cube root can be extracted with the help of the following table:

Table No.4

Left-most pair of cube root	1 To 7	8 To 26	27 To 63	64 To 124	125 To 215	216 To 342	343 To 511	512 To 728	729 To 1000
Nearest cube roots	1	2	3	4	5	6	7	8	9

There are three cases in the Vedic method to find out Cube Roots of Perfect Cubes [9, 17, 18, 19]:

1. Cube roots of a number having less than 7-digits
2. Cube roots of a number having more than 7-digits but less than 10-digits
3. Cube roots of a number greater than 7-digits but ending with even numbers.

Make the groups of 3-digits, starting from the right then

- a. The number having less than 4 digits will have a 1-digit cube root.
- b. The numbers having 4, 5 to 6 digits will have a 2-digits cube root.
- c. The numbers having 7, 8 or 9 digits will have a 3-digits cube root.
- d. The number having 10, 11 or 12 digits will have a 4-digits cube root.

Illustration for Case I [9, 17, 18]
In Vedic mathematics, Vilokanam (by mere observation) method is used to extract the cube root of a number having less than 7 digits. It is to be noted that, the Vilokanam is one of the formulae among Vedic formulae.

To find the cube root of 21952, making a group of three from the right as $\overline{21}\ \overline{952}$. So, it will have a 2-digits cube root. From the above facts and table No. 1, the unit digit of the cube root is 8. From the table No.2, it determined the left-digit of the cube root is 2. Hence, $\sqrt[3]{21952} = 28$.

Illustration for Case II [9, 17, 18]
The cube root of more than 7-digits and less than 10-digits number will contain 3-digits. Let it be denoted

by the unit digit (R), left digit (L) and middle digit (M). L and R can be determined by the Vilokanam method, whereas M can be determined by the help of digital roots.

Suppose we have to find the cube root of 12977875. Making groups of three from the right side as $\overline{12}\ \overline{977}\ \overline{875}$. As in the case I, R = 5 and L = 2. Subtracting R^3 from the given number 12977875, i.e., 12977875 – 125 = 12977750 and eliminating always last zero will be 1297775. The middle digit of the cube root is obtained by $3R^2M$ = 75 M. We should be looking for a suitable value of M so that the unit digit of 75M becomes equal to the unit digit of 1297775. If we obtain more than one value of M, we use the digital root method and test which value of M is best suited in this case.

In this problem, here is 5 options for M, i.e. 1, 3, 5, 7 or 9 because digital roots of $(215)^3 = 8, (235)^3 = 1, (255)^3 = 0, (275)^3 = 8, (295)^3 = 1$ and digital root of the given number 12977875 = 1. Here, digital root of $(235)^3 = (295)^3 = 12977875 = 1$. Again, by Vilokanam rule of Vedic method, 235 is the best suited. Hence, $\sqrt[3]{12977875} = 235$.

Illustration for Case III [9, 17, 18]
The cube root of a number greater than 7 digits but ending with an even number can be obtained by dividing the number whose cube root has to be extracted by 8 until odd cubs to be obtained and can be used digital root method to ascertain the cube roots as in case II.

Consider the number 10-digits even number 2674043072 such that which is a perfect cube. We need to divide the given number by 8 until the odd cube to be obtained.

8	2674043072
8	334255384
	41781923

Here, the cube root of 41781923 can be obtained as in case II. i.e. $\sqrt[3]{41781923} = 347$.
Hence, $\sqrt[3]{2674043072} = 2 \times 2 \times 347 = 1388$.

3 Findings

- Except for the scholars Pande and Tirthaji, many classical mathematicians prior and of a much later period than Bhaskaracarya followed the same method with some minor twist and turns to find the cube roots of a number.
- Bhaskaracarya takes one-digit at a time whereas Pande and Tirthji take groups of three in their procedures.
- Bhaskaracarya's method is direct related to the formula $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
- In all procedures, if the number has n-digits then the cube roots will have
  $= \begin{cases} \frac{n}{3} \text{ digits, if n is multiple of 3.} \\ \frac{n}{3} + 1 \text{ digits, if n is not multiple of 3.} \end{cases}$
  (It should be careful that the decimal part of the result would be neglected.)
- In all procedures, the first digit of the cube root can be found by observing the first part of the number (it is either one-digit or two-digit or three-digit numbers).
- Pande's idea for finding cube roots by the unitary method in each step of operation can be formulated as follows:

  Each number of the required cube roots from the left can be obtained (for each step) by using the formula

$$= \frac{\text{The number formed by } (x - a^3) \text{ with the number of next three digits of given dividend}}{100\{(a+1)^3 - a^3\}} + 1$$

where x and a are varying in each step and hence varying the cube roots. $x =$ The number (taken from the left side of the given dividend) to be taken at that time. In the above worked out example, for the first step $x = 13$, for the second step, $x = (13–2^3 = 5)$ with the number 824 (which is the next three digits of the given number from the left side) becomes 5824.
a = digit of the cube root of previous steps.

- In Pande's method, a^3 = maximum cube to be taken from left. It should be noted that x must be greater than a^3. If there occurred negative, there must readjust by deducting in the value of a (deducting by an adjustable number from a).
- In Pande's method, the numbers which are not a perfect cube can be obtained from the cube roots of a decimal number by introducing three zeros to the remainder obtained in the previous steps and proceed as above.
- Cube roots of non-exact cube numbers can also be extracted from all three scholar's methods.
- In the Vedic method, there should be remembered the cube table from 1 to 10, the provided facts and its left-most pair with the nearest cube roots to determine the required cube roots.
- In the Vedic method, Vilokanam is a unique one, which determines the result of a number having less than 7 digits quickly.
- As mentioned in this paper, the case I and III are similar except dividing the number by 8 in the Vedic method.
- On the basis of this paper, the case II is comparatively more difficult to ascertain the exact value of M because both the sutras Vilokanam and Digital Roots are used simultaneously in the Vedic method.

4 Conclusion

From the pieces of evidence and findings, it can be concluded that Bhaskaracarya's method for determining cube roots represents all the methods of classical scholars before and after his period. Nepali mathematician Gopal Pande has followed all the ideas of Bhaskaracarya in his teaching profession. And this attests that they are interrelated in some major aspects. But Pande did not agree with the Bhaskaracarya's verse concerning find the cube roots by using his unitary method. Pande expressed his techniques innovatively and differently by breaking the verse of Bhaskaracarya. After the period of Pande, another emerging mathematician Bharati Krishna Tirthaji known as the father of Vedic mathematics in India developed an independent and constructive method. His idea of determining cube roots is less influenced by the previously developed methods. To study the importance and effectiveness of the methods it is better to compare it with conventional methods. The real beauty and effectiveness of these methods cannot be fully appreciated without actually practicing the system. Although in comparison among the methods, the Vedic method is interesting and fast, people must pay attention to some facts and basic cubing table. In a modern mathematical system, people require a calculator for their numerical calculations. The practice of Vedic Mathematics may be very fruitful for the teaching-learning process of basic mathematical operations without using a calculator because Various investigation shows that calculator dependency deteriorates the logical thinking of the learner.

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