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Multi-constraint size optimization of planar trusses using a modified firefly algorithm

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Abstract

Truss size optimization is computationally a very demanding structural engineering problem due to its non-linear and non-convex nature. Despite being substantially efficient, metaheuristic algorithms like the Firefly Algorithm (FA), converge slowly for big problems. Herein, an Accelerated Firefly Algorithm (AFA) is presented to make the simple FA more efficient and reliable for the optimization of truss size. The AFA enhances performance by utilizing a dynamic randomization parameter and a scaled search strategy to expedite convergence. To evaluate its efficacy, the proposed algorithm was applied to three standard benchmark problems. The outcomes demonstrate that the AFA consistently yields optimal designs that either match or surpass the best-published results found in existing literature. Remarkably, the algorithm exhibited exceptional computational efficiency, converging rapidly while meeting all constraints related to stress and displacement. The findings confirm that the AFA is a promising, effective, and efficient approach for solving complex structural optimization problems and, therefore, well-suited to real-life engineering design problems.

1. Introduction

Trusses are structural members of civil engineering that are widely utilized, owing to their efficiency in distributing loads and a high strength-to-weight ratio. They are employed in bridges, roofs, towers, and large-span buildings since they provide excellent stability and durability. The truss members' geometrical arrangement in the form of connected triangles aids in transferring forces effectively, thus forming strong as well as economic structures. As real-world engineering projects have grown in complexity and demand more resources, optimizing truss structures has become a prominent area of research. This is because, with growing demands, designs are no longer required to be merely safe and reliable but also sustainable and economical.

Optimizing a truss is a challenging issue that can be tackled more efficiently by altering the dimensions of its members, the overall configuration, and the connectivity of its elements (topology) [1]. The focus of this investigation is size optimization, a specific approach that involves modifying the cross-sectional areas of the truss elements. This particular problem has been extensively addressed in numerous previous studies [2][3][4]. The primary aim is to minimize material usage, thus weight and cost, while designing the structure to be sufficiently strong to withstand all the loads safely and satisfy all the constraints. The constraints typically include restrictions on member stress, nodal displacement, and buckling, which render the optimization problem non-linear and non-convex.

In recent years, various metaheuristic algorithms have been effectively employed to solve truss optimization problems. Notable examples include Genetic Algorithms (GA) [5][6][7][8][9], Particle Swarm Optimization (PSO) [10], and the Firefly Algorithm (FA) [11][12][13]. All of these techniques are adept at navigating complex search spaces to identify optimal or near-optimal solutions. However, a notable drawback is its potential for slow convergence, particularly when addressing large-scale optimization tasks [14]. This paper explores enhancements to the FA to make it more robust and efficient.

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To evaluate its performance, we implemented the proposed algorithm on several standard benchmark problems. This validation process aims to confirm the algorithm's ability to produce optimal, structurally sound designs that adhere to all specified performance criteria.

The primary objective of this study is to demonstrate the improved performance of the accelerated Firefly Algorithm (AFA) over the conventional Firefly Algorithm (FA), and this comparison is explicitly presented in the results. In addition, comparisons with previously published solutions obtained using different metaheuristic algorithms are included because the selected truss problems are widely recognized benchmark examples in structural optimization. Such benchmark comparisons are essential to verify and validate the effectiveness, competitiveness, and robustness of the proposed AFA within the broader context of truss optimization research. Since both FA and AFA are applied to the same standard benchmark problems under identical constraints, the improved performance of AFA can be reliably attributed to the acceleration strategy rather than problem formulation differences.

2. Problem formulation

The fundamental goal in truss size optimization is to reduce the structure's overall weight, which is directly proportional to the material volume. This objective is achieved by identifying the ideal combination of cross-sectional areas for the truss members while satisfying a series of performance and geometric constraints.

2.1. Objective function

The formulation of the objective function for reducing the weight of the truss structure (W) is given by:

$$\min W = \sum_{i=1}^{m} A_i \, \rho_i \, L_i \tag{1}$$

where m represents the total number of members in the truss, ρ_i denotes the material density of the i^{th} member, A_i signifies the cross-sectional area of the i^{th} member (design variable), and L_i is the length of the i^{th} member.

2.2. Design constraints

The optimization process must adhere to the following constraints to ensure the structural integrity and serviceability of the truss:

Stress constraints: The axial stress (σ_i) in each member should not surpass the maximum permissible stress ($\sigma_i^{\max}$). This is expressed as:

$$|\sigma_i| - \sigma_i^{\max} \leq 0 \tag{2}$$

Displacement constraints: The displacement (δ_i) at each node i in any direction must be within the permissible limit ($\delta_i^{\max}$). This is given by:

$$|\delta_i| - \delta_i^{\max} \leq 0 \tag{3}$$

Kinematic stability: A truss must be kinematically stable to be viable and must not form a mechanism. This is verified using two primary criteria as proposed by Deb and Gulati [6].

Grubler's criterion: The degree of freedom (DOF) for a two-dimensional truss is determined by:

$$\text{DOF} = 2n - m - r \tag{4}$$

where n denotes the number of nodes, m denotes the number of members, and r denotes the number of reaction components. A non-positive DOF indicates a stable structure.

Cholesky decomposition: The global stiffness matrix (K) is evaluated for positive definiteness. A positive definite matrix K indicates that the truss design is stable.

In this study, the kinematic stability of candidate truss designs is verified through a sequential dual-criterion approach to ensure robustness of the Firefly algorithm–based optimization. Grubler's criterion (Eq. 4) is first applied as a necessary condition to eliminate geometrically unstable configurations and potential mechanisms. Cholesky decomposition is then employed to confirm the positive definiteness of the global stiffness matrix and numerical stability of the system. The combined use of both criteria is essential, as Grubler's criterion does not capture stiffness degradation effects, while Cholesky decomposition alone may admit near-singular or ill-conditioned designs during optimization. This strategy effectively prevents convergence toward physically infeasible solutions and ensures structurally meaningful optimal designs.

Sizing constraints: The cross-sectional area (A_i) of each member must lie within a practical range defined by a minimum ($A_i^{\min}$) and maximum ($A_i^{\max}$) value:

$$A_i^{\min} \leq A_i \leq A_i^{\max} \tag{5}$$

The optimization problem is solved using the Firefly Algorithm, where solutions that violate any of these constraints are penalized, guiding the search toward feasible and optimal designs.

3. Firefly Algorithm (FA)

The Firefly Algorithm (FA) is a metaheuristic method inspired by nature, specifically, the flashing behavior of fireflies, proposed by X-S. Yang [14]. The algorithm employs a population of fireflies to explore the solution space, where each firefly represents a candidate solution. Firefly movement is regulated by its relative brightness, which directly affects the objective function's fitness, i.e., truss weight in our situation. A firefly's brightness is directly correlated with the quality of its associated solution (objective function value), enabling brighter individuals to attract those that are less bright.

The algorithm operates based on the following three idealized rules, as originally proposed by X-S. Yang [14]:

1. All fireflies are considered unisex, meaning any firefly can be attracted to any other brighter firefly, irrespective of their sex.
2. The attraction of a firefly is associated with its luminosity. Since brightness decreases as the distance between fireflies increases due to light absorption by the medium, attractiveness is also inversely proportional to distance. A dimmer firefly will approach one that glows more intensely. In the absence of a brighter firefly, it will travel randomly.
3. The intensity of a firefly's glow is influenced by the form of the objective function. In the context of this study's minimization problem, a lower truss weight corresponds to a higher brightness.

The fundamental mathematical representation of the standard Firefly Algorithm employs these equations:

Attractiveness function: The attractiveness (β) of one firefly (i) to another (j) is a function of the distance r_{ij} separating them. This relationship is modeled using a negative exponential function to represent the decrease in light intensity over distance:

$$\beta = \beta_0 e^{-\gamma r_{ij}^2} \tag{6}$$

where β_0 represents the attractiveness at zero distance, and the light absorption coefficient γ reflects the rate at which the algorithm moves towards convergence.

Movement of fireflies: A firefly i moves towards a more attractive firefly j by updating its position within the search space. This movement combines both attraction and randomization components:

$$X_i^{t+1} = X_i^t + \Delta X_i^t = X_i^t + \beta(X_j^t - X_i^t) + \alpha\epsilon_i^t \tag{7}$$

where X_i and X_j represent the positions of fireflies i and j, respectively. The second term governs attraction, while the third term introduces stochastic behavior. The randomization is controlled by the parameter α, and ϵ_i^t is a vector of random numbers, typically drawn from a Gaussian or uniform distribution.

3.1. Accelerated Firefly Algorithm (AFA)

In this study, the terms *Simple* or *Standard* Firefly Algorithm refer to the original formulation proposed by Yang [14], governed by Eqs. 6 and 7. The term *Accelerated* Firefly Algorithm (AFA) refers to the modified version used in this research, which introduces enhancements to the randomization parameter (α) and the scaling term (λ). These modifications distinguish AFA from the Standard FA by replacing the constant randomization in Eq. 7 with the dynamic approach detailed in Eq. 10.

While the standard FA is a powerful optimization tool, it can suffer from a slow rate of convergence, particularly when the optimization problem is large-scale and complex. The Accelerated Firefly Algorithm (AFA), hence, is an enhanced variant designed to overcome this limitation by improving the convergence rate and reducing computational overhead. This is achieved through two primary modifications to the standard FA formulation, as proposed by Baghlani et al. [12].

Gradual randomness reduction: In the standard Firefly Algorithm (FA), the randomization parameter α is typically constant. In the Accelerated Firefly Algorithm (AFA), α is dynamically reduced as iterations progress. This strategy encourages global exploration in early stages and facilitates local exploitation near optimal solutions [15][16][17]. The parameter is updated as follows:

$$\alpha = \alpha_0\, \theta^t \tag{8}$$

where α_0 is the initial randomization parameter, θ is the randomization reduction constant ($0 < \theta < 1$), and t is the current iteration number.

Scaling the random term: In the standard FA, the random term is independent of the scale of the design variables. The AFA introduces a scaling parameter λ to make the randomization step proportional to the bounds of the design variables [12]. This ensures that the exploration is appropriately scaled to the problem's search space. The parameter is defined as:

$$\lambda = A_{\text{up}} - A_{\text{low}} \tag{9}$$

where A_{up} and A_{low} are the maximum and minimum limits of the design variables (e.g., the cross-sectional areas).

Updated movement equation: By integrating these modifications, the updated movement equation for the AFA becomes:

$$X_i^{t+1} = X_i^t+\Delta X_i^t = X_i^t+\beta_0 e^{-\gamma r_{ij}^2}(X_j^t-X_i^t)+\lambda\alpha_0(\epsilon_i^t-0.5) \tag{10}$$

This improved formulation enables the AFA to explore the solution space more efficiently. By gradually reducing randomness and scaling the search steps, the algorithm achieves faster convergence while maintaining a balance between exploration and exploitation, making it highly suitable for computationally demanding structural optimization tasks.

3.2. Constraint handling

There is a need for an effective constraint handling algorithm so that the resulting optimized design is physically reliable, safe, and satisfies all engineering constraints. The static penalty function method, a simple and effective approach for excluding the optimization search from infeasible regions of the design space, has been applied in numerous works [15][16][17][18], including this as well. This technique transforms the constrained optimization problem into an unconstrained form by using a significant penalty to the objective function of any solution that violates the design constraint.

For each candidate solution generated by the algorithm, a list of checks is performed to measure its feasibility against the aforementioned critical constraints. Any candidate solution violating any one or more of these constraints is declared infeasible. The algorithm punishes this solution by augmenting its objective function with a large, constant penalty value. The penalized objective function, $W_{\text{Penalized}}$, is therefore calculated as follows:

$$W_{\text{Penalized}} = \begin{cases} W(A), & \text{if feasible} \\ W(A) + P, & \text{if infeasible} \end{cases} \tag{11}$$

where $W(A)$ represents the computed weight of the truss, and P is the static penalty value. In this study, a penalty of $P = 10^6$ is applied.

The penalty factor $P = 10^6$ was selected based on preliminary sensitivity analysis and standard practices in metaheuristic structural optimization [15]. This value is sufficiently large to ensure that any feasible design, regardless of its weight, will always have a better fitness

Table 1: The pseudo code for AFA

```
Define the objective function $f(x)$, where $x$ is the vector of member areas.
Evaluate the fitness $f(x_i)$ for each firefly.
Set parameters:
    • $\beta_0$: Initial attractiveness
    • $\gamma$: Light absorption coefficient
    • $\alpha_0$: Initial randomization parameter
    • $\theta$: Randomness decay constant
Define design variable bounds and calculate the scaling factor $\lambda = A_{\text{up}} - A_{\text{low}}$.
While $t <$ MaxIteration:
    Update randomness: $\alpha = \alpha_0\theta^t$.
    For $i = 1$ to $n$ (all fireflies):
        For $j = 1$ to $n$ (all fireflies):
            If $f(x_j) < f(x_i)$:
                Compute distance: $r_{ij} = \|x_i - x_j\|$.
                Update attractiveness: $\beta = \beta_0 e^{-\gamma r_{ij}^2}$.
                Move firefly $i$: $x_i = x_i + \beta(x_j - x_i) + \lambda\alpha(\epsilon - 0.5)$.
                Enforce boundary constraints for the new $x_i$.
                Evaluate the new solution and update its fitness $f(x_i)$.
            End If
        End for $j$
    End for $i$
    Rank the fireflies and find the current best solution.
End While
Post-process the best solution found.
```

value (lower weight) than an infeasible design, thereby guiding the algorithm away from invalid regions without causing numerical overflow.

This penalty value is essentially a large positive quantity that renders any infeasible design extremely costly to the algorithm during the selection process. Hence, the AFA is largely motivated to discard infeasible solutions and perform all search operations entirely within the possible area of the design space so that the result is an optimized and valid structure.

4. Test problems

The performance of the presented Accelerated Firefly Algorithm was tested on three classic benchmark challenges well known from the literature. They were chosen to validate the accuracy, convergence rate, and scalability of the algorithm for problems of different complexities. To set the experiments uniform, the AFA was run with the same set of control parameters as the comparative studies. To set the experiments uniform, the AFA was run with a set of control parameters selected based on preliminary parameter tuning and sensitivity analysis. The population size was set to 20; our testing indicated that larger populations increased computational time without significant accuracy gains, while smaller populations reduced the diversity of the search space. Similarly, the decay constant ($\theta = 0.97$) was determined empirically; trials showed that values below 0.95 caused a rapid loss of exploration capability (leading to local optima), while values above 0.99 slowed convergence unnecessarily. The initial attractiveness ($\beta_0 = 2$) and light absorption coefficient ($\gamma = 1$) were tuned to balance the attraction mechanics. These empirically derived values also align closely with those found in the literature [12][17], confirming their robustness for truss optimization tasks.

4.1. 10-bar planar truss

The first case study involved the 6-node, 10-element truss, a commonly used standard for assessing optimization algorithms. The structural layout and loading conditions were based on the problem described by Baghlani et al. [12].

The design variables, representing the cross-sectional areas of the members, were treated as continuous variables with a minimum limit ($A_i^{\min}$) of 0.1 in^2 and a maximum limit ($A_i^{\max}$) of 35 in^2. Key parameters include a material density of 0.1 lb/in^3, a Young's modulus of 10,000 ksi, and an allowable stress of ±25 ksi. Furthermore, a permissible displacement limit of ±2.0 inches is enforced for every node, as strictly defined in the standard benchmark specifications by Baghlani et al. [12]. The optimization results from the AFA are summarized in Tables 2 and 3, which list the final member areas and nodal displacements.

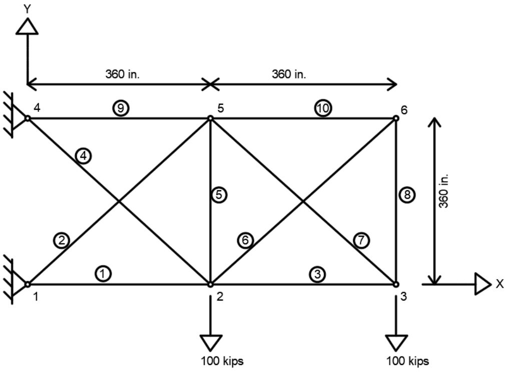

Figure 1: Optimized design of the 10-bar planar truss based on the work of Baghlani et al. [12]

Table 2: Statistical results for the optimization of a 10-bar planar truss Using AFA

Statistic	Value
Minimum weight (lb)	5055.475
Average weight (lb)	5063.080
SD of weights (lb)	91.768
Percentage of run solutions that are not more than 2% heavier than the minimum weight	98.7%

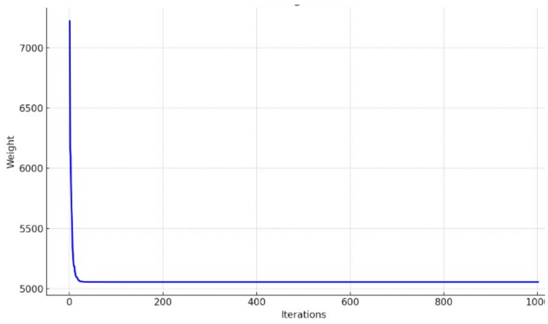

Figure 2: Convergence rate of AFA for the 10-bar planar truss

Figure 2 illustrates a rapid convergence within the first 20 iterations. However, the algorithm is run for 1000 iterations to ensure global stability. While the visual change in weight is negligible after iteration 50, the algorithm continues to perform micro-adjustments to the member areas to ensure that all stress and displacement constraints are strictly satisfied, preventing premature convergence to a local optimum.

Table 3: AFA-based comparative assessment of optimal 10-bar planar truss designs

Element	Node i - Node j	Member area (in^2)	
		AFA	**Baghlani et al. [12]**
1	Node1-Node 2	23.182	23.215
2	Node1-Node 5	21.047	20.866
3	Node2-Node 3	15.359	15.043
4	Node2-Node 4	7.495	7.453
5	Node2-Node 5	0.090	0.100
6	Node2-Node 6	0.090	0.100
7	Node3-Node 5	21.296	21.461
8	Node3-Node 6	0.499	0.591
9	Node4-Node 5	30.603	30.968
10	Node4-Node 6	0.090	0.100
Weight (lb)		**5055.475**	**5059.220**

The AFA optimization yielded a final weight of 5055.475 lb. As shown in the comparative results in Table 2, this solution is slightly superior to the published benchmark weight of 5059.220 lb, demonstrating the algorithm's capability to find high-quality solutions. The final design satisfied all constraints, confirming the robustness and efficiency of the AFA.

4.2. 11-member–6-node structure

The second reference challenge was an 11-member truss, initially defined as a ground structure, based on the work of Deb & Gulati [6]. The properties of the material incorporated a density of 0.1 lb/in^3 and a Young's modulus of 10,000,000 psi. The design is constrained by a maximum allowable stress of ±25,000 psi and a nodal displacement restriction of ±2.0 inches, which is adopted directly from the problem formulation presented by Deb & Gulati [6]. For the optimization process, the cross-sectional areas of all members were bounded within the range of 0.1 in^2 to 35 in^2.

The AFA successfully optimized the initial structure to a more efficient 6-member truss.

The final optimized design resulted in a minimum weight of 4898.35 lb. As depicted in Table 5, the result weighed 0.80 lb less than the optimal solution reported. All design constraints related to stress and nodal displacement were satisfied, thereby further validating the performance of the algorithm.

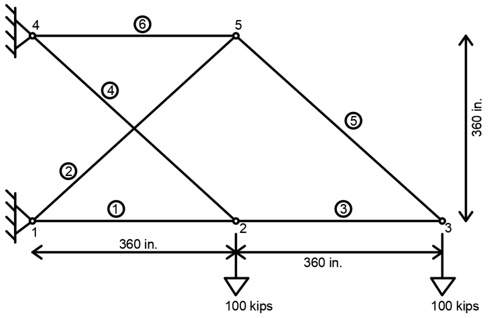

Figure 3: Optimized truss for an 11-member, 6-node structure based on the work of Deb and Gulati [6]

Table 4: Statistical results for the optimization of an 11-member, 6-node structure Using AFA

Statistic	Value
Minimum weight (lb)	4898.36
Average weight (lb)	4996.33
SD of weights (lb)	34.35
Percentage of run solutions that are not more than 2% heavier than the minimum weight	99.50%

Table 5: A Comparative assessment of optimal 11-member, 6-node truss designs

Element	Node i - Node j	Member area (in^2)	
		AFA	**Baghlani et al. [12]**
1	Node1-Node 2	22.21	22.07
2	Node1-Node 5	21.23	21.44
3	Node2-Node 3	15.08	15.3
4	Node2-Node 4	6.08	6.09
5	Node3-Node 5	21.20	21.29
6	Node4-Node 5	30.18	29.68
Weight (lb)		**4898.35**	**4899.15**

4.3. 6-node, 8-element truss

The final benchmark problem selected was the 6-node, 8-element truss. While Shakya et al. [10] discuss ground structures containing up to 30,381 elements, the specific validation case utilized here is the simplified 8-element topology derived from that study. To assess the scalability and performance of the AFA on a large-scale optimization task, the final benchmark problem selected was a 30,381-element ground structure based on the work of Shakya et al.[10]. This problem features more stringent constraints, including an allowable stress of ±5 ksi and a displacement limit of ±0.6 in, consistent with the constraints outlined in the reference work by

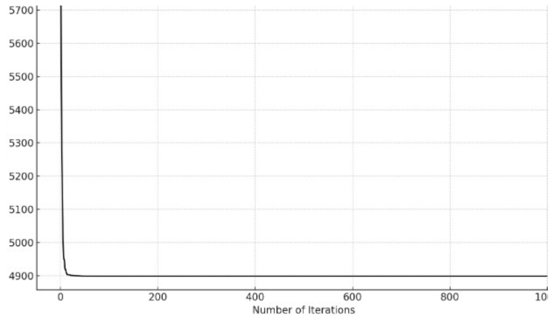

Figure 4: Convergence rate of AFA for the optimized 6-member- 5-node truss

Shakya et al.[10]. The search space for the member cross-sections was defined by a lower bound of 0.1 in^2 and an upper bound of 35 in^2, ensuring practical sizing limits.

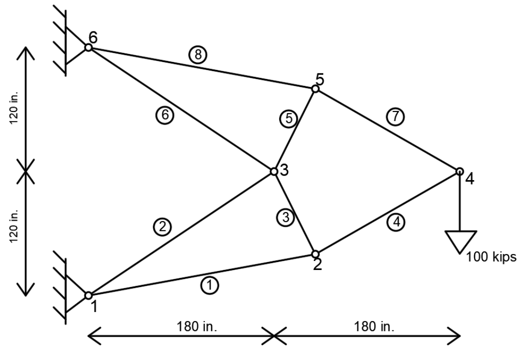

Figure 5: Optimized truss of a 6-node, 8-element truss based on the work of Shakya et al. [10]

Table 6: Statistical findings for the 6-node, 8-element truss Using AFA

Statistic	Value
Minimum weight (lb)	2200.000
Average weight (lb)	2205.306
SD of weights (lb)	57.629
Percentage of run solutions that are not more than 2% heavier than the minimum weight	98.60%

The AFA obtained the ideal solution of weight 2200.006 lb for this intricate, large-scale problem. This resolution aligns with the standard set forth by Shakya et al. [10], as evident from Table 7. This exact correlation demonstrates the capability of AFA for solving computationally demanding, real-life structural optimization problems, showcasing its potential to achieve the global optimum with very high precision.

Table 7: Comparative assessment of optimal 6-node, 8-element truss designs

Element	**Node i - Node j**	**Member area (in^2)**	
		AFA	**Shakya et al. [10]**
1	Node1-Node 2	20.963	20.963
2	Node1-Node 3	11.267	11.267
3	Node2-Node 3	6.988	6.988
4	Node2-Node 4	20.156	20.156
5	Node3-Node 5	6.988	6.988
6	Node3-Node 6	11.267	11.267
7	Node4-Node 6	20.156	20.156
8	Node5-Node 6	20.963	20.963
Weight (lb)		**2200.006**	**2200.006**

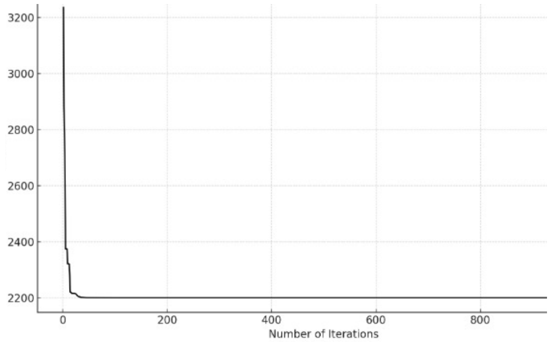

Figure 6: Optimized design of a 6-node, 8-element truss based on the work of Shakya et al. [10]

5. Discussion and conclusion

The results achieved with the three benchmark problems demonstrate a slightly superior performance and stability of the suggested Accelerated Firefly Algorithm (AFA) in truss size optimization. For all the test cases, the AFA produced solutions that were either superior to or fully consistent with the best results previously published in the literature. In the cases of the 10-bar and 11-member trusses, the proposed algorithm produced results with slightly lower weights than the current benchmarks, showcasing its capacity to navigate the solution space and progress towards better optima. More significantly, for the 6-node, 8-element truss problem, the AFA achieved a final weight that was identical to the known optimum solution. This outcome highlights the algorithm's high precision and its effectiveness at avoiding premature convergence, even with a very large and highly complex search space.

Additionally, this research concluded that the AFA exhibits high computational efficiency. A significantly high convergence rate, ranging from 7 to 12 seconds, was observed for all the test problems. Gradually diminishing the randomization factor (α) and scaling down the random search term (λ) creates an intelligent search process that results in the optimal, outweighing the capabilities of the classical Firefly Algorithm. In the initial stages, an elevated randomization factor facilitates a broad, global exploration of the design space. As the algorithm proceeds, the randomization aspect is gradually reduced, the search is more focused, boosting the exploitation and enhancing the final solution. This dynamic balance is crucial in achieving rapid convergence while avoiding being trapped in local optima.

Based on these findings, the primary conclusions of this work may be succinctly stated as follows:

1. The AFA was highly successful, often yielding best designs that were consistent with, and in some cases superior to, the best-published results from the literature.
2. The algorithm was seen to have excellent computational efficiency and exhibited good convergence behavior with very little computational time for all the test cases. This ensures that the modifications introduced to the original FA were successful in fixing its issue of slow convergence.
3. The results validate the reliability and strength of the approach. In each case, the optimized final structures satisfied the member stress and nodal displacement constraints completely and minimized the structural weight effectively.

This study validates that precise control and problem-oriented tuning of parameters of an algorithm are necessary in order to enhance its performance. The Accelerated Firefly Algorithm, thus formulated, proves to be an efficient and effective computational tool in truss optimization with the capability of creating efficient and optimal truss structures with high speed and accuracy.

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References

[1] Dominguez A, Stiharu I, Sedaghati R. Practical design optimization of truss structures using the genetic algorithms[J]. Research in Engineering Design, 2006, 17: 73-84.

[2] Hemp W S. Optimum structures[M]. Oxford: Clarendon, 1973.

[3] Pedersen P. Topology optimization of three-dimensional trusses[M]// Bendsøe M P, Mota Soares C A. Topology Design of Structures. Dordrecht: Kluwer, 1993: 19-30.

[4] Adeli H, Kamal O. Efficient optimization of plane trusses[J]. Advances in Engineering Software, 1991, 13(3): 116-122.

[5] Rajan S D. Sizing, shape, and topology design optimization of trusses using genetic algorithms[J]. Journal of Structural Engineering, 1995, 121(10): 1480-1487.

[6] Deb K, Gulati S. Design of truss structures for minimum weight using genetic algorithms[J]. Finite Elements in Analysis and Design, 2001, 37(5): 447-465.

[7] Hajela P, Lee E. Genetic algorithms in truss topological optimization[J]. International Journal of Solids and Structures, 1995, 32: 3341-3357.

[8] Rajeev S, Krishnamoorthy C S. Genetic algorithms-based methodologies for design optimization of trusses[J]. Journal of Structural Engineering, 1997, 123: 350-358.

[9] Tang W, Tong L, Gu Y. Improved genetic algorithm for design optimization of truss structures with size, shape and topology variables[J]. International Journal for Numerical Methods in Engineering, 2005, 62: 1737-1762.

[10] Shakya A, Nanakorn P, Petprakob W. A ground-structure-based representation with an element-removal algorithm for truss topology optimization[J]. Structural and Multidisciplinary Optimization, 2018, 58(2): 657-675.

[11] Miguel L F F, Lopez R H, Miguel L F F. Multimodal size, shape, and topology optimization of truss structures using the Firefly algorithm[J]. Advances in Engineering Software, 2013, 56: 23-37.

[12] Baghlani A, Makiabadi M H, Rahnema H. Optimal design of structures using an efficient firefly algorithm[J]. Asian Journal of Civil Engineering (Building and Housing), 2013, 14(2): 295-307.

[13] Tripathi S. A brief review of firefly algorithm: Application in structural optimization problem[J]. 2019.

[14] Yang X S. Studies in computational intelligence: volume 744 nature-inspired algorithms and applied optimization[M]. Springer International Publishing, 2018.

[15] Gomes H M. A firefly metaheuristic algorithm for structural size and shape optimization with dynamic constraints[J]. Mecánica Computacional, 2011, 76: 2059-2074.

[16] Kazemzadeh Azad S, Kazemzadeh Azad S. Optimum design of structures using an improved firefly algorithm[J]. International Journal of Optimization in Civil Engineering, 2011, 2: 327-340.

[17] Gandomi A H, Yang X S, Alavi A H. Firefly algorithm with chaos[J]. Communications in Nonlinear Science and Numerical Simulation, 2013, 18: 2059-2074.

[18] Perez R E, Behdinan K. Particle swarm approach for structural design optimization[J]. Computers & Structures, 2007, 85: 1579-1588.