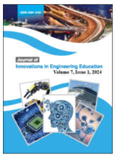

Journal of Innovations in Engineering Education
JIEE 2024, Vol. 7, Issue 1.

https://doi.org/10.3126/jiee.v7i1.72340

THAPATHALI CAMPUS
Institute of Engineering
Tribhuvan University

Transient response analysis of simply supported Pelton turbine during starting and shutdown

Bir Bahadur Chaudhary[a], Bikki Chhantyal[b,*] and Mahesh Chandra Luintel[a]

[a]*Institute of Engineering, Pulchowk Campus, Lalitpur*
[b]*Institute of Engineering, Thapathali Campus, Thapathali*

ARTICLE INFO

Article history:

Received 17 September 2024
Revised in 2 November 2024
Accepted 16 November 2024

Keywords:

Pelton turbine
Simply supported
Transient response
Mode

Abstract

Variations in speed of shaft-disc systems is due to starting, shutdown and maneuver. Starting and shutdown of hydraulic turbine is the condition in which system is subjected to dynamic load causing transient excitation. Previously, study on transient response of shaft-disc system is done by experimental approach. This study introduces integration of energy methods and Lagrange's equation to analyze transient dynamics in Pelton turbines, offering an alternative to expensive experimental approaches. So, mathematical model should be developed to define shaft-disc system for dynamic input. Development of a mathematical model for the study of vibration of a simply supported Pelton turbine under dynamic input is the main objective of this work. Mathematical model is developed based on rigid bearings, rigid disk and flexible shaft. As energy method is used in this work, total kinetic and potential energies of the shaft and disk are derived. By using assumed mode method, system's energies are formulated. Formulated energies are used in Lagrange's equation of motion from which system's governing equation of motion is obtained. MATLAB program is developed to solve governing equation of motion which gives peak amplitude of 58.34μm and 115.87μm for starting and shutdown respectively. Solution from mathematical model is compared with simulated solution from ANSYS which gives variations of 2.35 % and 0.58 %. The method's reliability is highlighted by its close agreement with ANSYS simulations, which offers crucial information for shaft-disc system design and stress analysis.

Nomenclature

Ω Spin speed of shaft

$\phi(z)$ Spatial function describing the transverse deflection

ρ_s Density of shaft

A Cross-sectional area of shaft

C_i Equivalent damping of the system of ith mode

E Modulus of elasticity

F_j Water jet force acting along X-axis

I Moment of inertia of shaft about X or Y-axis

I_d Moment of inertia of disk about X or Y-axis

J Polar moment of inertia of shaft about Z-axis

J_d Polar moment of inertia of disk about Z-axis

K_i Equivalent stiffness of the system of ith mode

L Length of shaft

M_i Equivalent mass of the system of ith mode

m_d Mass of disk

$U(t)$ **and** $V(t)$ Displacement vectors

1. Introduction

Shaft-disk system is widespread in engineering applications. For example, hydraulic turbine system, turbine engine system, rotor compressor system. Due to high demanding precision, study of vibration has become an important aspect in the field of mechanical engineering design. Due to high speed and high-performance requirements, study of coupling behavior on shaft-disc system have become more important. Speed variations in rotor system are typical during startup, shutdown and maneuver operations. Starting and shutdown of

*Corresponding author:
chhantyalbikki@tcioe.edu.np (B. Chhantyal)

hydraulic turbine is the condition in which system is subjected to dynamic load causing transient excitation. Numerous investigations on the transient dynamics of rotor systems have been conducted in the recent decades. Egusquiza et. al. [1]evaluated the start-up transient response of horizontal Pelton turbine prototypes experimentally to identify natural frequencies and mode shapes that were utilized to validate results from numerical model. Earlier researches have focused on dynamic response of a flexible shaft during steady state. For startup transient response on rotor dynamics, Egusquiza et al. [1] primarily focus on experimental approach, leaving a gap in robust analytical modeling for transient conditions in Pelton turbines. This study bridges this gap by developing a mathematical model compared with simulated solution, enabling validation of operational analysis. Depending upon the application in which the system is being used each principal components of the system i.e., disk, shaft and bearings can be assumed to be either rigid or flexible. If shaft can be assumed as massless flexible system, then a discrete model can be employed but if mass of the shaft cannot be ignored then a continuous model should be used for the dynamic analysis. Continuous shaft of any rotodynamic system can be modeled as either a rotating Euler-Bernoulli beam or a rotating Timoshenko beam [2]. Forced vibration of a system due to dynamic input with varying magnitude gives transient response. In many cases, the response of the system due to a dynamic input of a certain interval may exist for an interval greater that the duration of the dynamic excitation. Khanlo et al. [3] studied on chaotic vibration of a spinning flexible shaft-disk system including rub-impact in which they used Euler Bernoulli beam theory to obtain the governing partial differential equations.

2. Methodology

For this work, the axes x, y and z are chosen so that x is along the shaft's transverse axis on the horizontal plane, y is along the shaft's transverse axis on the vertical plane and z is along the shaft's longitudinal axis. Similar to this, each point on the shaft's transverse displacements in both the horizontal and vertical directions are u(z,t) and v(z,t), respectively. The x-axis of water jet force applies on the horizontal shaft Pelton turbine. In this study, disk and bearing are assumed as rigid and shaft as flexible. Total Kinetic and potential energy is calculated for shaft-disk system and with the help of assumed mode method and Lagrange's equation of motion, governing equation of motion is obtained which is second order coupled differential equation. For the development of mathematical model for simply supported Pelton turbine model, the test rig of Department of Mechanical and Aerospace Engineering, Pulchowk Campus is taken as given in Table 1. After substituting all the necessary data, the governing equations of motion are solved by using MATLAB ode45 solver to determine the responses. Response plots is obtained which helps to determine the peak amplitude of vibration. For verification; simulated solution is found from transient structural analysis in ANSYS software.

Table 1: Parameters and their values [2]

Parameters	**Value**
Pitch Circle Diameter of Runner (D_R)	155 mm
Rated RPM (N)	1500 rpm
Diameter of shaft (d_s)	32 mm
Length of shaft (L)	519 mm
Cross section area of shaft (A)	0.0008042 m^2
Density of shaft material (ρ_s)	7860 kg/m^3
Density of disc material (ρ_d)	8550 kg/m^3
Young's Modulus of Elasticity of shaft (E)	202 GPa
Mass of rotating runner (m_d)	10.564 kg
Thickness of runner (h)	0.035 m
Area polar moment of inertia of shaft about z-z axis (J_s)	1.0294×10^{-7} m^4
Area moment of inertia of shaft about x-x or y-y axis (I_s)	5.1472×10^{-8} m^4
Area polar moment of inertia of disk about z-z axis (J_d)	0.11053×10^{-3} m^4
Area moment of inertia of disk about x-x or y-y axis (I_d)	0.5527×10^{-4} m^4

3. Mathematical modelling

3.1. Kinematics of Shaft-Disc system

Velocity vector of any point on neutral axis of the flexible shaft is:

$$v_s = (\dot{u} - \Omega v)\mathbf{j} + (\dot{v} + \Omega u)\mathbf{k}$$

Angular velocity of disk is:

$$\omega_d = \left[(\Omega + u'\dot{v}')\mathbf{i} + (-\Omega u' - \dot{v}')\mathbf{j} + (-\Omega v' + \dot{u}')\mathbf{k}\right]$$

3.2. Energy method

3.2.1. Total kinetic energy of Shaft-Disk system

Kinetic energy of shaft is sum of translational and rotational kinetic energy which is given by following relation [4].

$$T_s = \frac{1}{2}\rho_s A \int_0^L \left[(\dot{u} - \Omega v)^2 + (\dot{v} + \Omega u)^2\right] dz + \frac{1}{2}\rho_s J_s \int_0^L \left[(\Omega + u'\dot{v}')^2\right] dz + \frac{1}{2}\rho_s I_s \int_0^L \left[(-\Omega u' - \dot{v}')^2 + (-\Omega v' + \dot{u}')^2\right] dz$$

Kinetic energy of disk sum of translational and rotational kinetic energy which is given by following relation [4].

$$T_d = \Bigg[\frac{1}{2}m_d\left[(\dot{u} - \Omega v)^2 + (\dot{v} + \Omega u)^2\right] + \frac{1}{2}\rho_d h J_d (\Omega + u'\dot{v}')^2 + \frac{1}{2}\rho_d h I_d\left[(-\Omega u' - \dot{v}')^2\right] + \frac{1}{2}\rho_d h I_d\left[(-\Omega v' + \dot{u}')^2\right]\Bigg]\Bigg|_{z=L/2}$$

Total kinetic energy of the shaft-disc system is sum of kinetic energy of shaft and disc which is given by following relation.

$$T = T_s + T_d$$

3.2.2. Total potential energy of Shaft-Disk system

Since shaft is assumed as flexible, the strain energy of shaft due to bending is given by following relation [4].

$$V_s = \frac{1}{2}EI_s \int_0^L \left[(u'')^2 + (v'')^2\right] dz$$

Since disk is assumed as rigid, potential energy of disk is zero.

Mathematically, $V_d = 0$Total potential energy of the shaft-disc system is the sum of the potential energy of the shaft and the disk, which is given by the following relation:

$$V = V_s + V_d$$

3.2.3. External excitation

For same model, water jet force is directly taken from [5].

$F_j = 193$ N Assumption for external excitation: Change in force during starting is linear.

Mathematically, starting excitation is defined as:

$$F = \frac{F_j}{t_0} t\left(1 - u(t - t_0)\right) + F_j u(t - t_0)$$

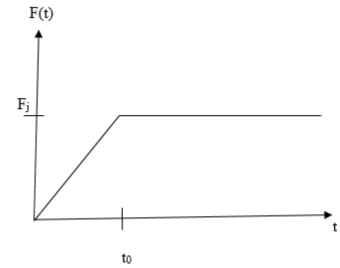

Figure 1: Starting excitation

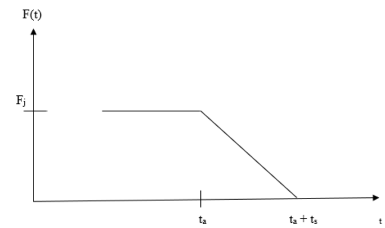

Figure 2: Shutdown excitation

The start-up transient period is the period between the opening of the nozzle and the turbine reaching nominal speed i.e., t_o. Since test rig of this work is manually operated, measuring of starting period depends upon speed of operator to open spear valve which gives average reading of $t_o = 5$s for start-up transient period.

Mathematically, shutdown excitation is defined as:

$$F = -\frac{F_j}{t_s}\left(t - (t_a + t_s)\right)\left(1 - u(t - (t_a + t_s))\right)$$

For shutdown period, initially turbine is operated for t_a = 300s in test rig which on shutdown by closing spear valve manually, system shutdowns in t_s = 10s.

3.2.4. External work done

Work done by the impact of jet is given by:

$$W_{\text{ext}} = F(t)\Big|_{z=L/2}$$

Assumed mode method

Using assumed mode method, displacement variable is [3].

$$u = \{\boldsymbol{\phi}(z)\}^T\{U(t)\} = \{\boldsymbol{\phi}\}^T\{U\}$$

$$v = \{\boldsymbol{\phi}(z)\}^T\{V(t)\} = \{\boldsymbol{\phi}\}^T\{V\}$$

Total kinetic and potential energies and external work done is discretized using above equations then applied in Lagrange's equation of motion.

3.2.5. *Lagrange's Equation of Motion*

Simplified equation of motion can be derived using Lagrange's equation as:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} - \frac{\partial W_{\text{ext}}}{\partial q} = 0$$

After simplification, we get following governing equation of motion:

$$M_i\ddot{U} + C_i\dot{V} + K_iU = F_i$$

$$M_i\ddot{V} - C_i\dot{U} + K_iV = 0$$

Where, equivalent parameters are as follows:

$$\begin{aligned}
M_i &= \rho_s A \int_0^L \left[\boldsymbol{\phi}\boldsymbol{\phi}^T\right] dz + \rho_s I_s \int_0^L \left[\boldsymbol{\phi}'\boldsymbol{\phi}'^T\right] dz \\
&\quad + m_d\left[\boldsymbol{\phi}_d\boldsymbol{\phi}_d^T\right] + \rho_d h I_d\left[\boldsymbol{\phi}'_d\boldsymbol{\phi}'^T_d\right], \\
C_i &= 2\rho_s A\Omega \int_0^L \left[\boldsymbol{\phi}\boldsymbol{\phi}^T\right] dz + 2\rho_s I_s\Omega \int_0^L \left[\boldsymbol{\phi}'\boldsymbol{\phi}'^T\right] dz \\
&\quad + 2m_d\Omega\left[\boldsymbol{\phi}_d\boldsymbol{\phi}_d^T\right] + 2\rho_d h I_d\Omega\left[\boldsymbol{\phi}'_d\boldsymbol{\phi}'^T_d\right] \\
&\quad + \rho_s J_s\Omega \int_0^L \left[\boldsymbol{\phi}'\boldsymbol{\phi}'^T\right] dz \\
&\quad + \rho_d h J_d\Omega\left[\boldsymbol{\phi}'_d\boldsymbol{\phi}'^T_d\right], \\
K_i &= -\rho_s A\Omega^2 \int_0^L \left[\boldsymbol{\phi}\boldsymbol{\phi}^T\right] dz - \rho_s I_s\Omega^2 \int_0^L \left[\boldsymbol{\phi}'\boldsymbol{\phi}'^T\right] dz \\
&\quad - m_d\Omega^2\left[\boldsymbol{\phi}_d\boldsymbol{\phi}_d^T\right] - \rho_d h I_d\Omega^2\left[\boldsymbol{\phi}'_d\boldsymbol{\phi}'^T_d\right] \\
&\quad + EI_s \int_0^L \left[\boldsymbol{\phi}''\boldsymbol{\phi}''^T\right] dz, \\
F_i &= F(t)\boldsymbol{\phi}_d
\end{aligned}$$

If shape function is taken as,

$$\phi_i = \sin\left(\frac{i\pi z}{L}\right)$$

For first mode

Coupled second order differential equation for first mode is:

$$M_1\ddot{U} - C_1\dot{V} + K_1U = F_1$$

$$M_1\ddot{V} + C_1\dot{U} + K_1V = 0$$

For second mode

Since in second mode excitation force is zero, no need for transient analysis during second mode.

For third mode

Since in third mode excitation force is present, transient analysis during third mode is done.

Since amplitude in the direction of jet is always higher than its perpendicular direction so total response is calculated only in the direction of jet. Here, total response in the direction of jet is sum of first and third response.

$$U = U_1\phi_1 + U_3\phi_3$$

Table 2: Equivalent parameters

For First Mode	**For Second Mode**	**For Third Mode**
$\phi_1 = \sin\left(\frac{\pi z}{L}\right)$	$\phi_2 = \sin\left(\frac{2\pi z}{L}\right)$	$\phi_3 = \sin\left(\frac{3\pi z}{L}\right)$
$M_1 = 12.208$ kg	$M_2 = 1.656$ kg	$M_3 = 12.239$ kg
$C_1 = 24.42\,\Omega\cdot$Ns/m	$C_2 = 3.342\,\Omega\cdot$Ns/m	$C_3 = 24.55\,\Omega\cdot$Ns/m
$K_1 = 3.622\times10^6 - 12.208\,\Omega^2\cdot$ N/m	$K_2 = 5.795\times10^7 - 1.656\,\Omega^2\cdot$N/m	$K_3 = 2.934\times10^8 - 12.239\,\Omega^2\cdot$ N/m
$F_1 = F(t)$	$F_2 = 0$	$F_3 = -F(t)$

3.3. Solution of mathematical model

Writing governing second order differential equation in the system of ODE

Assuming

$$U = x_1, \quad \dot{U} = \dot{x}_1 = x_2, \quad \ddot{U} = \dot{x}_2$$

$$V = y_1, \quad \dot{V} = \dot{y}_1 = y_2, \quad \ddot{V} = \dot{y}_2$$

System of ODEs are

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{K}{M}x_1 + \frac{C}{M}y_2 + \frac{F}{M} \\
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= -\frac{C}{M}x_2 - \frac{K}{M}y_1
\end{aligned}$$

Solving these four systems of ODE in MATLAB using ode45, we get response.

3.4. Verification With simulated solution from ANSYS

For simulated solution; material properties (density and Young's modulus of elasticity) of shaft are specified. For geometry; line body with circular cross section is created for shaft in design modeler. Since disk is assumed as rigid, point mass with same properties of disk is added instead of disk. Meshing is done with element size of 1 mm and medium smoothing for quality. Since bearing is assumed as rigid, two bearings at the end of shaft are added with rigid behavior. Displacement along shaft axis is set zero and about others axis set free. Similarly, rotation about shaft axis is set free while others are set zero. In transient structural analysis, dynamic force with varying magnitude at center of disk and varying rotational velocity are given in shaft disc system.

4. Result

4.1. Response during starting

4.1.1. Response from mathematical model during starting:

With the help of MATLAB program following response plot is obtained for first and third mode:

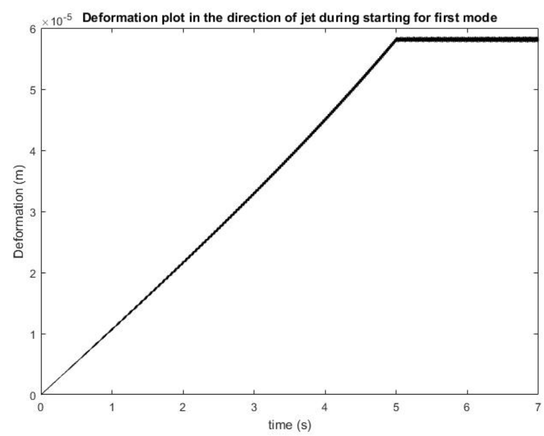

Figure 3: Response plot from mathematical model during starting for first mode

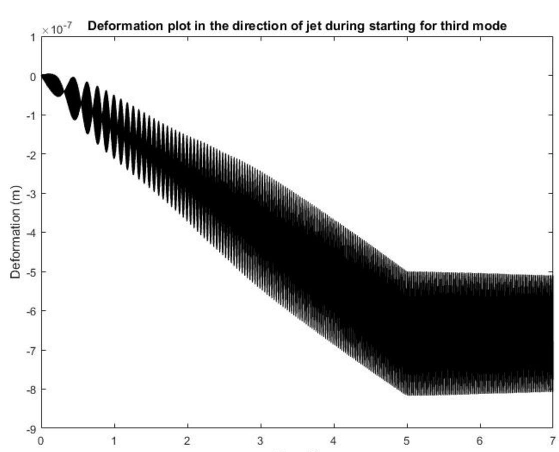

Figure 4: Response plot from mathematical model during starting for third mode

In above plot it is shown that response of shaft-disc system during starting in which peak amplitude of response during starting in the direction of jet for first mode is 58.34μm. In third mode, external excitation is opposite of first mode so nature of the response in third mode is also opposite to first mode. Peak amplitude of response during starting in the direction of jet for third mode is -0.82μm. As mentioned in Table 2, there is absence of forcing function in second mode, total response of the system is the sum of the responses in first and third mode. And as compared to first mode, effect of third mode is negligible. So, total response of the system during starting in the direction of jet is same as response of the first mode only. Higher modes were excluded from this analysis as their contribution to the overall response is minimal, given the dominant forcing frequency lies within the first mode only.

4.1.2. Response from simulated solution from ANSYS during starting:

With the help of ANSYS, following response plot is obtained:

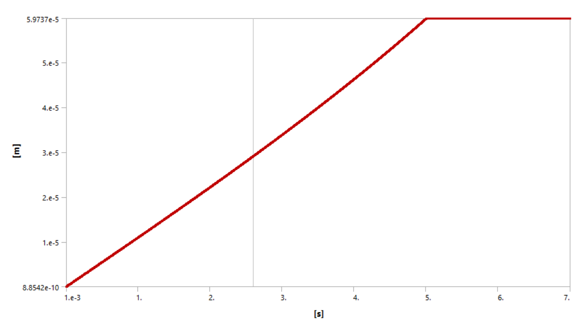

Figure 5: Response plot from simulated solution from ANSYS during starting

In above plot it is shown that response of shaft-disc system during starting in which peak amplitude of response during starting in the direction of jet for first mode is 59.71μm which is nearly same with the peak amplitude obtained from mathematical modeling of the system. After starting period, since excitation force is constant and continuously acting in the system to excite the system, response of the system is also in same nature of the forcing function. The peak values of responses from mathematical model and simulated solution from ANSYS are obtained during starting period of 5s which can be considered in designing phase of shaft-disc system to avoid these peak amplitudes or in stress analysis as well.

4.2. Response during shutdown

4.2.1. Response from mathematical model during shutdown:

With the help of MATLAB program, following response plot is obtained for first mode and third mode:

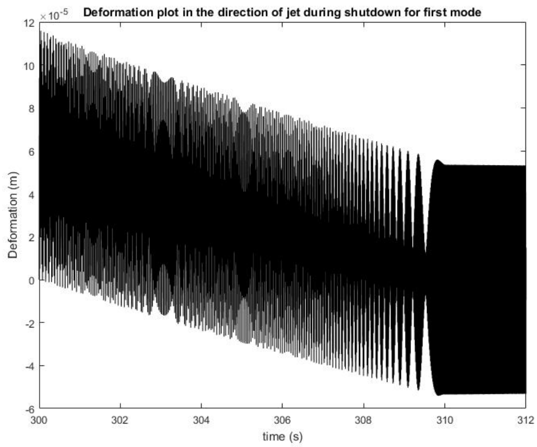

Figure 6: Response plot from mathematical model during shutdown for first mode

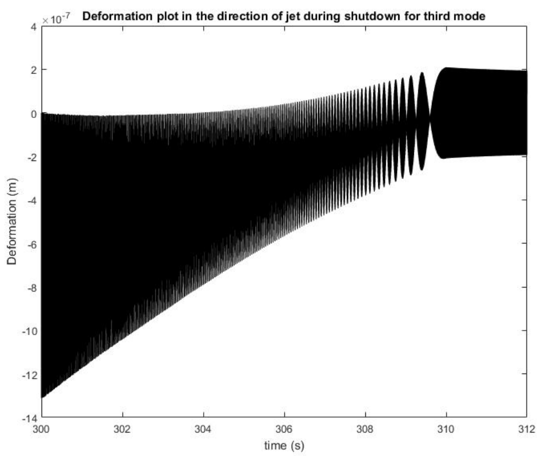

Figure 7: Response plot from mathematical model during shutdown for third mode

In above plot it is shown that response of shaft disc-system during shutdown in which peak amplitude of response during shutdown in the direction of jet for first mode is 115.87μm. In third mode, external excitation is opposite of first mode so nature of the response in third mode is also opposite to first mode. Peak amplitude of response during shutdown in the direction of jet for third mode is -1.30μm. As mentioned in Table 2, there is absence of forcing function in second mode, total response of the system is the sum of the responses in first and third mode. And as compared to first mode, effect of third mode is negligible. So, response of the system during shutdown in the direction of jet is same as response of the first mode.

4.2.2. Response from simulated solution from ANSYS during shutdown:

With the help of ANSYS, following response plot is obtained:

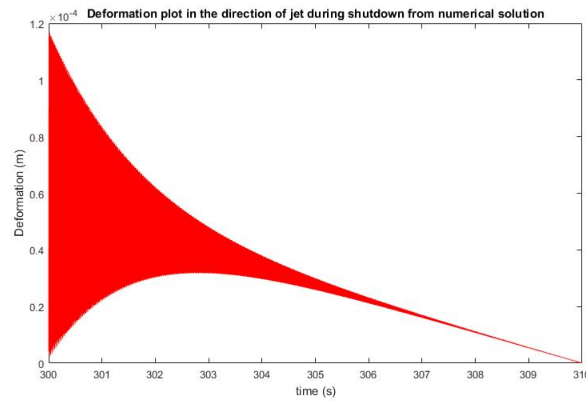

Figure 8: Response plot from simulated solution from ANSYS during shutdown

In above plot it is shown that response of shaft-disc system during shutdown in which peak amplitude of response during shutdown in the direction of jet is 116.54 μm which is nearly same with the peak amplitude obtained from mathematical modeling of the system. For shutdown of shaft-disc system, initially system was operated for 300s and then shutdown was initiated. The peak values of responses from mathematical model and simulated solution from ANSYS are obtained during shutdown period of 10s which can be considered in designing phase of shaft-disc system to avoid these peak amplitudes or in stress analysis as well.

5. Conclusions

From mathematical model, the governing equations of the system are found to be coupled systems of second order differential equation. Peak amplitude during starting and shutdown from mathematical model obtained is 58.34μm and 115.87μm respectively. Peak amplitude during starting and shutdown from simulated solution from ANSYS obtained is 59.71μm and 116.54μm respectively. The variation of peak amplitude of response from mathematical model and simulated solution from ANSYS during starting and shutdown is 2.35% and 0.58% respectively. While designing, if this shaft-disc system is used then these peak values of responses obtained, can be taken as a reference value. Since the size of shaft-disc systems and value of forcing function is very small as compared to real one, the values of displacement obtained are also very small but it can be used in stress analysis.

Acknowledgements

No financial aid has been received for this research.

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