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Harmonic analysis of cantilever beam with uniform cross section with a concentrated mass at its free end and with a uniformly distributed harmonic force

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Abstract

Cracks are one of the most catastrophic breakdowns that reduce useful life and contribute to the collapse of structural elements and structures. Today, component failure is the primary cause of structural element and structure failure. Depending on the depth and location of the crack, surface cracks on structural elements like beams result in significant differences in stiffness of the component. Certain vibrational characteristics can be used to identify beam fractures. The existence of fractures affects vibration factors including mode shapes and natural frequencies as well as the structural integrity. Modal analysis may be used to determine vibrational properties including modal frequencies and mode shapes for a structural component. In this work, harmonic analysis of a cantilever steel beam is conducted for various scenarios with cracks, no cracks at various locations using ANSYS software. Harmonic analysis is done within the study by taking a uniformly distributed force over the beam and the change in modal frequencies along with change in deformation pattern for different cases are visualized.

1. Introduction

Most of the members of engineering structures operate under loading conditions, which may cause damages or cracks in overstressed zones. Cracks in beams and other structural elements modify a structure's physical qualities, which in turn change how the structure responds dynamically. The monitoring of the changes in the response parameters of a structure has been widely used for the assessment of structural integrity, performance and safety[1].

Measurement of vibration and its analysis on engineering structures has gained quite a bit of popularity in engineering field because of the benefits it offers. Advancement in material science and technology have created an increasing requirement for reliable dynamic analysis of structural components. The behavior of a structure at "resonance" is a key aspect of study in civil as well as mechanical engineering field. Resonant frequency of vibration for a structure corresponds to that natural frequency at which we see maximum amplitude. Modal analysis has become a major alternative to provide a helpful contribution in understanding and in control of many vibrational phenomena's which are encountered in application. Determining the nature and extent of vibration response levels and verifying theoretical models and prediction are both major objectives[2].

A harmonic analysis is a type of periodic stimulation that is typically performed during steady-state operation to ascertain how a structure would react to a sinusoidal loading at a specific frequency[3]. After Modal analysis, harmonic analysis is done in this study to analyze the structure's frequency response.

2. Objectives

The main objectives of this analysis are:

- To perform modal analysis on both healthy and cracked cantilever beam to determine the first

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three mode shapes and three natural frequencies of transverse vibration.

- To conduct a harmonic analysis to determine the vibration's amplitude and phase angles for both cracked and un-cracked beam models.
- To understand how frequency and amplitude of vibration of a beam vary with the position of crack in the beam.

3. Methodology

For harmonic analysis, the geometry and mechanical characteristics indicated in the table below are considered. Structural Steel is taken as the material for the beam to carry out the analysis. The transverse crack under consideration is open in nature and has shifted places by 500 and 1000 mm. The crack depth is taken constant throughout our analysis which is 30mm. Comparison between amplitude and phase angle with respect to natural frequency is done within the scope of this work. The model is sketched in Design Modeler that is already available in ANSYS software and meshing is also done within ANSYS sequentially.

Table 1: Mechanical Properties

S.N.	Parameters	Specification
1.	Young's Modulus (E)	200×10^9 Pa
2.	Poisson's Ratio	0.3
3.	Mass Density (ρ)	7850 kg/m^3
4.	Length of Beam	2000 mm
5.	Width of Beam	100 mm
6.	Thickness	100 mm

4. Model geometry And crack modelling

Four different geometries were constructed for this analysis.

Case I:
The first geometry is of a healthy beam without any crack. The cross-section of the beam is taken as 100mm x 100mm and the length of the beam is taken 2000mm.

Case II:
Cross section of the beam as well as the length of the beam is unchanged. Crack is added at the position of 0.25L or 500mm from the beam's fixed end.

Case III:
Cross section of the beam as well as the length of the beam is unchanged. Crack is added at the position of 0.5L or 1000mm from the beam's fixed end.

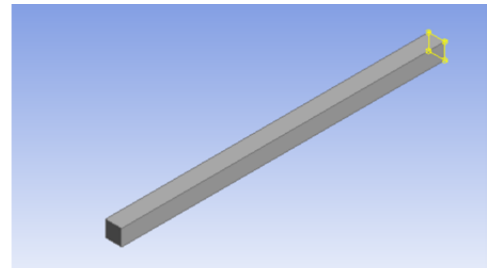

Figure 1: Geometry of the un-cracked beam

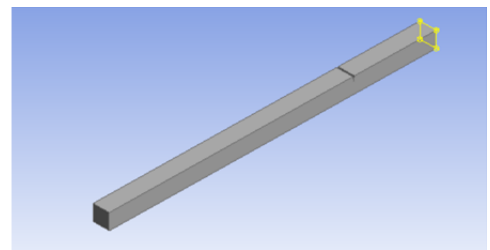

Figure 2: Geometry of beam at crack position of 0.25 L

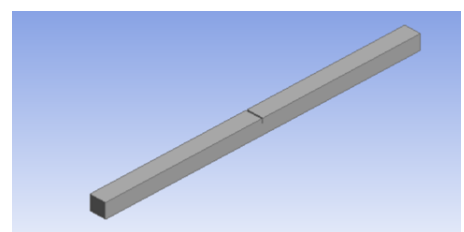

Figure 3: Geometry of beam with crack at 0.5L

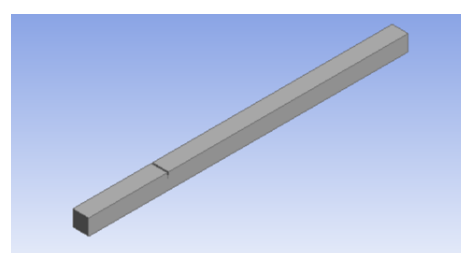

Figure 4: Geometry of beam with crack at 0.75L

Case IV:
Cross section of the beam as well as the length of the beam is unchanged. Crack is added at the position of 0.75L or 1500mm from the beam's fixed end.

4.1. Crack modelling

Studying the effects of beam cracks becomes more difficult since they alter the geometrical features of the beam. Thus, crack modeling is crucial. Free vibration analysis has been carried out considering the nonlinearity of the material and geometry in the cracked beam model[4]

A triangular crack of depth 30 mm and length 10 mm on the top face was constructed on ANSYS. Its distance from the fixed end was varied as 500 mm and 1000 mm to observe the change of crack position on amplitude of vibration.

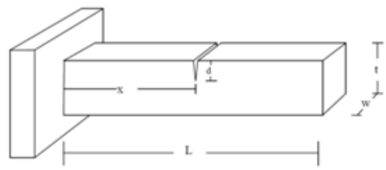

Figure 5: Cracked Cantilever beam

4.2. Finite element modelling

As it is exceedingly challenging to undertake model analysis using an analytical technique when there are discontinuities, the finite element analysis method is used to perform model analysis of an un-cracked and cracked cantilever beam. For the healthy beam, tetrahedral elements with an element size of 0.03 m were employed. Tetrahedral elements with an element size of 0.015 m were employed for the beam with the fractures the elements used were tetrahedral, with an element size of 0.03 m. For the beam with the fractures, tetrahedral elements with an element size of 0.015 m were used.

The beam's furthest left end was fixed to limit its degree of freedom, whereas the other end was free. Figure 6 shows that on the top face of cantilever beam 200N force was applied. Also, a mass of 10 kg was applied at the free end of the beam. First, using modal analysis, the first three mode forms and natural frequency of transverse vibration for both an un-cracked and a cracked cantilever beam were explored. Next, using harmonic analysis, the amplitude of vibration and associated frequency were determined.

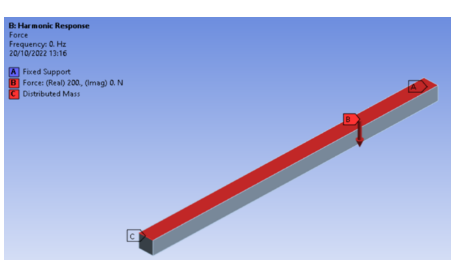

Figure 6: Loading Conditions on the Cantilever beam

5. Results and discussion

5.1. Modal analysis results for the un-cracked beam

For the purpose of conducting a modal analysis of the un-cracked cantilever beam initially, the problem geometry was developed, and the proper boundary conditions were enforced. Utilizing ANSYS Mechanical, the first three natural frequencies were discovered.

Figure 7, Figure 8 and Figure 9 shows the transverse deformation of the beam due to 1^{st}, 2^{nd} and 3^{rd} mode of frequency respectively. In the Figure 7 we have 1 stationary node, and the number of stationary nodes increases as the mode of frequency increases.

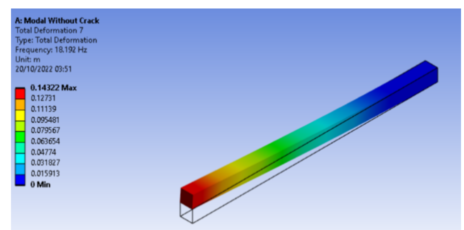

Figure 7: Deformation of beam due to 1^{st} mode of frequency

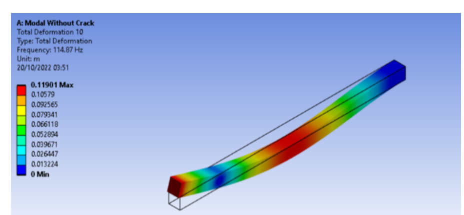

Figure 8: Deformation of beam due to 2^{nd} mode of frequency

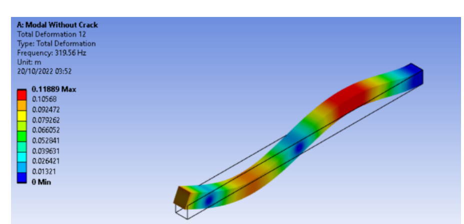

Figure 9: Deformation of beam due to 3^{rd} mode of frequency

5.2. Modal analysis results for the cracked beam

Crack was added within the beam at different location from the fixed end. The location of cracks was 0.25L, 0.5L and 0.75L. Figure 10, Figure 11 and Figure 12 shows the deformation of the beam when the position of crack was at 0.25L.

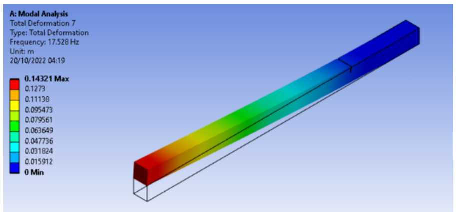

Figure 10: Deformation of beam with crack at 0.25L due to 1st mode of frequency

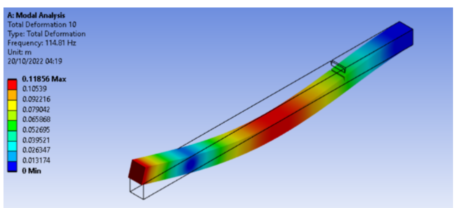

Figure 11: Deformation of beam with crack at 0.25L due to 2nd mode of frequency

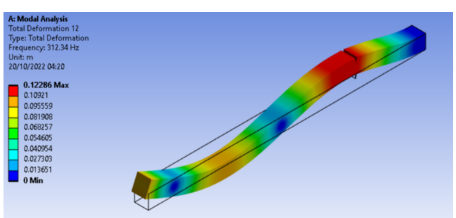

Figure 12: Deformation of beam with crack at 0.25L due to 3rd mode of frequency

Similarly, Figures 13, 14, and 15 shows the deformation of beam due to 1st, 2nd and third mode of frequency when position of the crack is at 0.5L.

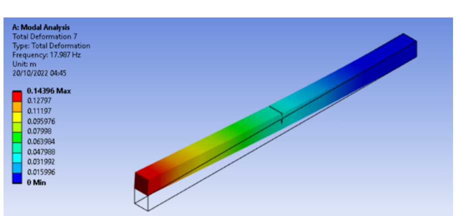

Figure 13: Deformation of beam with crack at 0.5L due to 1st mode of frequency

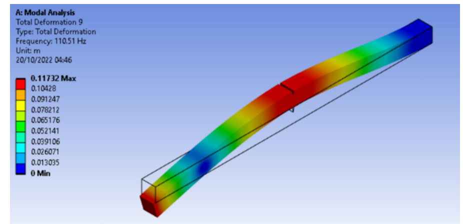

Figure 14: Deformation of beam with crack at 0.5L due to 2nd mode of frequency

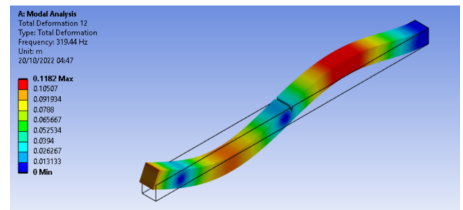

Figure 15: Deformation of beam with crack at 0.5L due to 3rd mode of frequency

When the position of the crack on the beam is shifted to 0.75L the deformation relating to 1st, 2nd and 3rd modes of frequency are shown in Figure 16,17,18.

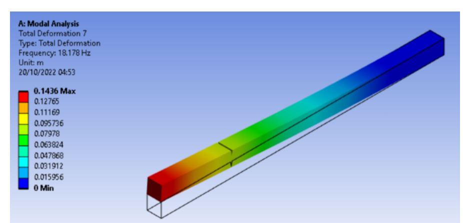

Figure 16: Deformation of beam with crack at 0.75L due to 1st mode of frequency

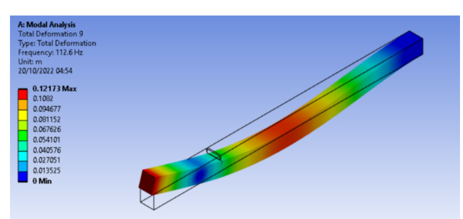

Figure 17: Deformation of beam with crack at 0.75L due to 2nd mode of frequency

5.3. Variation of frequency at different modes for different geometries

In the given table we can see how modal frequencies varies for beam without crack and beam with crack at different locations.

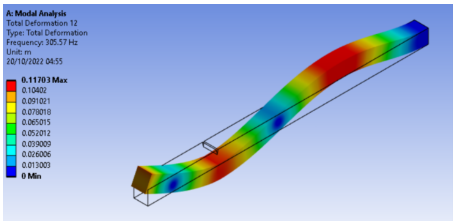

Figure 18: Deformation of beam with crack at 0.75L due to 3rd mode of frequency

The frequency corresponding to modes 1, 2, and 3 was observed to grow initially, and subsequently drop, as the crack was moved towards the free end of the cantilever beam.

Table 2: Variation of frequency at different modes for different geometries

Mode	Frequency (Hz)			
	Beam without Crack	Beam with crack at 0.25L	Beam with crack at 0.5L	Beam with crack at 0.75L
1st	18.192	17.528	17.987	18.178
2nd	114.87	114.81	110.51	112.6
3rd	319.56	312.34	319.44	305.57

5.4. Variation of total deformation at different modes for the geometries

The given table summarizes the deformation occurred for the beam at different modal frequencies. Maximum deformation is found to be when the crack is positioned at 0.5L and at 1st Mode.

Table 3: Variation of deformation at different modes for different geometries

Mode	Deformation (m)			
	Beam without Crack	Beam with crack at 0.25L	Beam with crack at 0.5L	Beam with crack at 0.75L
1st	0.14322	0.14321	0.14396	0.1436
2nd	0.11901	0.11856	0.11732	0.12173
3rd	0.11889	0.12286	0.1182	0.11703

The given table summarizes the deformation occurred for the beam at different modal frequencies. The deformation associated with modes 1 and 2 was shown to be growing up to 0.5L and then reducing when the crack was moved towards the cantilever beam's free end. However, mode 3 exhibits the opposite trend.

5.5. Harmonic analysis results for the un-cracked beam

After the modal frequencies are found we gear towards harmonic analysis. For the harmonic analysis the force is applied on the planar surface with a magnitude of 200N. For structural steel a constant damping coefficient of 0.02 was taken and the analysis was conducted. The frequency response of the beam without any crack can be seen in Figure 19.

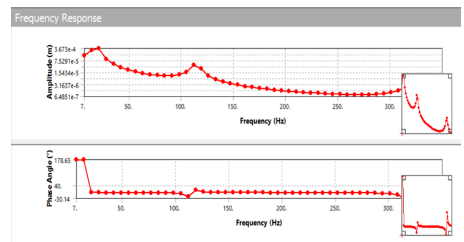

Figure 19: Frequency response of the beam without any crack

In Figure 19 we can see the maximum amplitude occurs at around 20-25 Hz.

5.6. Harmonic analysis results for the cracked beam

When crack is added within the beam, we see a change in frequency response with respect to the position of the crack. Figure 20, 21, 22 shows the frequency response of the beam when cracks are positioned at 0.25L, 0.5L and 0.75L respectively.

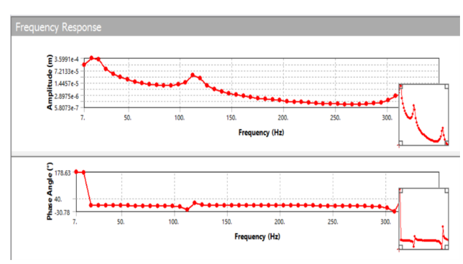

Figure 20: Frequency response of the beam with crack at 0.25L

From Figures 20, 21 and 22 we can see how the response of the beam is changing with change in position of the crack. Magnitude of amplitude is lower when the crack is positioned at 0.5L in comparison to when crack is positioned at 0.25L and 0.75L. This suggest the amplitude of the beam increases as the position of crack is away

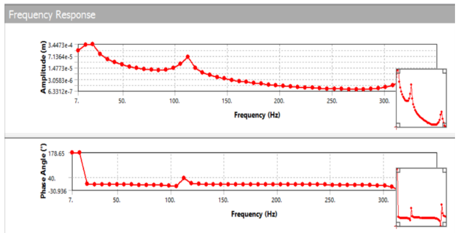

Figure 21: Frequency response of the beam with crack at 0.5L

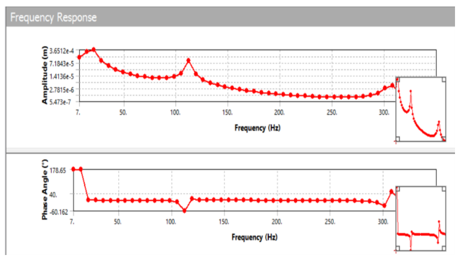

Figure 22: Frequency response of the beam with crack at 0.75L

from center. As the crack reaches near the center the amplitude decreases and increases again as the crack crosses the center.

6. Conclusion

Based on the results obtained from Modal and Harmonic Analysis the variations in Eigen frequency, frequency response and amplitude, following conclusions are made:

- Variations in modal frequencies depends not only on addition of crack to the beam but also on the position of the crack. Different position of crack seems to have relative effect on specific mode of frequency.
- Crack at 0.25L seems to have greater effect on 1st mode of frequency. When crack is at 0.5L it tends to have more effect on 2nd mode of frequency. Similarly, when crack is at 0.75L it seems to have more effect on 3rd mode of frequency. This suggest if crack is near to the stationary nodes within a mode of frequency it have more drastic effect and reduces the natural frequency of the beam.
- Deformation is decreasing as we move from 1st mode of frequency to 3rd which suggests our maximum amplitude or deformation of the beam will occur near the 1st mode of frequency, which can be seen from the frequency response of the beam.
- Crack at position of 0.5L seems to have the lowest magnitude of amplitude which suggests cracks at near the center of the beam reduces the maximum amplitude of the beam.

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