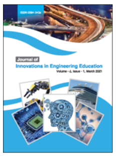

Journal of Innovations in Engineering Education
JIEE 2021, Vol. 4, Issue 1, pp 34-41.

https://doi.org/10.3126/jiee.v4i1.34642

THAPATHALI CAMPUS
Institute of Engineering
Tribhuvan University

Dynamic response of vertical shaft pelton turbine unit for forced vibration

Raman Koirala[a,*], Mahesh Chandra Luintel[b]

[a]*Department of Automobile and Mechanical Engineering, Thapathali Campus, Institute of Engineering, Tribhuvan University, Thapathali, Kathmandu, Nepal*
[b]*Department of Mechanical Engineering, Pulchowk Campus, Institute of Engineering, Tribhuvan University, Lalitpur, Nepal*

ARTICLE INFO

Article history:

Received 14 Dec 2020
Received in revised form
14 Jan 2021
Accepted 29 Jan 2021

Keywords:

Vertical shaft
Pelton
Forced vibration
Amplitude
Dynamic response

Abstract

This research work was carried out to model the excitation force imparted by water jet in the form of Fourier series and determine the forced response of Pelton turbine unit of Khulelhani-I hydropower analytically by developing mathematical mode. Amplitude of forced vibration form analytical was compared with simulation result. The mathematical model was developed by calculating the kinetic energy of disk and potential energy of both disk and shaft. Hamilton's principle was used to determine equation of motion and then Galerkin method was used to determine response of the system. Fourier analysis was done to obtain the function in its exact form. The developed methodologies were followed to find the analytical solution of Kulekhani-I unit of $3100kW$ rated at $600rpm$. A rigid disk (runner and bucket assembly) was situated along end of flexible shaft with fixed support at the shaft. First five Fourier components are to be considered in analysis for meaningful representation of forcing function. The amplitude of vibration of Pelton turbine unit with single nozzle in Y-direction (the direction of water jet) obtained by analytical method was closed with that obtained from the ANSYS simulation.

1. Introduction

Pelton turbine is an impulse which transform potential energy of water into kinetic energy in form of a water jet by which impacts and drive Pelton runner. Vibrations are oscillations in mechanical dynamic systems. Although any system can oscillate when it is forced to do so externally, the term "vibration" in mechanical engineering is often reserved for systems that can oscillate freely without applied forces. Sometimes these vibrations cause minor or critical effects on the performance and safety of many in engineered systems. whenever an aircraft wing vibrates, passengers in the aircraft become uncomfortable especially when the frequencies of vibration matches with natural frequencies of the human body.

Rotors are the rotating parts having a various engineering application like turbines, compressors, pumps, fans, etc. Dynamic behavior of rotor and its components should be study to understand its mechanism and failures. Rotor dynamics study the lateral/ transverse(bending), longitudinal(axial) and torsional vibration of rotating shafts with the objectives of limiting the vibration under acceptable range. Transverse is mode of rotor dynamics associated with bending of rotor and similarly longitudinal in the motion in axial direction and torsional is twisting around its own axis of rotor.

In turbine system rotor and blades are subjected to extreme loading conditions. Mechanical and electrical forces are the major concerns to increase the vibrations in hydro turbine. Dynamic response is the response of structure under dynamic loading condition and it will give an idea about how the system will behave under such conditions. Thus, the behavior of rotor system can be analyzed to some extent by studying their dynamic response by using approximated mathematical modeling and simulation techniques.

1.1. Literature review

Vibration that occurs under excitation of external force is called forced vibration. When the oscillatory excitation

*Corresponding author:
koiralaraman437@gmail.com (R. Koirala)

is given, the system is forced to vibrate at excitation frequency. If the frequency of excitation corresponds with the one of the natural frequencies of the system then resonance will occur. Hence the calculations of natural frequencies are very important in the study of vibration [1]

Rotor dynamics is an engineering that studies the vibrations of rotating shaft, with the objective to predict the behavior of system under the vibrational condition and the maintaining the level of vibration under an acceptable range. The vibration theory for rotor-dynamics was first proposed by August Föppl in 1895 and similarly Henry Homan Jeffcot in 1919. By using a simplified rotor/bearing system they proposed the basic theory on prediction of rotor vibration. This simplified rotor/bearing system is known to be the Föppl/Jeffcot rotor which is often used to evaluate more complex rotor-dynamic systems [2]

Bai et al. used ANSYS finite element in their research paper to model the main shaft system in the hydro-turbine unit. They carried out modal analysis to calculate the critical speed of rotation [3]. Egusquiza et al. proposed a theoretical model using finite element methods (FEM) in order to simulate the dynamic behavior of the Pelton turbine. Experimental verification was done under that were carried out in an existing hydro-turbine unit [4].

Rajak et al. researched on dynamic analysis of the Pelton turbine to obtain the natural frequency of the system, mathematical model was developed to calculate the kinetic energy and the strain energy, the equations of motion were derived using Lagrange equations and the Rayleigh-Ritz method to study the basic phenomena of cylindrical mode of rotor and the validation carried out in Mechanical APDL 14.5 to obtain the critical frequency [5].

Research by Panthee et al. presented the computational fluid dynamics (CFD) analysis of Pelton turbine of Khimti Hydropower in Nepal. The runner was scaled down by meeting IEC 60193 standard. Whole simulation was performed in ANSYS-CFX. The results obtained from simulation showed high pressure in splitter and deep face of the bucket. The torque calculation was further used to calculate the efficiency and analytical validation of the runner [6].

Motra and Luintel carried out the research work which was basically focused on the modeling of the Pelton turbine unit and had covered dynamic behavior of the centrally located rigid runner on the circular flexible shaft, which was supported by the rigid bearings on both ends and enabled to determine the natural frequency of the system by using different models and compared with the continuous system model. The unit was modeled as a discrete and continuous system. Considering the Foppl/Jeffcot rotor models and Rayleigh's energy method. The model for the continuous system was developed by calculating the kinetic and potential energy of the runner buckets assembly and shaft. The governing equations were formulated by using Lagrange's equation and solved analytically for natural frequency by using Rayleigh-Ritz method [7].

Karki et al. carried out the research by modeling of excitation force imparted by water jet in the form of Fourier series and determined the forced response of Pelton turbine unit analytically. The mathematical model was developed by calculating the kinetic energy and potential energy of the disk and shaft. Lagrange's equation, Rayleigh-Ritz method and virtual work method were used to derive the equation of motion of forced vibration condition. The developed methodologies were followed to find the analytical solution of dynamic response of selected Pelton turbine unit of 2 kW with single nozzle rated at $1500 RPM$ [8].

Research by Luintel and Bajrachayra present the method to study the dynamic response of the shaft of a Pelton turbine. Mathematical models for bending vibration of Pelton turbine, was developed by using Lagrange equation of motion followed by assumed modes method. Where Pelton wheel is assumed as a rigid disk attached on a Euler-Bernoulli shaft. The force by the water jet is represented in the form of Fourier series [9].

2. Mathematical model development

The mathematical model was developed by using various equations. Equations of motion was developed by using energy method. Hence at very first mathematical model for finding total energy of system was derived. Then by using Hamilton principle and Galerkin method the equation of motion was derived for dynamic forced response. The force imparted by water jet to bucket was estimated and the force function was modelled in the Fourier series. Final equation of motion for system was solved to find the amplitude of vibration.

2.1. Total energy of system

Since bearings are considered as rigid and undamped. The system consists of only shaft and disk. For disk only kinetic energy (K.E) is calculated while for shaft both kinetic and potential energy (P.E) is calculated because it is considered as flexible element. Thus, three reference frames will be taken into account i.e., fixed inertial frame, frame fixed in center of disk and frame fixed with shaft.

2.2. Euler angles and rotational matrix

The orientation of a rigid body with respect to fixed coordinate system is describe by using Euler angles. It is used to relate the rotation of one frame inside another frame of reference. There is different order of Euler angles which transform the inertial frame XYZ to final position of the shaft and disk. The consecutive rotation of the same axis is not considered for the Euler angles. The order in which the rotations is done is important. There are several conventions for Euler angles, depending on the axes about which the rotations are carried out. Hence twelve unique meaningful ordered sequence of rotation exists. The twelve Euler angles conventions are: XYX, XYZ, XZX, XZY, YXY, YXZ, YZX, YZY, ZXY, ZXZ, ZYX, ZYZ. 3-1-2 Euler angle are used to obtain Rotational matrix. The xyz rotated in relation to the XYZ system according to set of angles shown in Fig. 1. Where general orientation of cross section of the beam element can be obtained by rotation around X axis with angle φ, then by an angle θ_y around new axis y_1 and subsequently by an angle θ_z around z_2.

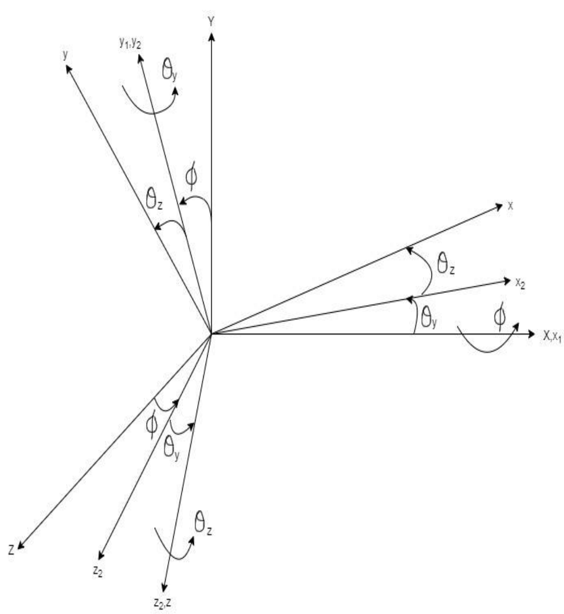

Figure 1: Rotational angles

Where, ω is an instantaneous angular speed which is related to coordinate system XYZ can be seen in Equation (1).

$$\omega = \dot{\varphi}_i + \dot{\theta}_{yj} + \dot{\theta}_k \tag{1}$$

Where: i, j and k are the unit vector along the axis x, y_1 and z_2. Equation (1) is transformed for the XYZ coordinate system and assuming the small angles to θ_z, θ_y will get: [10]

$$\omega = \begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} = \begin{bmatrix} \dot{\varphi} - \dot{\theta}_z \theta_y \\ \dot{\theta}_y cost\varphi - \dot{\theta}_z sin\varphi \\ \dot{\theta}_z cost\varphi - \dot{\theta}_y sin\varphi \end{bmatrix} \tag{2}$$

2.3. The Shaft and Disk

Kinetic energy of shaft is given by sum of translator and rotary in axial and transverse direction [10]

$$\begin{aligned} T_s &= \frac{1}{2}\rho_s A_s \int_0^L \left(V_x^2 + V_y^2 + V_z^2\right) dx \\ &+ \frac{1}{2} J_p \int_0^L \omega_x^2 \, dx \\ &+ \frac{1}{2} \int_0^L \left(\omega_y^2 + \omega_x^2\right) dx \end{aligned} \tag{3}$$

For strain energy of shaft combine axial deformation and bending can be written as below [10].

$$\begin{aligned} U_s &= \frac{1}{2} \int_0^L \left[EA \left(\frac{\mathrm{d}u}{\mathrm{d}x}\right)^2 + EI \left(\frac{\partial^2 v}{\partial x^2}\right)^2 \right. \\ &\left. + EI \left(\frac{\partial^2 w}{\partial x^2}\right)^2 \right] dx \end{aligned} \tag{4}$$

Now, kinetic energy of disk is given by:

$$\begin{aligned} T_d &= \left[\frac{1}{2} M_D \left(\dot{V}_x^2 + \dot{V}_y^2 + \dot{V}_z^2\right) \right. \\ &+ \frac{1}{2} J_{pd} \left(\dot{\varphi} - 2\dot{\varphi}\dot{\theta}_z \theta_y\right) \\ &\left. + \frac{1}{2} J_{dd} \left(\dot{\theta_y}^2 + \dot{\theta_z}^2\right) \right]_{x=L} \end{aligned} \tag{5}$$

Where,

$$\begin{aligned} \dot{V}_x^2 + \dot{V}_y^2 + \dot{V}_z^2 &= \dot{u}^2 - 2\dot{u}v\dot{\theta}_z cos\varphi - 2\dot{u}v\dot{\theta}_y sin\varphi \\ &+ 2\dot{u}w\dot{\theta}_y cos\varphi - 2\dot{u}w\dot{\theta}_z sin\varphi \\ &+ \dot{v}^2 + w^2\dot{\varphi}^2 - 2\dot{v}u\dot{\theta}_z cos\varphi \\ &- 2\dot{v}u\dot{\theta}_y sin\varphi + 2\dot{v}w\dot{\varphi} + \dot{w}^2 \\ &+ v^2\dot{\varphi}^2 - 2\dot{w}u\dot{\theta}_y cos\varphi \\ &- 2\dot{w}u\dot{\theta}_z sin\varphi + 2\dot{w}v\dot{\varphi} \end{aligned}$$

2.4. Hamilton's principle

The Hamilton's principle expresses that the motion of the system between a given initial configuration t0 and a given final configuration t1 is such that it extremizes the action integral[11].

$$I = \int_{t_0}^{t_1} L dt$$

Where 'L' is the Lagrange function which is expressed as $L = T - U$ where 'T' is kinetic energy and 'U' is potential energy of system. Hence after using Hamilton principle each term associated with δu, δv, δw, $\delta\theta_y$, and $\delta\theta_z$ from the above equations are collected.

For u,

$$0 = \rho_s A_s \ddot{u} - \rho_s A_s \dot{v}\dot{\theta}_z cos\phi - \rho_s A_s v\dot{\theta}_z \dot{\phi} sin\phi + \rho_s A_s \dot{v}\dot{\theta}_y (cos\phi - sin\phi) - \rho_s A_s v\ddot{\theta}_y sin\phi - \rho_s A_s phiv\dot{\theta}_y cos\phi + 2\rho_s A_s \dot{w}\dot{\theta}_y cos\phi + \rho_s A_s w\ddot{\theta}_y cos\phi - \rho_s A_s w\dot{\theta}_y \dot{\phi} sin\phi - \rho_s A_s w\ddot{\theta}_z sin\phi - \rho_s A_s w\dot{\theta}_z \dot{\phi} cos\phi - EAu'' + \left[M_D \ddot{u} - M_D \dot{v}\dot{\theta}_z cos\phi - M_D v\dot{\theta}_z \dot{\phi} sin\phi + M_D \dot{v}\dot{\theta}_y (cos\phi - sin\phi) - M_D v\ddot{\theta}_y sin\phi - M_D \dot{\phi} v\dot{\theta}_y cos\phi + 2M_D \dot{w}\dot{\theta}_y cos\phi + M_D w\ddot{\theta}_y cos\phi - M_D w\dot{\theta}_y \dot{\phi} sin\phi - M_D w\ddot{\theta}_z sin\phi - M_D w\dot{\theta}_z \dot{\phi} cos\phi\right] \delta_y (x - L)$$

For v,

$$0 = \rho_s A_s \ddot{v} - \rho_s A_s u\ddot{\theta}_z cos\phi + \rho_s A_s u\dot{\phi}\dot{\theta}_z sin\phi - \rho_s Au\ddot{\theta}_y sin\phi - \rho_s A_s u\dot{\phi}\dot{\theta}_y cos\phi - \rho_s A_s v\dot{\phi}^2 + EIV'''' + \left[M_D \ddot{v} - M_D u\ddot{\theta}_z cos\phi + M_D u\dot{\phi}\dot{\theta}_z sin\phi - M_D u\ddot{\theta}_y sin\phi - M_D u\dot{\phi}\dot{\theta}_y cos\phi - M_D v\dot{\phi}^2\right] \delta_f (x - L) - F(t)\delta_f (x - L)$$

For w,

$$0 = \rho_s A_s \ddot{w} - \rho_s A_s w\dot{\phi}^2 - 2\rho_s A_s \dot{u}\dot{\theta}_y cos\phi - \rho_s A_s u\ddot{\theta}_y cos\phi + \rho_s A_s u\dot{\phi}\dot{\theta}_y sin\phi - \rho_s A_s u\ddot{\theta}_z sin\phi - \rho_s A_s u\dot{\phi}\dot{\theta}_z cos\phi + EIw'''' + \left[M_D \ddot{w} - M_D w\dot{\phi}^2 - 2M_D \dot{u}\dot{\theta}_y cos\phi - M_D u\ddot{\theta}_y cos\phi + M_D u\dot{\phi}\dot{\theta}_y sin\phi - M_D u\ddot{\theta}_z sin\phi - M_D u\dot{\phi}\dot{\theta}_z cos\phi\right] \delta_f (x - L)$$

For θ_y

$$0 = \ddot{\theta}_y + J_p \dot{\phi}\dot{\theta}_z - 2\rho_s A_s \dot{u}\dot{v} sin\phi - \rho_s A_s \ddot{u} v sin\phi - \rho_s A_s \dot{u}\dot{\phi} v cos\phi + \rho_s A_s \ddot{u} w cos\phi - \rho_s A_s \dot{u} w\dot{\phi} sin\phi - \rho_s A_s u\ddot{v} sin\phi - \rho_s A_s u\dot{v}\dot{\phi} cos\phi - \rho_s A_s u\ddot{w} cos\phi + \rho_s A_s u\dot{w}\dot{\phi} sin\phi + [J_{dd}\ddot{\theta}_y + J_{pd}\dot{\phi}\dot{\theta}_z - 2M_D \dot{u}\dot{v} sin\phi - M_D \ddot{u} v sin\phi - M_D \dot{u}\dot{\phi} v cos\phi + M_D \ddot{u} w cos\phi - M_D \dot{u} w\dot{\phi} sin\phi - M_D u\ddot{v} sin\phi - M_D u\dot{v}\dot{\phi} cos\phi - M_D u\ddot{w} cos\phi + M_D u\dot{w}\dot{\phi} sin\phi]\delta_f (x - L)$$

For θ_z

$$0 = J_d \ddot{\theta}_z - J_p \dot{\phi}\dot{\theta}_y - \rho_s A_s \ddot{u} v cos\phi + \rho_s A_s \dot{u}\dot{\phi} v sin\phi - \rho_s A_s \ddot{u} w - \rho_s A_s \dot{u} w\dot{\phi} cos\phi - \rho_s A_s \ddot{v} u cos\phi + \rho_s A_s u\dot{v}\dot{\phi} sin\phi - \rho_s A_s u\ddot{w} sin\phi - \rho_s A_s u\dot{w}\dot{\phi} cos\phi - 2\rho_s A_s \dot{u}\dot{v} cos\phi - 2\rho_s A_s \dot{u}\dot{w} sin\phi + \left[J_{dd}\ddot{\theta}_z - J_{pd}\dot{\phi}\dot{\theta}_y - M_D \ddot{u} v cos\phi + M_D \dot{u}\dot{\phi} v sin\phi - M_D \ddot{u} w - M_D \dot{u} w\dot{\phi} cos\phi - M_D \ddot{v} u cos\phi + M_D u\dot{v}\dot{\phi} sin\phi - M_D u\ddot{w} sin\phi - M_D u\dot{w}\dot{\phi} cos\phi - 2M_D \dot{u}\dot{v} cos\phi - 2M_D \dot{u}\dot{w} sin\phi\right] \delta_f (x - L)$$

Since the above equation of motions are in non-linear form so that we use Perturbation method for above system equation of motion such that

$$ua = u_0 + \epsilon u_1 + \epsilon^2 u_2 + \epsilon^3 x_3 + \ldots \quad (6)$$

Where ϵ is the parameter. After solving for above equations. We get the equations of motion for 1^{st} mode are obtained as below.

$$\rho_s A_s \ddot{u}_1 + M_D \ddot{u}_1 \delta_f (x - L) - EA_s u_1'' = 0 \quad (7)$$

$$\rho_s A_s \ddot{v}_1 - \rho_s A_s v_1 \dot{\varphi}^2 + M_D \ddot{v}_1 \delta_f (x - L) - M_D v_1 \dot{\varphi}^2 \delta_f (x - L) + EIv_1'''' - F(t)\delta_f \times (x - L) = 0 \quad (8)$$

$$\rho_s A_s \ddot{w}_1 - \rho_s A_s w_1 \dot{\varphi}^2 + M_D \ddot{w}_1 \delta_f (x - L) - M_D w_1 \dot{\varphi}^2 \delta_f (x - L) + EIw_1'''' = 0 \quad (9)$$

$$J_d \ddot{\theta}_{z_1} - J_p \dot{\varphi}\dot{\theta}_{y_1} + J_{dd}\ddot{\theta}_{z_1}\delta_f (x - L) - J_{pd}\dot{\varphi}\dot{\theta}_{y_1}\delta_f (x - L) = 0 \quad (10)$$

$$J_d \ddot{\theta}_{y_1} + J_p \dot{\varphi}\dot{\theta}_{z_1} + J_{dd}\ddot{\theta}_{y_1}\delta_f (x - L) + J_{pd}\dot{\varphi}\dot{\theta}_{z_1}\delta_f (x - L) = 0 \quad (11)$$

Where, $\delta_f (x - L)$ is the Dirac delta function.

3. Analytical solution of system

For analytical solutions, we used Galerkin method. By determining shape function $\varnothing$ and further calculating residual function (R), analytical solution is obtained. Galerkin method is expressed as:

$$\int_0^L \varnothing_1 R dx = 0 \quad (12)$$

For shape function Assuming the polynomial function as,

$$\varnothing_1 = C_0 + C_1 \bar{x} + C_2 \bar{x}^2 + C_3 \bar{x}^3 + C_4 \bar{x}^4 \quad (13)$$

Where, Calculating $\varnothing_1'$, $\varnothing_1''$ & $\varnothing_1'''$ and applying the boundary conditions;

$$\varnothing(0) = \varnothing'(0) = \varnothing''(L) = \varnothing'''(L) = 0$$

We ge, the value of constants as:

$$C_0 = 0, \quad C_1 = 0, \quad C_2 = 6, \quad C_3 = -4, \quad C_4 = 1$$

Substituting the values for constants in Eq.13, the mode shave for v & w is obtained as,

$$\varnothing_1 = x^4 - 4Lx^3 + 6L^2 x^2 \quad (14)$$

For calculating the mode shape for u, the boundary conditions is:

$u(0) = 0, \quad at \quad x = 0$

$u^{'}(0) = 0, \quad at \quad x = L$

The shape function for first mode of vibration for u is obtained as,

$$\varnothing_2 = x^2 - 2xL \tag{15}$$

Residual function for u

$$R = \rho_s A_s \ddot{u}_1(t)\varnothing_2 + M_D \ddot{u}_1(t)\delta_f(x-L)\varnothing_2 - EA_s u_1(t)\varnothing_2^{''} \tag{16}$$

Similarly, for v,

$$R = \rho_s A_s \ddot{v}_1(t)\varnothing_1 - \rho_s A_s v_1(t)\Omega^2\varnothing_1 + M_D \ddot{v}_1(t)\delta_f(x-L)\varnothing_1 - M_D v_1(t)\Omega^2\phi_1\delta_f(x-L) + EIv_1(t)\varnothing_1^{''''} - F(t)\delta_f(x-L)\varnothing_1 \tag{17}$$

Similarly, for w,

$$R = \rho_s A_s \ddot{w}_1(t)\varnothing_1 - \rho_s A_s w_1(t)\Omega^2\varnothing_1 + M_D \ddot{w}_1(t)\delta_f(x-L)\varnothing_1 - M_D w_1(t)\Omega^2\delta_f(x-L) + EIw_1(t)\varnothing_1^{''''} \tag{18}$$

After solving we get

For U_1

$$\rho_s A_s L^2 \ddot{u}_1(t) + M_D L \ddot{u}_1(t) + \frac{4}{3}EA_s u_1(t) = 0 \tag{19}$$

For V_1

$$\frac{104}{45}\rho_s A_s \ddot{v}_1(t)L^4 - \frac{104}{45}\rho_s A_s v_1(t)\Omega^2 L^4 + 9M_D\ddot{v}_1(t)L^3 - 9M_D v_1(t)\Omega^2 L^3 + \frac{144}{5}EIv_1(t) - 9L^3F(t) = 0 \tag{20}$$

For W_1

$$\frac{104}{45}\rho_s A_s \ddot{w}_1(t)L^4 - \frac{104}{45}\rho_s A_s w_1(t)\Omega^2 L^4 + 9M_D ddotw_1(t)L^3 - 9M_D w_1(t)\Omega^2 L^3 + \frac{144}{5}EIw_1(t) = 0 \tag{21}$$

Solving above Eq. 14, 15 and 16 by putting all values

$$\begin{aligned}
\rho_s &= 7850\,kg/m^3 \\
L &= 3.241m \\
\Omega &= \frac{2\pi N}{60} = 62.832 \\
M_D &= 3.5\,ton = 3500\,kg \\
E &= 200\,Gpa = 200\times10^9\,Pa \\
A_s &= \frac{\pi d^2}{4} = \pi\times\frac{0.425^2}{4} \\
&= 0.14186\,m^2 \\
I &= \frac{\pi d^4}{64} = \pi\times\frac{0.425^4}{64} \\
&= 1.6015\times10^{-3}m^4
\end{aligned}$$

We get,

$$\ddot{u}_1(t) + 1.52\times10^7 u_1(t) = 0 \tag{22}$$

$$\ddot{v}_1(t) + 2.8531\times10^3 v_1(t) - 2.2589\times 1\times10^{-4}F(T) = 0 \tag{23}$$

$$\ddot{w}_1(t) + 2.8531\times10^3 w_1(t) = 0 \tag{24}$$

3.1. External forced applied by jet and fourier series representation

In Pelton turbine force exerted by water jet can be determine easily. In mechanical system often excitation forces are not harmonic function but periodic in nature. Hence excitation force is converted to periodic function by Fourier expansion. For calculation of forces the data was obtained from Kulekhani-I hydropower is shown in Table 1. Force imparted by water jet in bucket can be given as:

$$F_j = \rho_w A_j V_1(V_{w1} + V_{w2})$$

Where,

ρ_w : density of water

A_j : area of water jet

V_1 : velocity of water jet

W_{w1} : component velocity in the direction of jet

W_{w2} : component velocity in the direction of vane

And,

$$V_{w1} = V_1$$

$$W_{w1} = k(V_1 - U)cos\varphi - U$$

Where,

'k' : blade friction coefficient = 0.95

'φ' : vane angle at outlet is 15deg

'U' : circumferential velocity of runner

$$U = \frac{\pi D_w N}{60}$$

D_w : diameter of Pelton wheel equals to 1562 mm

N : rated rpm of turbine which is 600 rpm

V_1 : $c_v\sqrt{2gH}$

C_v : velocity coefficient of turbine i.e,

H : net head of turbine

Plugging in all value the force is calculated as F_j = 3311.825 KN

For Fourier series representation of the function can be written as 25.

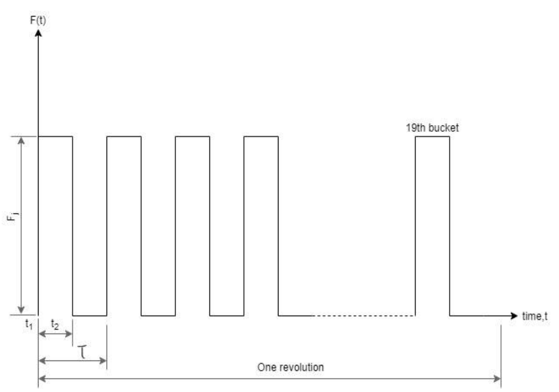

Figure 2: Excitation force on rotating turbine bucket

$$F(t) = \frac{a_0}{2} + \sum_{n=0}^{\infty}(a_n cos\omega t + b_n sin\omega t) \quad (25)$$

The coefficients of the Fourier series can be found as

$$a_0 = \frac{2}{\tau} + \int_{t_1}^{t_2} F(t)d(t)$$

$$a_n = \frac{2}{\tau} + \int_{t_1}^{t_2} F(t)cosn\omega td(t)$$

$$a_n = \frac{2}{\tau} + \int_{t_1}^{t_2} F(t)cosn\omega t\, d(t)$$

$$b_n = \frac{2}{\tau} + \int_{t_1}^{t_2} F(t)sinn\omega t\, d(t)$$

We have

t_1 = 0

t_2 = 39.18/1000 Ω

τ = 1/19 Ω

where t_1 and t_2 time of hitting and leaving the bucket. We get,

a_0 = 1.488 F_j

a_n = $\frac{0.3183}{2}$ F_j sin4.667n

b_n = $\frac{0.3183}{2}$ F_j [1- cos4.667n]

Hence Fourier series can be expressed as:

$$F(t) = \frac{1.488F_j}{2} + \sum_{n=1}^{\infty}\left[\frac{0.3183}{n}F_j \cdot \sin(4.667n)\cos(n\omega t) + \frac{0.3183}{n}F_j \cdot [1-\cos(4.667n)]\sin(n\omega t)\right] \quad (26)$$

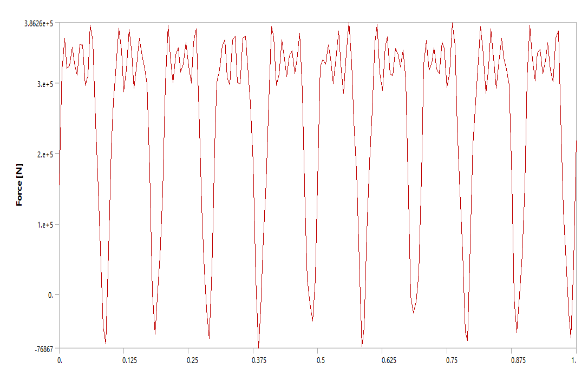

Figure 3: First five Fourier resultant for all nineteen buckets

Assuming the general solution for

$$v(t) = A_0 + A_1 sin\omega t + A_2 cos\omega t + A_3 sin2\omega t + A_4 cos2\omega t + A_5 sin3\omega t + A_6 cos3\omega t + A_7 sin4\omega t + A_8 cos4\omega t + A_9 sin5\omega t + A_{10} cos5\omega t$$

After solving Eq. 23 and 26 we get

$$v_1(t) = 19.546\times10^{-3} + 1.753\times10^{-5} sin\omega t - 1.6755\times10^{-5} cos\omega t + 4.1784\times10^{-6} sin2\omega t + 1.8972 \times10^{-7} cos2\omega t + 5.358\times10^{-7} sin3\omega t + 6.143\times10^{-7} cos3\omega t + 4.2978\times10^{-9} sin4\omega t + 1.8886\times10^{-7} cos4\omega t + 1.640\times10^{-7} sin5\omega t - 1.3603\times10^{-7} cos5\omega t \quad (27)$$

4. Simulation

For verification of mathematical solution simulation work was done in ANSYS 2019R3 workbench. For simulation data for the geometry was taken from Kulekhani-I HEP. The specifications used for geometry construction were taken from the table of parameter value of the vertical shaft Pelton turbine unit of Kulekhani-I HEP. After that automatic meshing was applied in ANSYS workbench. Static structure analysis was done for time response where one end was fixed and periodic force was applied at the other end along Y-axis.

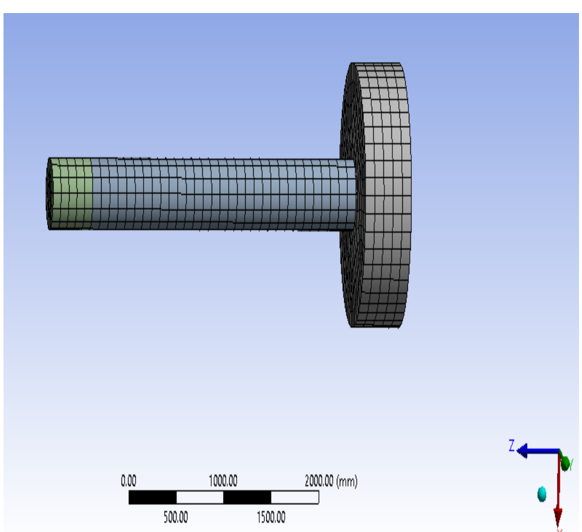

Figure 4: ANSYS Model

5. Result and discussion

For verification of mathematical model and comparison with simulation work carried out in ANSYS by using the data of Pelton turbine of Kulekhani-I HEP. The various technical specifications for Kulekhani –I HPS used for the research purpose are as listed in Table 1:

Table 1: parameter value from Kulekhani-I HP

Parameters	Value
Output power	31000 kw
Rated rpm	600 rpm
Runner diameter tip to tip	2045 mm
Pitch circle diameter	1562 mm
Density of runner material	8050 kg/m3
Youngs modulus of runner material	195 GPa
No of buckets	19
Width of bucket	490 mm
Density of bucket material	8050 kg/m3
Weight of runner-bucket assembly	3.5 ton
Diameter of shaft	425 mm
Length of shaft	3241 mm
Density of shaft	7850 kg/m3
Youngs modulus of shaft	200 GPa (mild steel)
Weight of shaft	4.43 ton
Diameter of nozzle	190 mm
No. of nozzle	4

For Analytical result Eq.27 was solved. Where 'v_1 (t)' is an amplitude of vibration depending upon time. As time increases corresponding values of amplitude was obtained. After plotting graph for those values of time and amplitude sinusoidal curve was formed as shown in Figure 5, thus maximum amplitude of vibration along Y-Axis was found to be 19.567mm. From simulation

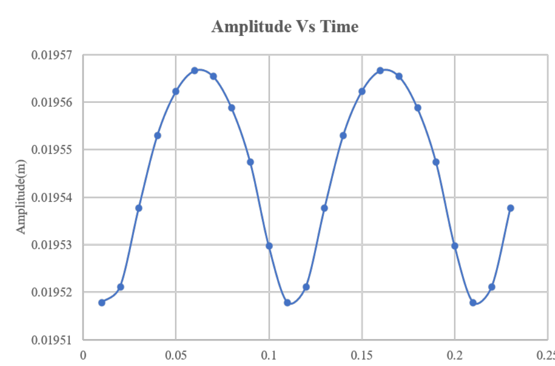

Figure 5: Amplitude vs time (along Y-axis)- analytically

corresponding values of time amplitude was obtained in tabular data. Hence after plotting a graph sinusoidal curve was obtained as shown in Figure 6, from which maximum vibrational amplitude along Y-axis was found to be 18.068mm. As we compare the graph between analytical and simulation for each response is sinusoidal whereas period of each graph was found to be $\frac{\pi}{5}$. Maximum amplitude for Analytical solution and Simulation

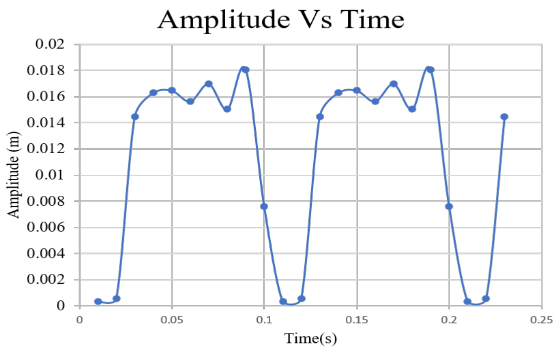

Figure 6: Amplitude vs time (along Y-axis)-ANSYS simulation

solution was found to be 19.74mm and 18.068mm respectively.

6. Conclusion

This research paper presented the methodologies to study the dynamic response of Pelton turbine unit of shaft disk system. The mathematical model for dynamic response of the Pelton turbine unit was formulated and the analytical solution of amplitude of forced vibration was found. The Fourier analysis for the excitation force showed the minimum number of Fourier components to be considered to obtain the solution in its exact form. Thus, the analysis showed the summation of first five Fourier components began to represent the actual shape of pressure pulse. The first five Fourier components have been considered in analysis for meaningful representation of a forcing function. The amplitudes of forced vibration of the selected Pelton turbine unit nozzle rated at 600 RPM in Y (the direction of water jet) direction was found to be 19.567mm analytically. Similarly, the amplitude of vibration of in Y (the direction of water jet) direction was found to be 18.068mm by ANSYS simulation. This methodology can be applied to find the dynamic force response and other hydropower plants to calculate the acceptance level of vibration analytically and can compare the vibration level during the operation period in long run by measuring the amplitudes using vibration measuring devices.

7. Acknowledgement

The authors like to thank Er. Subodh Kumar Ghimire, Head of Department, and Er. Sunil Adhikari, Coordinator, Department of Automobile and Mechanical Engineering, Thapathali Campus, IOE, Tribhuvan University, Nepal for their continuous technical support for conducting this research. The authors are also thankful to Assistant Professor Er. Sudan Neupane, Head of Department, Department of Industrial Engineering, Thapathali Campus, IOE, Tribhuvan University, Nepal for providing guidelines on research paper writing.

References

[1] Thomson W T, Dahleh M D. Theory of Vibration with Applications-Prentice Hall (1997).pdf[M]. 1997.

[2] Surovec R, Bocko J, Šarloši J. Lateral Rotor Vibration Analysis Model[J/OL]. American Journal of Mechanical Engineering, 2014, 2(7): 282-285. DOI: https://doi.org/10.12691/ajme-2-7-23.

[3] Bai B, Zhang L, Guo T, et al. Analysis of dynamic characteristics of the main shaft system in a hydro-turbine based on ANSYS[J]. Procedia Engineering, 2012, 31(2011): 654-658.

[4] Egusquiza M, Egusquiza E, Valero C, et al. Advanced condition monitoring of Pelton turbines[J/OL]. Measurement: Journal of the International Measurement Confederation, 2018, 119(October 2017): 46-55. https://doi.org/10.1016/j.measurement.2018.01.030.

[5] Rajak A, Shrestha P, Rijal M, et al. Dynamic Analysis of Pelton Turbine and Assembly[J]. IOE Graduate Conference Proceeding, 2014: 103-109.

[6] Pudasaini S, Neopane H P, Panthee A, et al. Computational Fluid Dynamics (CFD) analysis of Pelton runner of Khimti Hydropower Project of Nepal[J]. International Journal of Scientific and Research Publications, 2014, 4(September): 18-21.

[7] Motra L, Luintel M C. Free Vibration Analysis of Selected Pelton Turbine using Dynamic Approach[J]. IOE Graduate Conference Proceeding, 2017, 8914: 229-236.

[8] Karki, Sanjeev Luintel M C. Dynamic Response of Peltion Turbine Unit for Forced Vibration[J]. IOE Graduate Conference Proceeding, 2017, 8914: 509-517.

[9] Luintel M, Bajracharya T R. Dynamic response of a shaft of a Pelton turbine due to impact of water jet[J]. journal of computational and applied research in meachanical engineering, 2019.

[10] Agostini C E, Sousa E A C, Paulista U E. COMPLEX MODAL ANALYSIS OF A SLENDER VERTICAL ROTOR[C]// 15th international conference on Experimental Mechanics. 2012: 1-19.

[11] O'Reilly O M. Intermediate dynamics for Engineers[M]. 2008: 300.