Meaning of Numeral and Number System and Formulation of Roman numerals Corresponding to Decimal Numbers

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Abstract

The primary objective of the paper is to review the definition of number system adopting the basic and general way of formation of number in any existing number system like Egyptian, Roman, and Hindu-Arabic number system which played and are playing vital role for development of today's existing number system. But secondary aim of this article is to analyze and synthesize on Roman number system in various aspects and then to formulate structural stands to create Roman numbers corresponding to Hindu-Arabic (Decimal) numerals/numbers.

Keywords: Glyphs, numerals, Hindu-Arabic numeral system, Egyptian numeral system,

Introduction

Any numeral originates from symbols like digit(s) or glyphs or letters in Hindu-Arabic numeral system or in Egyptian numeral system or in Roman numeral system respectively and .number from numeral with particular sense. So digit(s) is/are vital element(s) to give rise any numeral(s) and number(s). Digit is a single symbol used to make any numeral(s).

In Hindu-Arabic system (decimal system).0,1,2,3,4,5,6,7,8 and 9 as ten digits(symbols) used to represents any numeral and number Eg:-5 is numeral made by using single digit 5 ,45 is numeral made by using two digits 4 and 5 so on. Hindu-Arabic system is characterized as:- (i) System is based on "base 10" and has use of zero (ii)The system uses only ten digits(symbols or glyphs) as 0,1,2,3,4,5,6,7,8,9 to create any numeral and number. (iii)In this system place value can be ascertained as:-, Cardinal value(face value) of each symbol in a numeral is multiplied by its positional value to get total value of each symbol .Finally adding all these total values, number (i.e. Hindu–Arabic number) corresponding to that numeral is found. Then

Formation of any other numeral and number (beside single digit of decimal system):-

For a numeral 3572,

Total value of symbol 3 is =3×it's positional value=3×10^3=3000,

Total value of symbol 5 is =5×it's positional value=5×10^2=500,

Total value of symbol 7 is =7×it's positional value=$7\times 10^1=70$ and
Total value of symbol 2 is =2×it's positional value=$2\times 10^0=2\times 1$,
Then number (i.e. Hindu–Arabic number) corresponding to numeral 3572 is
$3\times 10^3+5\times 10^2+7\times 10^1+2\times 10^0=3000+500+70+2=3572$(in sense)
Note:-Zero symbol in any numeral has face value and total value same as zero.

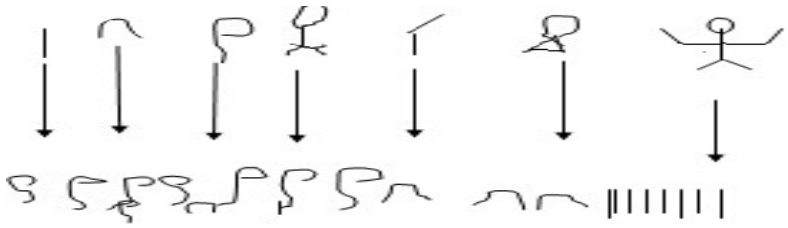
Similarly, in Egyptian System**:-**Egyptian civilization for development and application of numeral system was seen in 3ooo BC. On the basis of Hieroglyphs (a little figures), in which words are represented by little figures. Eg. to represents a bird, a little figure of bird is sketched out. They used decimal system(bases 10 system of hieroglyphs for numerals) using different symbols to represent 1^{th}, 10^{th},100^{th},1000^{th},10000^{th},100000^{th} and 1000000^{th} position by $1(10^0)$,$10(10^1)$,$100(10^2)$,$1000(10^3)$,$10000(10^4)$,$100000(10^5)$,$1000000(10^6)$ respectively. Rhind and Moscow were credited as sources in Egyptian mathematical civilization it was convinced that they used two forms of symbols to represent as a digit. They are: - (i) Hieroglyphs which uses seven symbols and (ii)Hieratic and demotic which uses more symbols.

Numeral and number construction due to Egyptians:-

Egyptian Hieroglyphs or seven symbols and numerals throughout history are described as:-

Egyptian seven symbols (digits):-

strok arc coiled lotus pointed Fish Astonished



rope flower finger or Tadpole man

Decimal symbol: - 1 10 100 1000 10000 100000 1000000

Formation of any other numeral and number (beside single glyph of Egyptian system):-

If we take any numeral:-

Then it's corresponding number is: 738=7 +3 + 8 =7×100+3×10+8×1

To make up the number 738 **in hieroglyphs**, eighteen symbols were required: seven "hundred" symbols, three "ten" symbols and eight "unit" symbols.

This system resemble with Hindu-Arabic(Decimal system) in the sense that Egyptian symbol has positional value of $10^2 or 100^{th}$ position of decimal system with value 100, the symbol has positional value of $10^1 or 10^{th}$ position of

decimal system with value 10 and the symbol has positional value of $10^0 or 1^{th}$ position of decimal system with value 1. But there is no use of zero in it. And very large numbers can't be viewed by Egyptian numeral system.

Likewise, in Roman system, I, V, X, L, C, D, and M as symbols used to represent any numeral in Roman system. Eg:-V is one symbol numeral corresponding to number 5, XV is two symbol numeral corresponding to number 15, XVII is three symbol numeral corresponding to number 17, so on…The Roman numeral system uses only seven symbols as digits

Roman symbols (digits): I, V, X, L, C, D, and M in the sense of

Hindu-Arabic numbers:-1, 5, 10, 50,100,500 and 1000 respectively i.e.

I represents the number 1, V represents 5, *X* is 10, L is 50 which was traditionally written by XXXXX [Kennedy, Benjamin Hall (1923)], *C* is 100, *D* is 500, and *M* is 1,000 M=1000 was traditionally written by CIↃ [Asimov, Isaac (1966)]. Different arrangements of these seven symbols represent different numbers. The numbers 1–10 are:

- 1 = I
- 2 = II
- 3 = III
- 4 = IV which was traditionally written by IIII on Roman numeral clocks[Judkins, Maura (2011)]
- 5 = V which was traditionally written by IIIII [Kennedy, Benjamin Hall (1923)]
- 6 = VI
- 7 = VII
- 8 = VIII
- 9 = IX
- 10=X which was traditionally written by L

are obviously new Roman numerals constructed from basic Roman symbols. But Basic symbols (I, V, X, L, C, D and M) are self constructed Roman numerals.

Formation of any other numeral and number (beside single symbol in Roman system):-

Roman Symbols are I, V, X, L, C, D and M

If we take any numerals CXV1 and IV, then their corresponding numbers are: C+X+V1=100+10+6=106(sense). And V-I=5-1=4(sense) respectively.

Adopting **above** ways **(**methods) of formation any numeral and numbers in any existing number system of any civilization in the world, primary researched version about it is:-

Primary Researched Article for Definition of Numeral

Any numeral that originates from glyphs or symbols or letters and number from numeral with particular sense. So digit(s) is/are vital element(s) to give rise any numeral(s) and numbers.

If we Consider any system, for example, Roman numeral system ,seven symbols I, V, X, L, C, D, and Mare used to represents any numeral in this system so that X is numeral made by one symbol X, CI is numeral made by symbols C and I. so on... Same way is valid in Egyptian and Roman system.

So, our researched conclusion about formation of numerals and numbers is:- Numeral is an arrangement (combination) of individual symbol(s) (i.e. digits or glyphs) that stands for particular number (view). Or Numeral is a new symbol (or name) generated from each individual symbols that stand for a particular number (sense). Number is an idea or sense or view or (a concrete or abstract concept or quantity) or mathematical way to count and measure a quantity (totality) of thing(s). Illustration:-Numeral is only arrangement of symbols but number is arrangement of symbols with particular sense.

So the number is an idea and the numeral is how we write and arrange it.

Symbols (digits or letters or glyphs) -> Numerals -> Numbers

So digits make up numerals, and numerals stand for an **idea** of a number as:- Digits (symbol);- 1, 3 ⇒Numeral (arrangement):- 13 or 31

⇒number (idea or view):- 𝌆 ¯ or 𝌇 ≡ respectively.

Just like letters (symbols) make up words (numerals), and words (numerals) stand for an idea of the thing (number) as:-Letters (symbols) g, o and d

⇒word (numerals):-god or dog →

⇒idea of thing (sense or view or number):-religiously accepted and worship able body or four footed domesticated animal tamed for security respectively.

Hence, Numeral is only arrangement of symbols but number is arrangement of symbols with particular sense.

NOTE:-Symbol in equivalent to letter in Roman numeral system (also letter in linguistic way), digit in Decimal numeral system and glyph in Egyptian numeral system.

Roman Number System and Historical Overview

Roman numeral system is that system in which any particular numeral in this system is composed of two or more symbols, even a single symbol itself taken from Roman symbols (or letters) i.e. I, V, X, L, C, D, M. that make up the number system that was used by the ancient Romans and succeeding civilizations in Europe to represent numbers before the adoption of Arabic numerals. Today, Roman numerals are more commonly used in titles, to number parts of works, in music theory, on clock faces and numbering the preliminary pages of a book and numbering paragraphs or subparagraphs in a document.

The numeral system developed by the Romans was used by most Europeans for nearly 1800 years, far longer than the current Hindu-Arabic system has been in existence. Although the Roman numeral system provided for easy addition and subtraction, other arithmetic operations proved more difficult. Due to lack of an effective system for utilizing fractions and the absence of the concept of zero, the cumbersome nature of the Roman numeral system, while it served most of the needs of the Romans, hindered future mathematical advances.

Roman numeral system for representing numbers was developed around 500 B.C. As the Romans conquered many countries of the world that was known to them, their numeral system spread throughout Europe, where Roman numerals remained the primary manner for representing numbers for centuries. Around A.D. 1300, Roman numerals were replaced throughout most of Europe by more effective Hindu-Arabic system which is still used there.

Impact and Limitation

The Romans adopted the symbols that they used for their numerals from a variety of sources, including their Greek counterparts. The origin of I derived from counting on one's hand or one finger, The V came to represent five fingers counted on the hand which was traditionally written by IIIII [Kennedy, Benjamin Hall (1923)], it's shape is by the space between the thumb and first finger. Originally the Romans adopted the Greek letter X, or chi, to represent 50. Through the study of monument transcriptions historians have been able to determine that L (replaced X) as 50 which was traditionally written by XXXXX [Kennedy, Benjamin Hall (1923)], and X came to represent 10. X was derived from one V, or five, placed on top of another, upside-down V. Eventually the Romans adopted just X to be the numeral for 10. The symbol C came to represent 100, due to its first letter of the Latin word:-centum (for one hundred). Likewise, M was adopted for 1000, due to its first letter of the Latin word:-mile (for one thousand).Likewise D for 500 was adopted

Unlike the Greeks, the Romans were not concerned with pure mathematics, such as number theory, geometric proofs, and other abstract ideas. Instead, the Romans preferred utilitarian mathematics. The Romans primarily used mathematics to figure personal and government accounts, keep military records, and maintain the construction of aqueducts and buildings. The Roman numeral system accepted simple addition and subtraction.

The fact that multiplication and division were fairly difficult operations. The counting boards, which resembled the familiar abacus, could also be used for addition and subtraction. Even with these counting boards, multiplication and division of large numbers remained a difficult task.

. One flaw of the Roman numeral system was the absence of a way to numerically express fractions. Romans were aware of fractions, but putting them

to use was difficult, as they were expressed in written form. The Romans usually expressed fractions in terms of the uncia [Maher, David W.; Makowski, John F. (2013)] an uncia originally meant 1/12 of the Roman measure of weight (English derived the word "ounce" from uncial inch or dozen). Soon, however, uncia evolved to mean 1/12 of anything. Although basing the use of fractions on 1/12, the Romans were able to express one-sixth, one-fourth, one-third, and half. While the modern numerical expression of one-fourth is ¼, the Romans would have expressed one-fourth as three unciae ($^3/_{12}$ = ¼). This system allowed the Romans to approximate measures, but they could not easily express exact measures.

Another flaw that limited Roman mathematics was the absence of the concept of zero [Byrhtferth's Enchiridion (1016)]. This forced the Romans to accept the cumbersome (weakness) in the system. Unlike the ancient Greeks, the Romans also did not understand or explore the concept of irrational numbers. This severely limited the Romans in geometry, because much of geometry rests on an understanding of π, these flaws in the Roman mathematical systems limited the advancement of mathematical theory in Rome. Due Roman's conquests, most of Europe adopted the Roman numeral system and used it throughout the middle Ages. Accordingly, theoretical mathematical advances were likewise also stunted throughout most of Western civilization for nearly 1,000 years. The absence of zero [Byrhtferth's Enchiridion (1016)] and irrational numbers, impractical and inaccurate fractions, and difficulties with multiplication and division prevented the Romans and the Europeans who later used the system from making advances in number theory and geometry as the Greeks had done in the Pythagorean and Euclidean schools.

During these mathematical Dark Ages, advancements in these fields were made by Middle Eastern and Indian subcontinent civilizations. With the innovation of zero place use within the Hindu-Arabic place-value system, great advances were made in these regions in the fields of geometry, number theory, and the invention and advancement of algebra.

Although Roman numerals are no longer a necessary component of mathematics, they are an important part of the history of the development of Western civilization. Modern numerals remain aesthetically important because of their widespread artistic use in art, architecture, and printing.

Application of Roman Numerals

The ancients used Roman numerals for commerce and mathematics only. Modern applications include: - widespread artistic use in art, architecture, and printing. The numerals are commonly used in Monarchy titles, in repetition of people's name of same generation, to number parts of works, in the year of construction on building faces and cornerstones, in page numbering of prefaces and introductions of books, and sometimes of appendices and annexes, too ,in

Book for volume and chapter numbers, as well as the several acts within a play, to outline that use numbers to show hierarchical relationships, in recurring grand event like summer and winter Olympic games as XXI winter Olympic games, in WrestleMania, the annual professional wrestling event , in music theory,films,videos, on clock faces[Pickover, cliffords A.(2003)] , numbering the preliminary pages of a book and numbering paragraphs or subparagraphs in a document, in specific disciplines as in astronomy, the natural satellites or "moons" of the planets E.g., Titan's designation is Saturn VI , in chemistry to denote the groups of the periodic table , in education, school grades Eg., "grade IX" is sometimes seen for "grade 9" ,in entomology, in graphic design, in law, in advanced mathematics (including trigonometry, statistics, calculus etc.),naming I, II, III, and IV, in military, in pharmacy, in photography, in sports representing a nation or province, a club or a school at the highest level , in tarot Roman numerals (with zero) are used to denote the cards of the Major Arcana,

Special characteristics Roman Numeral System:-

(i)This system uses at most 7 symbols I,V,X,L,C,D,M to arise any number.

(ii)It does not resemble with decimal system in the sense of positional value of its symbol to get resultant number. It's any symbols are repetitive at most 3 times. A new combined symbol (on the basis of subtraction) is assigned instead of 4 times repetition of any symbol.

(iii)There is no place of zero [Byrhtferth's Enchiridion (1016)] sense (view) in Roman system.

(iv)It is based on simple addition and subtraction to give resultant number.

(v)It can be applied to generate even large numbers on the basis of multiplication principle.

Weakness of Roman Numeral System

(i) The absence of zero [Byrhtferth's Enchiridion (1016)] and irrational numbers, impractical and inaccurate fractions, and difficulties with multiplication and division..

(ii) As the systems are repetitive. So value of number will also repeats as symbols do repetitive. So it is more tedious and time taking due to composition of large number from their symbols

(iii)There is no fixed positional value in these systems.

General (basic) Rule for Formation of Roman Numeral System:-

From Roman symbols to create any new numerals and numbers, there are a few guidelines to keep in mind.

For any numeral,

(i) Addition rule:-we add numbers together by putting numbers corresponding to the symbols in descending order from left to right (i.e.if numeral is placed in such a way that symbol with a smaller value is right of one

with a larger value) to get the total value. For example, numeral XVI is 10 + 5 + 1, or 16*(*number*)*. Numeral XXXIII is 10 + 10 + 10 + 1 + 1 + 1, or *33*(numeral*)*.

(ii) Subtraction rule:-we also subtract number of value of the smaller symbol from value of the larger symbol in ascending order from left to right (i.e. if numeral is placed in such a way that symbol with a smaller value is left of one with a larger value) to get the total value. For instance, by subtractive rule, numeral IV is 5 – 1, or *4(*number) and IX is 10-1=9 [Stanislas Dehaene (1997)]. Further, we can only place one smaller numeral in front of a larger one for subtractive purposes.

(iii)Combination of addition and multiplication rule:- For instance, a numeral can only be placed before two numerals that are closest to it in the Roman numeral system. That is, I can only be placed before *V* (e.g. IV, or 4) which was traditionally written by IIII on Roman numeral clocks [Judkins, Maura (4 November 2011)] and X (e.g. IX, or 9). It can't be placed before L, C, D, or M. But the symbols V, L, D are never subtracted. For example, in Roman numerals, 46 would be XLVI ({50 – 10}+{5+1}=40+6 = 4*6*).

(iv)Multiplication rule:- Vinculum is Roman system to get numerals multiplied by 1000[Ifrah (2000)].We use a bar over each symbol to multiply it by 1,000. For example, an *X* with a line over it i.e.$\bar{X}$=10× 1000 = 10,000, while $\bar{M}$ means =1000× 1000 = 1,000,000, $\bar{V}$=5× *1000*=5000 which was traditionally written by Q[Gordon, Arthur E. (1982)]. Where M=1000 was traditionally written by CIↃ [Asimov, Isaac (1966)].

NOTE:-

(i)If numeral with same symbol(only I,X,C and M, if necessary but not V,L and D) are repeated at most up to 3 times, then value of numeral is found by adding the symbol up to repeated times(or the symbol is multiplied by repeated time) to get a Hindu-Arabic number.eg:-

Numeral III stands for 1+1+1 or 3×1=3(number),

Numeral XX stands for 10+10 or 2×10=20

Numeral CCC stands for 100+100+100 or 3×100=300

(ii) In any numeral, the symbol V, L, D are never subtracted where as I can be subtracted from V and X only, X from L and C only, C from D and M only

(iii)In any numeral, symbol of smaller value, between two symbols of greater value, is subtracted from symbol on it's right to get Hindu-Arabic number, eg:-

XIV stands for X+IV=10+(5-1)=14, XCIX stands for XL+IX=(100-10)+(10-1)=99 etc. A single (not two or three) smaller symbol is placed before larger symbol. For example, the correct way to write 9 is IX and 8 can't not be written by IIX instead by VIII.

However, above method of formation of any Roman numerals are described, the search activity says that the numeral formation may be formulated as below

Secondary (crucial) Researched Article

Adopting above notational view of the Roman symbols and formation of new numerals on the basis of prescribed rules of addition, subtraction and multiplication and assumptions, any Roman numeral corresponding to particular Hindu Arabic number can be formulated by formula derived as:-

Remark:- I^x==x.I= So that I^3 =3.I=I+I+I=III for x=3 and I^0=0.I=0 for x=0 (for x times repetition of Roman symbol .But x may be at most three times for only repeatable symbol I or X or C or M but not for V or L or D) Also considering I,V,X,L,C,D and M as perfect i.e. self formed (formulated or constructed symbols) in the sense of number 1,5,10,50,100,500,1000 respectively. Although exploration pattern up to four digit(i.e. 9999corresponding to decimal number)is established in this paper, it can be continued up to any large Roman numerals by same induction method as applied in this article.

Methodology:-Mathematical induction

(a) One digit numeral(x) creating Roman numerals, I, II, III, IV, V, VI, VII, VIII IX

(i)$x=I^x$=xI for x=1,2,3 (ii) x=5-1=V-I=IV for x=4, (subtraction principle on the symbol)

(iii)$x=5+I^{x-5}$ =V+I^{x-5} ,for x=5,6,7,8 (Applying addition principle on the symbol)

(iv)x=10-1=X-I=IX ,for x=9 (Applying subtraction principle on the symbol)

(b)Two digit numeral (xy) creating Roman numerals X,XI…..XCIX

(i)$xy=10^x+ y=X^x+ y$ for x=1,2,3 and y=0,1,2,….9

(ii) xy=(50-10)+ y=(L-X)+y=XL+ y for x=4 and y=0,1,2,….9

(iii) $xy=50+10^{x-5}+ y =L+X^{x-5}+ y$ for x=5,6,7,8 and y=0,1,2,….9

(iv) xy=(100-10)+ y=(C-X)+y=XC+ y for x=9 and y=0,1,2,….9

(c)Three digit numeral (xyz) creating Roman numerals C, CI…..CMXCIX

Under x=1,2,3,

(i)$xyz=100^x+10^y+ z=C^x+X^y+ z$ for y=0,1,2,3 and z=0,1,2,….9

(ii) $xyz=100^x+(50-10)+ z=C^x+XL+z$ for y=4 and z=0,1,2,….9

(iii) $xyz=100^x+50+10^{y-5}+ z=C^x+L+X^{y-5}+z$ for y=5,6,7,8 and z=0,1,2,….9

(ii) $xyz=100^x+(100-10)+ z=C^x+XC+z$ for y=9 and z=0,1,2,….9

Under x=4

(i)$xyz=(500-100)+10^y+ z=CD+X^y+ z$ for y=0,1,2,3 and z=0,1,2,….9

(ii) xyz=(500-100)+(50-10)+ z=CD+XL+ z for y=4 and z=0,1,2,….9

(iii) $xyz=(500-100)+50+10^{y-5}+ z=CD+L+X^{y-5}+ z$ for y=5,6,7,8 and z=0,1,2,….9

(iv) xyz=CD+(100-10)+ z=CD+XC+z for y=9 and z=0,1,2,….9

Under x=5,6,7,8,

(i)xyz=500+100^{x-5}+10^{y}+ z=D+C^{x-5}+X^{y}+ z for y=0,1,2,3 and z=0,1,2,….9

(ii) xyz=500+100^{x-5}+(50-10)+ z=D+C^{x-5}+XL+z for y=4 and z=0,1,2,….9

(iii) xyz=500+100^{x-5}+50+10^{y-5}+ z=D+C^{x-5}+L + X^{y-5} +z for y=5,6,7,8 and z=0,1,2,….9

(iv) xyz=500+100^{x-5} +(100-10)+ z= D+C^{x-5}+XC+z for y=9 and z=0,1,2,….9

Under x=9

(i)xyz=(1000-100) +10^{y}+ z=CM+X^{y}+ z for y=0,1,2,3 and z=0,1,2,….9

(ii) xyz=(1000-100)+(50-10)+ z=CM+XL+z for y=4 and z=0,1,2,….9

(iii) xyz=(1000-100)+50+10^{y-5}+ z=CM+L+X^{y-5}+z for y=5,6,7,8 and z=0,1,2,….9

(iv) xyz=(1000-100)+(100-10) + z=CM+XC +z for y=9 and z=0,1,2,….9

(d)Four digit numeral (xyzw) creating Roman numerals of 1000(M)…..up to 9999

Under x=1,2,3,and y=0,1,2,3

(A)xyzw=1000^{x}+100^{y}+10^{z} +z=M^{x}+C^{y}+ X^{z}+z for z=0,1,2,3 and w=0,1,2,….9

(B)xyzw=1000^{x}+100^{y}+(50-10)+ w=M^{x}+C^{y}+XL+w for z=4 and w=0,1,2,….9

(C) xyzw=1000^{x}+100^{y}+50+10^{z-5}+w =M^{x}+ C^{x}+L + X^{z-5} +w for z=5,6,7,8 and w=0,1,2,….9

(D) xyzw= 1000^{x}+100^{y}+(1000-100)+w =M^{x}+ C^{x}+XC + w for z=9 and w=0,1,2,….9

Under x=1,2,3,and y=4

(A)xyzw=1000^{x}+(500-100)+10^{Z}+ w=M^{x}+CD+X^{z}+w for z=0,1,2,3 and w=0,1,2,….9

(B) xyzw=1000^{x}+(500-100)+(50-10) + w=M^{x}+CD+XL+w for z=4 and w=0,1,2,….9

(C)xyzw=1000^{x}+(500-100)+50+10^{z-5}+w=M^{x}+CD+L+X^{z-5}+w forz=5,6,7,8 and w=0,1,2,….9

(D) xyzw=1000^{x}+(500-100)+(100-10) + w=M^{x}+CD+XC+w for z=9and w=0,1,2,….9

Under x=1,2,3,and y=5,6,7,8

(A)xyzw=1000^{x}+500+100^{y-5}+10^{Z}+ w=M^{x}+D+C^{y-5}+X^{z}+w for z=0,1,2,3 and w=0,1,2,….9

(B) xyzw=1000^{x}+500+100^{y-5}+(50-10) + w=M^{x}+D+C^{y-5}+XL+w for z=4 and w=0,1,2,….9

(C)xyzw=1000^{x}+500+100^{y-5}+50+10^{z-5}+w=M^{x}+D+C^{y-5}+L+X^{z-5}+w,if z=5,6,7,8and w=0,1,2,..9

(D)xyzw= 1000^{x}+500+100^{y-5}+(100-10)+w=M^{x}+D+C^{y-5}+XC+w for z=9and w=0,1,2,….9

Under x=1,2,3, and y=9

(A)xyzw=1000^{x}+(1000-100) +10^{z}+w=M^{x}+CM+X^{z}+w for z=0,1,2,3 and w=0,1,2,….9

(B) xyzw=1000^{x}+(1000-100) +(50-10)+w=M^{x}+CM+XL+w for z=4 and w=0,1,2,….9

(C)xyzw=1000^{x}+(1000-100) +50+10^{z-5}+w=M^{x}+CM+L+X^{z-5}+w, if z=5,6,7,8and w=0,1,2,..9

(D)xyzw= 1000^{x}+(1000-100) +(100-10)+w=M^{x}+CM+XC +w for z=9and w=0,1,2,….9

Under x=4 and y=0,1,2,3

(A)xyzw=($\bar{V}$-M) +C^{y}+ X^{z}+z for z=0,1,2,3 and w=0,1,2,….9

=M$\bar{V}$+C^{y}+ X^{z}+z for,z=0,1,2,3 and w=0,1,2,….9

(B) xyzw=($\bar{V}$-1000)+100^{y}+(50-10)+ w=M$\bar{V}$+C^{y}+XL+w for z=4 and w=0,1,2,….9

(C) xyzw =M$\bar{V}$ + C^{y}+L + X^{z-5} +w for z=5,6,7,8 and w=0,1,2,….9

(D) xyzw== M$\bar{V}$+ C^{y}+XC +w for z=9 and w=0,1,2,….9

Under x=4 and y=4

(A)xyzw=($\bar{V}$-1000)+(500-100)+100^{y}+(50-10)+w=M$\bar{V}$+CD+X^{z}+w for z=0,1,2,3and w=0,1,2..9

(B) xyzw=($\bar{V}$-1000)+100^{y}+(50-10)+ w=M$\bar{V}$+CD+XL+w for z=4 and w=0,1,2,….9

(C) xyzw =($\bar{V}$-1000)+100^{y}+50+10^{z-5}+ w=M$\bar{V}$+CD+L+X^{z-5}+w for z=5,6,7,8and w=0,1,2..9

(D) xyzw=($\bar{V}$-1000)+100^{y}+(100-10)+ w=M$\bar{V}$+CD+XC+w for z=9 and w=0,1,2, ….9

Under x=4 and y=5,6,7,8

(A)xyzw=($\bar{V}$-1000)+500+100^{y-5}+10^{z}+w=M$\bar{V}$+D+C^{y-5}+X^{z}+w for z=0,1,2,3and w=0,1,2,9

(B) xyzw=($\bar{V}$-1000)+500+100^{y-5}+(50-10)+w = M$\bar{V}$+D+C^{y-5}+XL+w for z=4 and w=0,1,2,….9

(C)xyzw=($\bar{V}$-1000)+500+100^{y-5}+50+10^{z-5}+w=M$\bar{V}$+D+C^{y-5}+L+X^{z-5}+w for z=5,6,7,8 and w=0,1,2..9

(D)xyzw= ($\bar{V}$-1000)+500+100^{y-5}+(100-10)+w=M$\bar{V}$+D+C^{y-5}+XC+w for z=9 and w=0,1,2,….9

Under x=4 and y=9

(A)xyzw=($\bar{V}$-1000)+(1000-100) +10^z+w=M$\bar{V}$+CM +X^z+w for z=0,1,2,3and w=0,1,2..9

(B) xyzw= ($\bar{V}$-1000)+(1000-100) +(50 − 10)+w=M$\bar{V}$+CM +XL+w for z=4 and w=0,1,2,….9

(C)xyzw=($\bar{V}$-1000)+(1000-100) +50+10^{z-5}+w=M$\bar{V}$+CM+L+X^{z-5}+w for z=5,6,7,8and w=0,1,2..9

(D) xyzw=($\bar{V}$-1000)+(1000-100)+(100-10)+w=M$\bar{V}$+CM +XC+w for z=9 and w=0,1,2,….9

Under x=5,6,7,8 and y=0,1,2,3,

(A)xyzw=$\bar{V}$ +1000^{x-5}+C^y+ X^z+w for z=0,1,2,3 and w=0,1,2,….9

=$\bar{V}$+M^{x-5}+C^y+ X^z+w for z=0,1,2,3 and w=0,1,2,….9

(B)xyzw= $\bar{V}$+M^{x-5}+C^y+ (50-10)+w= $\bar{V}$+M^{x-5}+C^y +XL+w for,z=4 and w=0,1,2,….9

(C)xyzw=$\bar{V}$+M^{x-5}+C^y+ 50+10^{z-5}+w=$\bar{V}$+M^{x-5}+C^y+L+X^{z-5}+w for,z=5,6,7,8and w=0,1,2,….9

(D)xyzw=$\bar{V}$+M^{x-5}+C^y+ (100-10)+w= $\bar{V}$+M^{x-5}+C^y+XC +w for,z=9and w=0,1,2,….9

Under x=5,6,7,8 and y=4

(A)xyzw= $\bar{V}$+M^{x-5}+(500 − 100)+10^z+w=$\bar{V}$+M^{x-5}+CD+X^z+wfor,z=0,1,2,3and w=0,1,2,….9

(B)xyzw= $\bar{V}$+M^{x-5}+(500 − 100)+(50-10)+w=$\bar{V}$+M^{x-5}+CD+XL +w for,z=4 and w=0,1,2,….9

(C)xyzw=$\bar{V}$+M^{x-5}+(500 − 100)+50+10^{z-5}+w=$\bar{V}$+M^{x-5}+CD+L+X^{z-5}+w for,z=5,6,7,8and w=0,1,2,….9

(D)xyzw=$\bar{V}$+M^{x-5}+(500 − 100)+(100-10)+w=$\bar{V}$+M^{x-5}+CD+XC +w for z=9and w=0,1,2,….9

Under x=5,6,7,8 and y=5,6,7,8

(A)xyzw=$\bar{V}$+M^{x-5}+500+100^{y-5}+10^z+w=$\bar{V}$+M^{x-5}+D+C^{y-5}+10^z+w for z=0,1,2,3and w=0,1,2,….9

(B)xyzw=$\bar{V}$+M^{x-5}+500+100^{y-5}+(50-10)+w=$\bar{V}$+M^{x-5}+D+C^{y-5}+XL+ w for z=4and w=0,1,2,….9

(C)xyzw=$\bar{V}$+M^{x-5}+500+100^{y-5}+50+10^{z-5}+w=$\bar{V}$+M^{x-5}+D+C^{y-5}+L+X^{z-5}+ w for,z=5,6,7,8and w=0,1,2,….9

(D) xyzw=$\bar{V}$+M^{x-5}+500+100^{y-5}+(100-10)+w=$\bar{V}$+M^{x-5}+D+C^{y-5}+XC+w for,z=9 and

w=0,1,2,….9

Under x=5,6,7,8 and y=9

(A)xyzw=$\bar{V}$+M^{x-5}+(1000-100)+10^z+w=$\bar{V}$+M^{x-5}+CM+10^z+wfor,z=0,1,2,3and w=0,1,2,….9

(B)xyzw= $\bar{V}+M^{x-5}$+(1000-100)+(50-10)+w=$\bar{V}+M^{x-5}$+CM+XL+w for,z=4and w=0,1,2,….9

(C)xyzw=$\bar{V}+M^{x-5}$+(1000-100)+50+10^{z-5}+w=$\bar{V}+M^{x-5}$+CM+L+X^{z-5}+w for,z=5,6,7,8and w=0,1,2,….9

(D)xyzw= $\bar{V}+M^{x-5}$+(1000-100)+(1000-10)+w=$\bar{V}+M^{x-5}$+CM+XC+w for,z=9and w=0,1,2,….9

Finally, Under x=9, and y=0,1,2,3,

(A)xyzw=(10000-1000)+C^y+ X^z+w for z=0,1,2,3 and w=0,1,2,….9

=($\bar{X}$-M) +C^y+ X^z+w for z=0,1,2,3 and w=0,1,2,….9

==M$\bar{X}$ +C^y+ X^z+w for z=0,1,2,3 and w=0,1,2,….9

(B) xyzw= M$\bar{X}$ +C^y+ (50-10)+w = M$\bar{X}$ +C^y+XL+w for ,z=4 and w=0,1,2,….9

(C) xyzw= M$\bar{X}$ +C^y+ 50+10^{z-5}+w = M$\bar{X}$ +C^y+L+X^{z-5}+w for,z=5,6,7,8 and w=0,1,2,….9

(D) xyzw= M$\bar{X}$ +C^y+ (100-10)+w = M$\bar{X}$ +C^y+XC+w for, z=9and w=0,1,2,….9

Under x=9, and y=4

(A)xyzw= M$\bar{X}$ +(500-100)+10^z+w = M$\bar{X}$ +CD+X^z+w for ,z=0,1,2,3 and w=0,1,2,….9

(B)xyzw= M$\bar{X}$ +(500-100)+(50-10)+w = M$\bar{X}$ +CD+XL+w for,z=4and w=0,1,2,….9

(C)xyzw= M$\bar{X}$ +(500-100)+50+10^z+w = M$\bar{X}$ +CD+L+X^z+w for ,z=5,6,7,8 and w=0,1,2,….9

(D)xyzw= M$\bar{X}$ +(500-100)+(100-10)+w = M$\bar{X}$ +CD+XC+w for,z=9and w=0,1,2,….9

Under x=9, and y=5,6,7,8

(A)xyzw= M$\bar{X}$ +500+100^{y-5}+10^z+w = M$\bar{X}$ +D+C^{y-5}+X^z+w for ,z=0,1,2,3 and w=0,1,2,….9

(B)xyzw= M$\bar{X}$ +500+100^{y-5}+(50-10)+w = M$\bar{X}$ +D+C^{y-5}+XL+w for,z=4and w=0,1,2,….9

(C)xyzw=M$\bar{X}$ +500+100^{y-5}+50+10^{z-5}+w =M$\bar{X}$ +D+C^{y-5}+L+x^{z-5}+w for, z=5, 6, 7, 8 and w=0,1,2,….9

(D)xyzw= M$\bar{X}$ +500+100^{y-5}+(100-10)+w = M$\bar{X}$ +D+C^{y-5}+XC+w for,z=9 and w=0,1,2,….9

Under x=9, and y=9

(A)xyzw= M$\bar{X}$ +(1000-100)+10^z+w = M$\bar{X}$ +CM+X^z+w for ,z=0,1,2,3 and w=0,1,2,….9

(B)xyzw= M$\bar{X}$ +(1000-100)+(50-10)+w = M$\bar{X}$ +CM++XL +w for,z=4and w=0, 1,2,….9

(C)xyzw= $M\overline{X}$+(1000-100)+50+10^{z-5}+w=$M\overline{X}$+CM+L+X^z+w for ,z=5,6,7,8 and w=0,1,2,….9

(D)xyzw= $M\overline{X}$ +(1000-100)+(100-10)+w = $M\overline{X}$ +CM+XC+w for,z=9and w=0,1,2,….9

Similar pattern of formula can be established for Roman numerals whose decimal number is 5 digits, 6 digits and so on….

We can verify above explored pattern of formulae to get Roman numerals corresponding to their respective decimal numbers. For example

Example Conversion of decimal number 461 into Roman number

Here 3 digit Decimal number 371 is compared with xyz s.t. x=3,y=7, z=1, then using our explored formula,

xyz=100^x+50+10^{y-5}+ z=C^x+L + X^{y-5} +z for x=1,2,3, y=5,6,7,8 and z=0,1,2,….9

i.e. 371= C^3+L + X^{7-5} +1 = CCC+L+X^2+1=CCC+L+XX+I=CCCLXXI, where 1=I. Hence, required Roman numeral corresponding to its Decimal number 371 is CCCLXXI. Moreover, by observing available table of Roman numbers, our explored formula stays validity.

Conclusions and Decision

Numeral is only arrangement of symbols but number is arrangement of symbols with particular sense. Any Roman numbers up to 4 digits can be found out from above appropriate generalization .Similarly, in this way Roman numbers up to any digits can be found out by above method of induction. As math is various ways for finding formula (generalization), one can find out other possible fourmula (s) to establish Roman numerals by their own concept of mathematical way.

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