Journal of Business and Social Sciences (ISSN: 2350-8876)
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Relation between Weighted Mean Matrices and Area of Application and Challenges

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Cite this paper:

Kumar, N.K., & Mahato , R.K. (2024). Relation between weighted mean matrices and area of application and challenges. *Journal of Business and Social Sciences, 6*(1), 85- 90

https://doi.org/10.3126/jbss.v6i1.78757

Abstract

This paper established relationship between ($\overline{N}, p_n$) and $\overline{N}, q_n$) weighted mean matrices, for l^p spaces as $1 < p < \infty$. This paper also contains its components, applications and Challenges and Considerations. A flexible tool that considers the importance of individual data points, weighted mean matrices provide more in-depth data analysis. For the purpose of making well-informed judgments based on weighted averages, they find applications in a variety of fields, including environmental science and finance. Weighted mean matrices can shed light on complicated data sets when used correctly.

Keywords: Weighted mean matrices, diagonal matrices, partial summation, Chebychev's inequality, Nurlund matrix

MSC 2020: 11C20,11J83,11K45,11K50

Introduction

In various fields of mathematics, statistics, and data analysis, weighted mean matrices play a crucial role in aggregating and summarizing data while considering the significance or importance of individual data points. This article explores the concept of weighted mean matrices, their applications [19,20,21].

Weighted Mean Matrix

A weighted mean matrix is a mathematical construct used to compute the weighted average of a set of values or data points. Unlike a traditional mean, which assigns equal importance to all data points, a weighted mean matrix assigns different weights to each data point based on their significance or relevance to the analysis [5,6].

Components of a Weighted Mean Matrix [15,16,17,18]

1. Data Points (X): These are the values you want to calculate the weighted mean for. They could represent various metrics, such as scores, prices, or quantities.

2. Weights (W): Weights are assigned to each data point to indicate their relative importance. Higher weights signify that a data point has a greater influence on the final weighted mean.

Preliminaries

Let s_n is partial sum of an infinite series $\sum a_n$.

$p_n = p_0 + p_1 + \cdots\cdots + p_n$ tends to $\propto$ as n tends to $\propto$, for $p_n \geq 0$, $p_0 > 0$.

$t_n = \frac{1}{p_n} \sum_{\nu=0}^{n} p_\nu s_\nu$ tends to s.

If n tends to $\propto$, then s_n tends to s $(\overline{N}, p_n)$ [1]

Let $p_n \geq 0$, $q_n > 0$ and $\sum p_n = \propto$, either $\sum q_n = \propto$ then

(i) $\frac{q_{n+1}}{q_n} \leq \frac{p_{n+1}}{p_n}$ (1)

(ii) $\frac{p_{n+1}}{p} \geq \frac{q_{n+1}}{q_n}$ (2)

Also gives $\frac{P_n}{p_n} \leq H \frac{Q_n}{q_n}$ (3)

Then $\sum a_n = s\ (\overline{N}, p_n) \Longrightarrow \sum a_n = s\ (\overline{N}, q_n)$.

Main Results Concerning Relationship [11,12,13,14]

Theorem 1. If p_n and q_n be +ve sequences satisfying conditions

$p_n \left(\frac{q_n+1}{p_n+1} - \frac{q_n}{p_n}\right) \approx (n+1)^{\alpha}$, for some $\alpha \geq 0$. (4)

$\frac{p_n}{p_n} \cdot \frac{q_n}{q_n} \leq$ H. (5)

$[p_n]$ is decreasing sequences (6)

$(\overline{N}, q_n)$ is a matrix of bounded linear operator on l^p. In view of equation given by Hady [2] is partially true for $\frac{q_{n+1}}{p_{n+1}} \geq \frac{q_n}{p_n}$ for l^p spaces as $1 < p < \infty$.

Rhoades [2] discovered sufficient conditions in 1994, when seen as bounded operators on l^p, 1 < p < o, some weighted mean matrices are identical to C,

Theorem 2. Let $[a_n]$ be a positive sequence that, for some $\alpha \geq 0$, satisfies the following condition:

(n + 1) $(a_{n+1} - a_n)$ almost equal to $(n+1)^{\alpha}$. For 1 < p < $\propto$, then ($\overline{N}$, a) and C are identical over l^p. The requirement of Theorem B is met if $q_n = a_n$ and $p_n = 1$ is used in theorem 2.

Rhoades' theorem 2 states that whenever a matrix (C, 1) is a bounded linear operator on l^p, ($\overline{N}$, a) matrix is also a bounded linear operator on l^p. The following Lemma 1 and 2 are required for the proof of theorem 2.

Lemma 1. If 1 < p < $\propto$, and q is the conjugate index of p. If $c_n, d_k \geq 0$ and

$c_n \sum_{k=0}^{n} d_k^q \leq K d_n^{\frac{1}{p-1}}$ for n = 0,1, --------- and for constant K, then A is a bounded operator on l^p [3].

Lemma 2. Let

(i) $a_1 \leq a_2 \leq$ ------$\leq a_n$ and $b_1 \leq b_2 \leq$ ------$\leq b_n$ then the Chebychev's inequality describes further,

(ii) $\left(\frac{1}{n}\sum_{\nu=1}^{n} a_\nu\right)\left(\frac{1}{n}\sum_{\nu=1}^{n} b_\nu\right) \leq \left(\frac{1}{n}\sum_{\nu=1}^{n} a_\nu b_\nu\right)$

(iii) $1^p + 2^p +$ --------$+ n^p > \frac{n^{p+1}}{p+1}$ where $p > 0$ [4].

Lemma 3. If $\{p_n\}$ and $\{q_n\}$ be the two positive sequences satisfying the equations (4), (5) and (6) of the theorem, then

$$Q_n \geq \frac{(n+1)^{\alpha+1}}{(\alpha+1)(\alpha+2)}, \text{ for } a \geq 0. \tag{7}$$

Proof. If $Q_n \to \infty$, $as\ n \to \infty$. $a \geq 0$, by equation (4)

$$P_k\left(\frac{q_{k+1}}{p_{k-1}} - \frac{q_k}{p_k}\right) \approx (k+1)^\alpha \Longrightarrow \frac{q_{k+1}}{p_{k-1}} - \frac{q_k}{p_k} \approx \frac{(k+1)^\alpha}{p_k}$$

Now, summation

$$\sum_{k=0}^{n-1}\left(\frac{q_{k+1}}{p_{k-1}} - \frac{q_k}{p_k}\right) \approx \sum_{k=0}^{n-1}\frac{(k+1)^\alpha}{p_k} \Longrightarrow \frac{q_n}{p_n} \approx \sum_{k=0}^{n-1}\frac{(k+1)^\alpha}{p_k}.$$

$\frac{q_k}{p_k} \approx \sum_{r=0}^{k}\frac{(r+1)^\alpha}{p_r}$ implies $q_k \approx p_k \sum_{r=0}^{k}\frac{(r+1)^\alpha}{p_r}$ also implies $\sum_{k=0}^{n} q_k \approx \sum_{k=0}^{n} p_k \sum_{r=0}^{k}\frac{(r+1)^\alpha}{p_r}$

From Chebychev's inequality,

$$Q_n \geq n\left(\frac{1}{n}\sum_{k=0}^{n} p_k\right)\left(\frac{1}{n}\sum_{k=0}^{n}\sum_{r=0}^{k}\frac{(r+1)^\alpha}{p_r}\right) = \frac{p_n}{n}\sum_{k=0}^{n}\sum_{r=0}^{k}\frac{(r+1)^\alpha}{p_r} \geq \frac{p_n}{n}\sum_{k=0}^{n}\frac{1}{p_k}\sum_{r=0}^{k}(r+1)^\alpha$$

$$\geq \frac{p_n}{n}\sum_{k=0}^{n}\frac{1}{p_k}\frac{(k+1)^{\alpha+1}}{(\alpha+1)} \text{ (from lemma 2)}$$

$$\geq \frac{p_n}{np_n}\sum_{k=0}^{n}\frac{(k+1)^{\alpha+1}}{(\alpha+1)} = \frac{1}{n(\alpha+1)}\sum_{k=0}^{n}(k+1)^{\alpha+1} \geq \frac{1}{n(\alpha+1)}\frac{(n+1)^{\alpha+2}}{(\alpha+2)} \text{ (from Lemma 2)}$$

Implies $Q_n \geq \frac{1}{(\alpha+1)}.\frac{(n+1)^{\alpha+2}}{(\alpha+2)}$ is the proof of Lemma 2

Now proof of the theorem 2. Let $t_{n,q} = \frac{1}{Q_n}\sum_{k=0}^{n} q_k s_k$ (8)

$$t_{n,p} = \frac{1}{P_n}\sum_{k=0}^{n} p_k s_k \tag{9}$$

Solving equations (9) for s_n then after putting in equation (8),

$$t_{n,q} = \frac{1}{Q_n} \sum_{k=0}^{n} [\, p_k t_{k,p} - p_{k-1} t_{k-1,p} \,] \frac{q_k}{p_k}.$$

From partial summation,

$$t_{n,q} = \frac{1}{Q_n} \left(\frac{q_{n+1}}{p_{n+1}} p_n t_{n,q} + \sum_{k=0}^{n} \left(\frac{q_k}{p_k} - \frac{q_{k+1}}{p_{k+1}} \right) p_k t_{k,p} \right) \quad (10)$$

Let Dis diagonal matrix, $d_{nn} = \frac{q_{n+1}}{p_{n+1}} \frac{P_n}{Q_n}$ (11)

It suffices to demonstrate that D is bounded and that C $\in$ B (l^p) in order to demonstrate that A $\in$ B (l^p). Considering the equation (5), D has boundaries. We utilize Lemma 1 to demonstrate that C is bounded. Let

$$c_n := \frac{1}{Q_n}$$

$$d_n := p_k \left(\frac{q_{k+1}}{p_{k+1}} - \frac{q_k}{p_k} \right) \geq 0$$

Let $\alpha \geq 0$. Then the equation (4) implies that $d_k \approx (k+1)^{\alpha}$.

From lemma 1,

$$c_n \sum_{k=0}^{n} d_k^q \leq k d_n^{1/p-1},$$

makes C is bounded.

Now

$$\frac{c_n}{d_n^{1/p-1}} \sum_{k=0}^{n} d_k^q = \frac{1}{Q_n [(n+1)^{\alpha}]^{q-1}} \sum_{k=0}^{n} (k+1)^{\alpha q} \leq k \text{, as constant k.}$$

So, $t_{n,q} \in l^p$ and also $t_{n,p} \in l^p$ is the proof o theorem 2 [5,6].

Applications [5, 7,8,9,10]

Financial Analysis: In finance, weighted mean matrices are used to calculate portfolio returns, where different stocks are given different weights based on their importance in the portfolio.

Statistics: In survey sampling, weighted means are used to account for different probabilities of selecting sample units, ensuring a more accurate representation of the population.

Environmental Science: Researchers often use weighted mean matrices to compute environmental indices, considering various factors' ecological importance.

Machine Learning: In machine learning, weighted mean matrices can be applied to create weighted averages of feature vectors, emphasizing important features while reducing noise.

Social Sciences: Weighted means are used in social sciences to calculate composite indices, like quality-of-life indexes, which assign different weights to various components based on societal priorities.

Customer Satisfaction Surveys: Companies use weighted mean matrices to calculate overall satisfaction scores, giving more weight to aspects that matter most to customers.

Challenges and Considerations

While weighted mean matrices offer powerful tools for data analysis, they come with some challenges:

Subjectivity: Assigning weights can be subjective and may vary depending on the context and the analyst's judgment.

Data Quality: The accuracy of the weighted mean matrix depends on the quality of the data and the appropriateness of weight assignments.

Interpretation: Interpreting the results requires a clear understanding of the weighting scheme used and its implications.

Conclusion

Weighted mean matrices are versatile tools that allow for more nuanced data analysis by considering the significance of individual data points. They find applications in various fields, from finance to environmental science, and are essential for making informed decisions based on weighted averages. When used appropriately, weighted mean matrices can provide valuable insights into complex data sets.

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