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A Comparative Study of Immediate Annuities and Ordinary Annuities in establishing the Phantom of Zero Liability Under the Trusteeship Pension Valuation Structure

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Abstract

Background: The liabilities of a pension scheme define the financial value to be paid at a definite period in the future. The underlying goal of pension plans is to provide retirees with sufficient stream of income to enable them to live a decent financially independent life post-employment period. The regulatory framework for occupational pension schemes necessitates the services of trustees as administrators who assume legal administrative responsibilities on the scheme and saddled to oversee actuarial valuations of the scheme's liabilities at definite points in time.

Objectives: The objective of this paper is (i) to empirically examine the drivers of pension liability and how they are evaluated by the trustee's model. (ii) Specifically, the study intends to use input parameters of the trustee model to establish the conditions for which the value of liability is zero under trusteeship annuity factor.

Methods: This study applies trustees' valuation model, the present values together with infinitesimal calculus. Salary data as well as demographic data were obtained from an agricultural production services company located in Jos-South, Nigeria.

Results: Computational evidence from our results proves that the total service liability under the conditions of the current model is vanishingly zero. However, when the annuity factor is replaced by life table annuity, the service liability does not vanish.

Conclusion: The total service liability obtained as zero therefore initiates inquiry as to whether this current valuation framework causes potential uncertainties for the pension trustees who are responsibly saddled with both administration and core decision-making responsibilities of the system.

Keywords: Annuity factor, liability, potential uncertainties, trustees' model, valuation

JEL Classification: C02, C13, C15, C63

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Introduction

The goal of this paper is to numerically examine the drivers of pension liability and determine how they are computationally evaluated under the trustee's model. In particular, it intends to employ the parameters of the trustee's model to compute the pension liability of a defined benefit pension scheme and subsequently prove that the value of liability is zero under the newly defined trusteeship annuity factor as functionally defined in McNally and O'Connor (2013). The liability for pension plan is an important form of employee's basic compensation package that has favourable bearing on scheme member's motivation and diligence to continue in the service of the plan sponsor. It further commits the plan members to achieve the set objectives on the job and arouses concern for the continued existence of the company. Therefore, a public pension plan defines a negotiated framework where the plan sponsor designs pension for members towards retirement.

Consequently, at retirement, the plan gives the plan members opportunities to secure a reasonable living standard to fall in line with what obtained during service. There seems to be variations in the manner in which pension assets are administered and benefits are disbursed to qualified members as a result of the problems connected to the previously existing pension plan. Defined benefits pension schemes present a type of deferred remuneration received beyond the different kinds of cash disbursements. Apart from the salaries earned on their official duties presently, scheme members would obtain promised benefits payable from retirement. In order carry out actuarial valuation on these benefits, it is sufficient to determine the level of the expected future pension benefits would be recognized now such that the present value of benefits captures the future expected benefit disbursements. This describes the net present-value accounting for the scheme member's expected future salaries as a current liability.

Review of Literature

Generally, the social security system recognizes pension liabilities employing the actuarial phenomenon of pension benefit obligation or the entry age normal approach. The two methods mentioned takes care of future salaries growth projection but would not account for future service and consequently only accounts for a certain percentage of the present value of benefits. The pension benefit obligation accounts for the present value of benefits in proportion to the percentage of a plan member's service received to date in relation to the expected total at retirement. However, the entry age normal pension benefit obligation accounts for the present value of benefits in proportion to the percentage of a scheme member's discounted total salaries received to the present in relation to the expected total at retirement.

In Novy-Marx and Rauh (2009), the least applied actuarial liability method on defined benefits pension schemes is the accrued benefit obligation. This technique recognizes the benefit payments which are received to the present and bases the projected benefit payments outgo on the current member's salary profile. We infer from Pugh (2006) that actuarial techniques of pension liabilities valuations which incorporate salary growth in relation to pension benefit obligation, entry age normal or present value of benefits should be actuarially discounted

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at bigger rates. Consequently, pension actuary observe that the discount rate must include a measure of uncertainty connected to pension liabilities and because pension benefits are usually guaranteed under public pension scheme, the precise discount factor is a risk-free rate. We observe in Chen and Matkin (2017) that since employee's salaries are inadvertently introduced to inflation uncertainties, salaries growth and the stock market should be positively correlated across a long-time horizon. When the employee's salary profile is reviewed upwards, then Brien (2020); Wang et al. (2021) argue that optimal investment on the risky asset declines although in order to earn robust returns, the pension manager should inject more funds to buy risky asset instruments. The authors noted that when the employee's salary increases, the living reserve fund markedly improves, and the risk tolerance will be strong such that the investment trajectories become more volatile.

Following Black (2006), the price of stocks rises where it seems that times look definitely favourable. During favourable times, salaries and benefits seems to grow exponentially faster than normal, consequently, the wider your perspective on the pension liability, the more stocks you would require for hedging against risk. Lucas and Zeldes (2006) obtained an actuarial structure for the numerical estimation of the correct risk-adjusted discount rates for defined benefits pension liabilities that takes care of the future pension benefit payments exposure to the market via the salaries growth profile.

These problems seem insignificant under the accrued benefits obligation that is not exposed to salary risk. However, Bulow (1982) observes that the accrued benefit obligation is usually an adequate liability technique for company's schemes since deeper actuarial methods could inadvertently and inadequately mean an intrinsic pension scheme under which young plan members acquiesce to low aggregate remuneration in exchange for an unofficial covenant that such employees would be heavily compensated in their career progression. As a result of the attendant consequences in addition to those inadvertently connected with which benefits are currently recognized precisely, Following Bulow (1982), a possible consequence of such an assumption is that it is erroneously believed that where an employee's benefit is tied to his final salary profile, he is insured against inflation risk till he retires. Therefore, the provision for inflation protection mechanism or the non-existence of such mechanism would definitely affect the correct discount rate to apply in discounting pension liabilities irrespective of which liabilities amount are recognized.

Nevertheless, Bodie (1990) observes that the inability of plan sponsors to exhibit marked interest in inflation protection investment vehicles through CPI-linked bonds apparently proves that the pension fund liabilities is not unit linked nor inflation-indexed. The basic goal of pension plan is to secure retirees against abject poverty, empower them to live required standard of living and guarantee an economically independent life at senescence. From the point where a scheme is incepted, achievable financial targets are set which should be satisfied as appropriate. The capacity of a pension plan to satisfy its pension liabilities objectives at a prescribed period under current regulatory framework can be actuarially appraised even though

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this could solely define a good estimation provided that the future is assumed certain. In order to achieve the plan's set objectives, it is necessary to perform actuarial valuation of liabilities at regular intervals under the framework of defined benefit scheme structure.

The valuation of a defined benefit plan necessitates a numerical appraisal of both the plan's assets and its liabilities. Essentially, valuation exercise of a defined benefit pension scheme's assets and liabilities under trusteeship scheme is performed under the following settings. (i) To establish both the approved pension scheme valuation funding technique and determine what techniques are applicable to the scheme being considered. (ii)To establish an approved assumption for valuation and estimate the impact of scheme's size on the tendency to recover from any perturbations (stability of its funding). (iii) To measure future cash flows of the scheme and value the assets and liabilities. (iv) To model the sensitivity of the plan to perturbations of actuarial parameters such as interest rates and mortality. The valuation could be performed where the actuary intends to confirm if the pension fund meets the minimum funding standard laid down under the legal framework. For an essential use of annual trust report to the plan members, the pension fund trustees particularly needs the valuation exercise to evaluate contribution rates.

The valuation exercise could be performed for the purpose of the financial statements of the plan sponsor to recognize the fair value of the surplus or deficit in the pension plan although there does not seem to exist definite conditions for consistency under the conditions (i)-(iv) valuation assumptions just enumerated in the valuation requirements. Consequently, a defined benefit plan could possess four key valuation results needed for the four differing scenarios each of which could be described as sufficiently adequate on its set objective at any given valuation date. However, the specified guidelines set for the four valuation procedures could orchestrate varied assumptions, computation bases and varied attention on the results produced. Since defined benefit scheme has long term liabilities, the valuation concept must embed uncertainties such as investment uncertainties and volatility uncertainties which are apparently connected with the claims together with investment returns which impact the adequacy of the reserved capital to cover such claims.

As a result of the marked risks connected with the incomplete financial markets, defined benefit pension liabilities is not usually fully hedgeable because a full hedging could lead to unbearable costs. We observe in Sundaresan & Zapatero (1997); Inkmann, Blake & Shi (2017), that the adoption of mortality rate in measuring valuation is a critical variable when computing replacement ratio of future funds. The estimation of liabilities already received necessitates accumulation as opposed to pension system which addresses discounting a stream of promised future benefits cash flows to the present with the goal of computations associated with the valuation exercise. McNally & O'Connor (2013) observe that a current challenge for trustees is to understand and reconcile the different valuation processes. Despite the appointment of financial market consultants on the scheme, trustees could still be conscripted into legal problems over non-performance of pension funds if they are not adequately informed as to the

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robustness of the valuation results.

Following Josa-Fombellidaa and Rincon-Zapatero (2004), and Josa and Navas (2014), an aggregate form of defined benefit pension scheme where the active scheme members instantaneously exist contemporarily with retirees is suggested such that the trustees determine in advance benefits receivable by the scheme members at retirement. However, it is assumed that such benefit being modelled through deterministic method would correlate with the financial market dynamics. As scheme members save for retirement benefits vide occupational pension schemes, there exist expectations on the structural value of benefits disbursement at qualification period and consequently, public pension plans possess explicit models for computing retirement benefits which functionally depends on the number of years of service and salary.

Apparently, both McNally and O'Connor (2013), and McNally and O'Connor (2018) developed alternative innovative models when performing actuarial valuation exercise coming from the Irish perspectives under three scenarios. From our keen observation, it seems the authors could not carry out further asymptotic investigations on their models. This lacuna observed in McNally & O'Connor (2018) presents serious gaps in actuarial literature. In attempting to partly solve this problem, Ogungbenle (2022) further investigated the actuarial conditions on pension liability under the International Accounting Standard $IAS19$ pension guidelines. The author found out that when certain mathematical annuity was imposed on $IAS19$ pension model, the total liability vanishes. Moreover, the actuarial valuation of pension liability under the minimum funding standard was investigated in Ogungbenle and Omede (2022). Consequently, when a different mathematical annuity was imposed on the model, the authors found out that the liability is vanishingly zero. In order to fill the gap identified, this study will approach our arguments in another actuarial direction different from those in Ogungbenle (2022); Ogungbenle and Omede (2022) and McNally and O'Connor (2018).

As observed above, it seems McNally and O'Connor (2018) could not carry out any further asymptotic investigations on their models. This lacuna observed in McNally and O'Connor (2018) represents serious waves of gaps in actuarial literature. In order to fill the gap identified, this study will approach our arguments in another actuarial direction using the Trustee's valuation model with another mathematical annuity factor defined on it**.**

Description of Notation

L_{Trust}	Total service liability
S_c	Number of years of pensionable service completed to date.
SAL	Current salary
k	The number of pensionable years
X	Number of years to retirement
N	Expected lifespan post-retirement
AF	Annuity factor
AR	Annuity rate

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P_g	Pension increase
N	Life expectancy after retirement 10 years
P_g	Pension increase from 1.0% to 4.7%
d	Discount rate 4.5%
$SALG$	Salary growth 5.0%

Life Annuities

The present value of an immediate yearly annuity of 1 unit of currency per annum due to a life aged x is defined as a_x and converges to the sum of a series of pure endowments of 1. The first payment of these annuities is disbursed towards the end of the payment period.

Following Neil (1979), $$a_x = \left(\frac{1}{\frac{1}{(1+i)^x}} \sum_{s=1}^{\infty} ({}_sP_x) \frac{1}{(1+i)^s} \right) \left[\frac{1}{(1+i)^x} \right] \quad (1)$$

${}_sP_x$ is the probability that a life aged x will survive to age $x+s$ and i is the interest rate. However, l_x defines the number of lives surviving to age x

$$a_x = \left(\frac{\frac{1}{(1+i)} l_{x+1} + \frac{1}{(1+i)^2} l_{x+2} + \frac{1}{(1+i)^3} l_{x+3} + \frac{1}{(1+i)^3} l_{x+3} \ldots}{l_x} \right) \left[\frac{\frac{1}{(1+i)^x}}{\frac{1}{(1+i)^x}} \right] \quad (2)$$

$$a_x = \left(\frac{\frac{1}{(1+i)^{x+1}} l_{x+1} + \frac{1}{(1+i)^{x+2}} l_{x+2} + \frac{1}{(1+i)^{x+3}} l_{x+3} + \frac{1}{(1+i)^{x+4}} l_{x+4} \ldots}{\frac{1}{(1+i)^x} l_x} \right) \quad (3)$$

$$A_{x:\overline{m}|}^{\;1} = v^m ({}_mP_x) = {}_mE_x \quad (4)$$

$$D_x = v^x l_x \quad (5)$$

D_x is the discounted deaths and $v = \frac{1}{1+i}$ (6)
is the discount function

$$A_{x:\overline{m}|}^{\;1} = \frac{D_{x+m}}{D_x} \quad (7)$$

$$a_x = \left(\frac{v^{x+1} l_{x+1} + v^{x+2} l_{x+2} + v^{x+3} l_{x+3} + v^{x+4} l_{x+4} \ldots}{v^x l_x} \right) \quad (8)$$

$$a_x = A_{x:\overline{1}|}^{\;1} + A_{x:\overline{2}|}^{\;1} + A_{x:\overline{3}|}^{\;1} + A_{x:\overline{4}|}^{\;1} + \ldots + A_{x:\overline{\Omega - x - 1}|}^{\;1} \quad (9)$$

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$$a_x = \left(\frac{D_{x+1} + D_{x+2} + D_{x+3} + D_{x+4} \ldots}{D_x} \right) \tag{10}$$

$$a_x = \frac{1}{D_x} \sum_{s=1}^{\Omega - x - 1} D_{x+s} \tag{11}$$

$$N_x = \sum_{s=0}^{\Omega - x - 1} D_{x+s} \tag{12}$$

N_x is the sum of discounted deaths and Ω is the highest age in the mortality table

$$a_x = \frac{N_{x+1}}{D_x} \tag{13}$$

However, when the first of the series of payments is disbursed towards the beginning of the period, then the present value of the annuity is defined by the following series.

$$\ddot{a}_x = \left(\frac{\frac{1}{(1+i)^0} l_x + \frac{1}{(1+i)} l_{x+1} + \frac{1}{(1+i)^2} l_{x+2} + \frac{1}{(1+i)^3} l_{x+3} + \frac{1}{(1+i)^4} l_{x+4} \ldots}{l_x} \right) \tag{14}$$

$$\ddot{a}_x = \left(\frac{1}{l_x} \sum_{s=0}^{\infty} \frac{l_{x+s}}{(1+i)^s} \right) \left[\frac{\frac{1}{(1+i)^x}}{\frac{1}{(1+i)^x}} \right] \tag{15}$$

$$a_x = \left(\sum_{s=0}^{\infty} e^{-\delta s} \left({}_sP_x \right) \right) \left[\frac{\frac{1}{(1+i)^x}}{\frac{1}{(1+i)^x}} \right] \tag{16}$$

$$\ddot{a}_x = \left(\frac{\frac{1}{(1+i)^0} l_x + \frac{1}{(1+i)} l_{x+1} + \frac{1}{(1+i)^2} l_{x+2} + \frac{1}{(1+i)^3} l_{x+3} + \frac{1}{(1+i)^4} l_{x+4} \ldots}{l_x} \right) \left[\frac{\frac{1}{(1+i)^x}}{\frac{1}{(1+i)^x}} \right] \tag{17}$$

$$\ddot{a}_x = \left(\frac{\frac{1}{(1+i)^x} l_x + \frac{1}{(1+i)^{x+1}} l_{x+1} + \frac{1}{(1+i)^{x+2}} l_{x+2} + \frac{1}{(1+i)^{x+3}} l_{x+3} + \frac{1}{(1+i)^{x+4}} l_{x+4} \ldots}{\frac{1}{(1+i)^x} l_x} \right) \tag{18}$$

$$\ddot{a}_x = \left(\frac{v^x l_x + v^{x+1} l_{x+1} + v^{x+2} l_{x+2} + v^{x+3} l_{x+3} + v^{x+4} l_{x+4} \ldots}{v^x l_x} \right) \tag{19}$$

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$$\ddot{a}_x = \left(\frac{D_x + D_{x+1} + D_{x+2} + D_{x+3} + D_{x+4} + \ldots}{D_x} \right) \quad (20)$$

$$\ddot{a}_x = \frac{N_x}{D_x} \quad (21)$$

$$\ddot{a}_x - a_x = \frac{N_x}{D_x} - \frac{N_{x+1}}{D_x} \quad (22)$$

$$\ddot{a}_x - a_x = \frac{N_x - N_{x+1}}{D_x} \quad (23)$$

$$N_x = D_x + N_{x+1} \quad (24)$$

$$\ddot{a}_x - a_x = \frac{D_x + N_{x+1} - N_{x+1}}{D_x} \quad (25)$$

$$\ddot{a}_x - a_x = \frac{D_x}{D_x} = 1 \quad (26)$$

Consequently, the actuarial deviation of $\ddot{a}_x$ from a_x defines the payment made immediately and hence $\ddot{a}_x - a_x = 1$

Material and Methods

Theoretical and Conceptual Framework

Let $\mathbf{R} < \infty$ be the retirement age and let $B(\mathbf{R})$ define the accrued benefits at age $\mathbf{R}$

In (Jordan, 1991), the commutation function ${}_{\xi}E_x = \frac{D_{x+\xi}}{D_x}$ (27)

where $D_x = v^x l_x$; $v = \frac{1}{1+i}$ is the discount factor ; i is the interest rate and l_x is the number of lives surviving to age x

The liability of the pension plan to the employee at $\mathbf{R}$ is $L(\mathbf{R})$

$$L(\mathbf{R}) = \frac{B(\mathbf{R})}{D_{\mathbf{R}}} \int_0^{\infty} D_{\mathbf{R}+\xi} d\xi = B(\mathbf{R}) \int_0^{\infty} \frac{D_{\mathbf{R}+\xi}}{D_{\mathbf{R}}} d\xi \quad (27a)$$

$$L(\mathbf{R}) = B(\mathbf{R}) \int_0^{\infty} {}_{\xi}E_{\mathbf{R}} d\xi \quad (27b)$$

Following Neil (1979; Bowers et al., (1997); Hudec (2017); Souza (2019); & Kara (2021),

$$\left({}_{\xi}E_{\mathbf{R}}\right) = e^{-\int_0^{\xi} \mu_{\mathbf{R}+u} + \delta du} \quad (27c)$$

Where $\mu_{\mathbf{R}+u} = -\frac{1}{l_{\mathbf{R}+u}} \frac{dl_{\mathbf{R}+u}}{du}$ (27d)

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is the mortality rate intensity
and $\delta = \log_e(i+1)$ is the force of interest. Consequently, putting (2a) in (2), the liability function becomes

$$L(\mathbf{R}) = B(\mathbf{R})\int_0^{\infty}\left\{e^{-\int_0^{\xi}\mu_{\mathbf{R}+u}+\delta\, du}\right\}d\xi \tag{27e}$$

We can now push the $B(\mathbf{R})$ into the integral to have

$$L(\mathbf{R}) = \int_0^{\infty} B(\mathbf{R})\left\{e^{-\int_0^{\xi}\mu_{\mathbf{R}+u}du}\times e^{-\int_0^{\xi}\delta\, du}\right\}d\xi \tag{27f}$$

$$L(\mathbf{R}) = \int_0^{\infty} B(\mathbf{R})\left\{e^{-\int_0^{\xi}\mu_{\mathbf{R}+u}\, du}\right\}e^{-\delta\xi}\, d\xi \tag{27g}$$

Note that $\left({}_{\xi}P_{\mathbf{R}}\right) = e^{-\int_0^{\xi}\mu_{\mathbf{R}+u}du}$ (27h)

$$L(\mathbf{R}) = \int_0^{\infty} B(\mathbf{R})\left({}_{\xi}P_{\mathbf{R}}\right)e^{-\delta\xi}d\xi \tag{27i}$$

Let x is any age such that $x > \mathbf{R}$ and e is the entry age at which the employee was employed.
Suppose $\xi = x - \mathbf{R}$ then $d\xi = dx$. When $\xi = 0$; $x = \mathbf{R}$ and if $\xi = \infty$; $x = \infty$
If $\mathbf{R} - e$ is the years of service with $\mathbf{R} - e > l$ where l is the length of service.
Then the expected present value of the future benefits promised by the pension plan sponsor connected with the retirement age $\mathbf{R}$ for the employee with $\mathbf{R} - e$ years of service is given by

$$L(\mathbf{R}) = \int_{\mathbf{R}}^{\infty} B(\mathbf{R})\left({}_{x-\mathbf{R}}P_{\mathbf{R}}\right)e^{-\delta(x-\mathbf{R})}dx = B(\mathbf{R})\int_{\mathbf{R}}^{\infty}\left({}_{x-\mathbf{R}}P_{\mathbf{R}}\right)e^{-\delta(x-\mathbf{R})}dx \tag{27j}$$

$$L(\mathbf{R}) = \int_{\mathbf{R}}^{\infty} B(\mathbf{R})\left({}_{x-\mathbf{R}}P_{\mathbf{R}}\right)e^{-\delta(x-\mathbf{R})}dx = B(\mathbf{R})\eta(\mathbf{R}) \tag{27k}$$

The continuous annuity factor representing the present value of retirement age $\mathbf{R}$ of a lifetime annuity of 1 unit of currency is therefore defined as

$$\eta(\mathbf{R}) = \int_{\mathbf{R}}^{\infty}\left({}_{x-\mathbf{R}}P_{\mathbf{R}}\right)e^{-\delta(x-\mathbf{R})}dx \tag{27l}$$

Empirical Framework

The empirical framework of advanced actuarial annuity factor under trusteeship model was applied to investigate the actuarial liability under study.

The Trustee's Valuation Model

Following the nomenclature in Ogungbenle and Omede (2022), we define the functions below

Model 1

$$L_{Trust} = \left(\frac{k}{\theta}\right)\times\left(SAL\times(1+SALG)_x\right)\times\left(\frac{1}{(1+d)^x}\right)\times\left(\frac{1-\left(\frac{1}{1+A}\right)^N}{A}\right) \tag{28}$$

$$L_{Trust} = f(\lambda, AF) \tag{29}$$

where $\lambda = \ddot{a}_x$ and $\ddot{a}_x = a_x + 1$ (30)

$$f(\lambda, AF) = \begin{cases} g(\lambda) \neq 0 \; for \lambda = a_x \\ g(\alpha) \to 0 \; for \alpha = AF \end{cases} \tag{31}$$

Model 2

$$AR = \frac{d - p_g}{1 + p_g} \tag{32}$$

Adding 1 to both sides in (32)

$$1 + AR = 1 + \frac{d - p_g}{1 + p_g} \tag{33}$$

$$1 + AR = \frac{1 + p_g + d - p_g}{1 + p_g} \tag{34}$$

$$1 + AR = \frac{1 + d}{1 + p_g} \tag{35}$$

$$(1 + AR)(1 + p_g) = 1 + d \tag{36}$$

$$d = (1 + AR)(1 + p_g) - 1 \tag{37}$$

$$\frac{1}{1 + AR} = \left(\frac{1 + p_g}{1 + d}\right) \tag{38}$$

Define $AF = \left[1 - \left(\frac{1 + p_g}{1 + d}\right)\right]^{\frac{N}{R}}$ (39)

Model 3

$$P_{Trust} = L_{Trust} \times \frac{S_c}{k} \tag{40}$$

Study Area, Study Population and Sampling

The research region is the Jos-South Local government area of Plateau state, Nigeria. However, Jos South is the region where viable industries are located. The salary data of 39 employees from an agricultural production service firm in Jos-South Local Government area of Plateau state, Nigeria was collected from the human resources department.

Research Instrument and Data Analysis Techniques

Method of Data Presentation and Analysis

The trusteeship model stated above is the main actuarial tool used. The salary data collected was cleaned for ease of computations. Along the salary data, the demographic data of each employee was also collected and fed into *R* language software to enable us to carry out full computation of the actuarial liability. The data was analysed using the trustee valuation model and findings were depicted in graphs and tables for easy understanding of numerical values.

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Table 1

Table of Liabilities (L_{TRUST})

PNC	*A*	*B*	*C*	*D*	*E*
001	0.6000	5763673.20	0.2340	0.0309	25041.78
002	0.6333	3952684.60	0.3797	0.0331	31488.02
003	0.6667	2577590.84	0.3327	0.0324	18549.97
004	0.7333	2941101.05	0.2790	0.0316	19039.15
005	0.6667	2565783.57	0.3327	0.0324	18465.00
006	0.6833	2632010.76	0.3184	0.0322	18457.26
007	0.8333	5087603.58	0.2050	0.0305	26516.59
008	0.8333	4694979.60	0.2050	0.0305	24470.24
009	0.7833	4168250.03	0.2340	0.0309	23643.67
010	0.6000	3073314.72	0.3797	0.0331	23194.19
011	0.5333	2501652.10	0.4528	0.0341	20624.43
012	0.6667	2896347.15	0.3184	0.0322	19815.56
013	0.7333	1406082.10	0.1878	0.0302	5854.49
014	0.7000	1550205.51	0.1719	0.0300	5597.20
015	0.5833	906373.06	0.2790	0.0316	4667.24
016	0.7000	1275357.91	0.2050	0.0305	5583.47
017	0.5500	1473970.79	0.2670	0.0315	6808.58
018	0.6500	1961983.51	0.1962	0.0304	7598.87
019	0.6667	2060082.68	0.1878	0.0302	7797.77
020	0.6500	1549897.03	0.3634	0.0329	12039.97
021	0.7000	2111996.61	0.1719	0.0300	7625.61
022	0.6667	1915643.19	0.1878	0.0302	7251.04
023	0.7000	2760666.94	0.3047	0.0320	18857.12
024	0.6833	1214626.58	0.2143	0.0306	5449.90
025	0.7333	1406082.10	0.1878	0.0302	5854.49
026	0.6167	973025.11	0.2050	0.0305	3752.84
027	0.5000	691510.78	0.2790	0.0316	3052.14
028	0.5000	691510.78	0.2790	0.0316	3052.14
029	0.6167	973025.11	0.2050	0.0305	3752.84
030	0.6833	1072760.18	0.1878	0.0302	4162.10
031	0.6500	973025.11	0.2050	0.0305	3955.70
032	0.5833	1101702.12	0.2340	0.0309	4653.67
033	0.6000	1156787.22	0.2239	0.0308	4785.50
034	0.6833	1126398.19	0.1797	0.0301	4165.04
035	0.7167	1241854.01	0.1645	0.0299	4376.64
036	0.6500	1021676.37	0.1962	0.0304	3957.01
037	0.6833	1126398.19	0.1797	0.0301	4165.04
038	0.4333	3460390.82	0.3327	0.0324	16187.05
039	0.6333	4129131.50	0.2239	0.0308	18030.75
Total	**25.5000**	**84187154.72**	**9.6913**	**1.2153**	**452340.08**

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In Table 1 above we have computed the liabilities of the scheme for each member. The total service liability of the scheme was computed using the trustee valuation model. In column E the sum of the total liability of the plan is 452, 340.08 based on the parameters we have set below

where:

$PCN = permanent\ code\ number$

$$A = \frac{k}{\theta} \tag{41}$$

$$B = SAL \times \left(1 + SALG\right)^{x} \tag{42}$$

$$C = \frac{1}{\left(1+d\right)^{x}} \tag{42a}$$

$$D = \frac{\left(1 - \left(\frac{1}{1+A}\right)^{N}\right)}{A} \tag{43}$$

$$E = L_{Trust} = \left(\frac{k}{\theta}\right) \times \left(SAL \times \left(1+SAL\right)_{x}\right) \times \left(\frac{1}{\left(1+d\right)^{x}}\right) \times \left(\frac{\left(1 - \left(\frac{1}{1+A}\right)^{N}\right)}{A}\right) \tag{44}$$

In Table 1, column A shows the pension accrual of each member of the sheme. This is the ratio of the number of pensionable years to the retirement age 65 . The total pensionable accrual is 25.50 Colunm B shows the projection of the annual salary figure of each member as compared to the current salary growth rate of 5% in Nigeria. This also depend on the number of years to retirement of each of the scheme members and total value sum up to 84,187,154.72 . Column C shows the discount rate of each member of the scheme and the total value is 9.6913 . Column D shows the compution of the annuity factor based on their respective age and on the number of expected years to live after retirement which is 10 years the total value is 1.2153 . Column E shows the service liability of each member of the sheme. The computation was based on the trustee valuation method and the total service laibility is 452, 340.08

Table 2

Table of Service Years Completed

PCN	*F*	*G*	*x*	*I*
001	1152000.00	36	32	33
002	1351224.72	38	43	22
003	761169.72	40	40	25
004	714529.68	44	36	29
005	757683.00	40	40	25
006	740228.64	41	39	26
007	878412.48	50	29	36
008	810623.04	50	29	36
009	833118.72	47	32	33
010	1050612.24	36	43	22

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011	1039488.12	32	47	18
012	814570.80	40	39	26
013	220200.00	44	27	38
014	220200.00	42	25	40
015	220200.00	35	36	29
016	220200.00	42	29	36
017	341043.60	33	35	30
018	322620.00	39	28	37
019	322620.00	40	27	38
020	504602.00	39	42	23
021	300000.00	42	25	40
022	300000.00	40	27	38
023	739440.00	42	38	27
024	220200.00	41	30	35
025	220200.00	44	27	38
026	168000.00	37	29	36
027	168000.00	30	36	29
028	168000.00	30	36	29
029	168000.00	37	29	36
030	168000.00	41	27	38
031	168000.00	39	29	36
032	220200.00	35	32	33
033	220200.00	36	31	34
034	168000.00	41	26	39
035	168000.00	43	24	41
036	168000.00	39	28	37
037	168000.00	41	26	39
038	1021863.00	26	40	25
039	786000.00	38	31	34
Total	**18983449.76**			

Table 2 above shows the computation of service years completed, the current age and the number of future service years. The total annual salary of each member sum up to $18,983,449.76$ based on the parameters set below

where:

$PCN = permanent\ code\ number$

$F = monthly\ salary \times 12$

$G = current\ age - entry\ age$

$x = current\ age$

$I = retirement\ age - current\ age$

In table 2 , Column F shows the annual salary of each member of the scheme. The total annual salary is 18, 983, 449.76. Column G shows the number of total pensionable years. Column

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x shows the current age of each member of the scheme the minimum is 24 years while the maximum is 47 years. Column *I* shows the number of years to retirement.

Table 3

Table of past service

PCN	*J*	*K*	*L*	*M*	*N*
001	25041.78	3	36	0.083333	2086.815
002	31488.02	16	38	0.421053	13258.11
003	18549.97	15	40	0.375	6956.239
004	19039.15	15	44	0.340909	6490.618
005	18465.00	15	40	0.375	6924.375
006	18457.26	15	41	0.365854	6752.656
007	26516.59	14	50	0.28	7424.646
008	24470.24	14	50	0.28	6851.666
009	23643.67	14	47	0.297872	7042.796
010	23194.19	14	36	0.388889	9019.961
011	20624.43	14	32	0.4375	9023.19
012	19815.56	14	40	0.35	6935.445
013	5854.49	6	44	0.136364	798.3393
014	5597.20	2	42	0.047619	266.5333
015	4667.24	6	35	0.171429	800.0978
016	5583.47	6	42	0.142857	797.6391
017	6808.58	3	33	0.090909	618.9622
018	7598.87	2	39	0.051282	389.6856
019	7797.77	2	40	0.05	389.8883
020	12039.97	16	39	0.410256	4939.476
021	7625.61	2	42	0.047619	363.1245
022	7251.04	2	40	0.05	362.5519
023	18857.12	15	42	0.357143	6734.687
024	5449.90	6	41	0.146341	797.5464
025	5854.49	6	44	0.136364	798.3393
026	3752.84	1	37	0.027027	101.4281
027	3052.14	1	30	0.033333	101.7381
028	3052.14	1	30	0.033333	101.7381
029	3752.84	1	37	0.027027	101.4281
030	4162.10	3	41	0.073171	304.5436
031	3955.70	3	39	0.076923	304.2844
032	4653.67	2	35	0.057143	265.924
033	4785.50	2	36	0.055556	265.8612
034	4165.04	2	41	0.04878	203.1729
035	4376.64	2	43	0.046512	203.5648
036	3957.01	2	39	0.051282	202.9235
037	4165.04	2	41	0.04878	203.1729
038	16187.05	1	26	0.038462	622.5789
039	18030.75	4	38	0.105263	1897.974
Total	**452340.08**				

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where $PCN = permanent\ code\ \ number$
$J\ =\ the\ \ pensionable\ \ years$
$K = current\ age - entry\ age$

Table 3 is the table of past services. Column *J* shows the service liability. Column *K* shows the present number of past service years the minimum is 1 while the maximum is16 Column *L* shows the number of total pensionable years of each member. Column *M* shows the ratio of past service years to total number of pensionable years. Column *N* shows the past service liability for each member of the scheme.

Table 4

Table of Ordinary Annuity

PCN	*P*	*Q*	*R*
001	0.01	0.0347	0.0000
002	0.011	0.0336	0.0000
003	0.012	0.0326	0.0000
004	0.013	0.0316	0.0000
005	0.014	0.0306	0.0000
006	0.015	0.0296	0.0000
007	0.016	0.0285	0.0000
008	0.017	0.0275	0.0000
009	0.018	0.0265	0.0000
010	0.019	0.0255	0.0000
011	0.02	0.0245	0.0000
012	0.021	0.0235	0.0000
013	0.022	0.0225	0.0000
014	0.023	0.0215	0.0000
015	0.024	0.0205	0.0000
016	0.025	0.0195	0.0000
017	0.026	0.0185	0.0000
018	0.027	0.0175	0.0000
019	0.028	0.0165	0.0000
020	0.029	0.0155	0.0000
021	0.03	0.0146	0.0000
022	0.031	0.0136	0.0000
023	0.032	0.0126	0.0000
024	0.033	0.0116	0.0000
025	0.034	0.0106	0.0000
026	0.035	0.0097	0.0000
027	0.036	0.0087	0.0000
028	0.037	0.0077	0.0000
029	0.038	0.0067	0.0000
030	0.039	0.0058	0.0000
031	0.04	0.0048	0.0000

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032	0.041	0.0038	0.0000
033	0.042	0.0029	0.0000
034	0.043	0.0019	0.0000
035	0.044	0.0010	0.0000
036	0.045	0.0000	0.0000
Total	**0.99**	**0.6167**	**0**

In Table 4 , the sensitivity analysis was carried out between annuity rate and the annuity factor, it is apparent that the annuity factor is zero. This was proven in the theorem, the total value if the annuity rate of the plan is 0.6167

Table 4 is a table of ordinary annuity. Column *P* shows the pension growth rate which varies with number of years of each employee in service and it ranges from 1% to 4.7% . Column *Q* shows the annuity rate which varies according to the number of years of each employee in service. Column *R* shows the annuity factor which equate to zero for each employee.

Table 5

Table of Life Annuity

PCN	X	$\ddot{a}_x$	a_x
001	32	20.686	19.686
002	43	18.100	17.1
003	40	18.894	17.894
004	36	19.894	18.849
005	40	18.894	17.894
006	39	19.144	18.144
007	29	21.238	20.238
008	29	21.238	20.238
009	32	19.686	19.686
010	43	18.100	17.1
011	47	16.941	15.941
012	39	19.144	18.144
013	27	21.570	20.571
014	25	21.877	20.877
015	36	19.894	18.849
016	29	21.239	20.239
017	35	20.069	19.069
018	28	21.408	20.408
019	27	21.571	20.571
020	42	18.327	17.372
021	25	21.877	20.877
022	27	21.571	20.571
023	38	19.386	18.386
024	30	21.061	20.061
025	27	21.571	20.571

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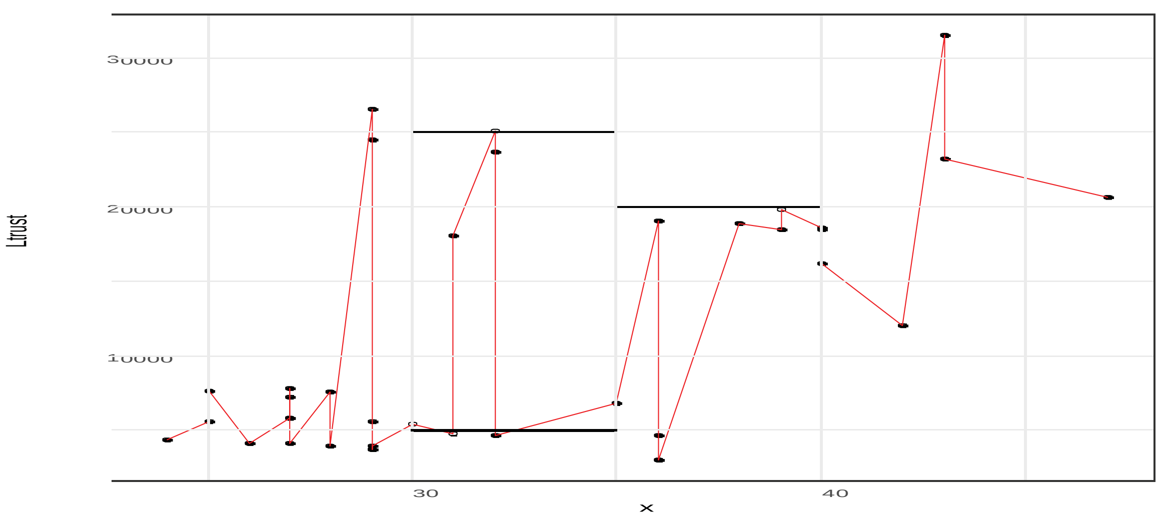
026	29	21.238	20.238
027	36	19.849	18.849
028	36	19.849	18.849
029	29	21.238	20.238
030	27	21.571	20.571
031	29	21.238	20.238
032	32	20.686	19.686
033	31	20.877	19.877
034	26	21.727	20.727
035	24	22.020	21.02
036	28	21.408	20.408
037	26	21.727	20.727
038	40	18.894	17.894
039	31	20.877	19.877

Source: Neil, 1979.

Table 5 above shows the ages of the employees and the corresponding values of immediate annuities and annuity due. Column X is the current age of each member of the scheme, colun $\ddot{a}_x$ shows annuity due and colunm a_x shows annuity immidiate for each member of the scheme. It is therefore amazing to note in column E of table 1 that when the ordinary annuity in McNally & O'Connor (2018) is changed to life annuities, the liability value is not zero.

Figure 1

Graph of L_{TRUST} *against x*



The Figure1 shows the graph of total service liability (L_{Trust}) against the respective age x for each member of the scheme. It displays wavy trajectories of service liability against age. The trajectories describes the value of the benefits of retirement at the ages of each of the 39 members of the scheme using the trustee valuation method. The graph further shows pension increase from the lowest to the highest age that is 24 years to 47 years. Futheremore, based

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on the applicable government regulations, the pension benefits chart is more likely to increase due to increase in pension benefits at every age attained by a member of the scheme.

Figure 2

Graph of L_{TRUST} *against K*

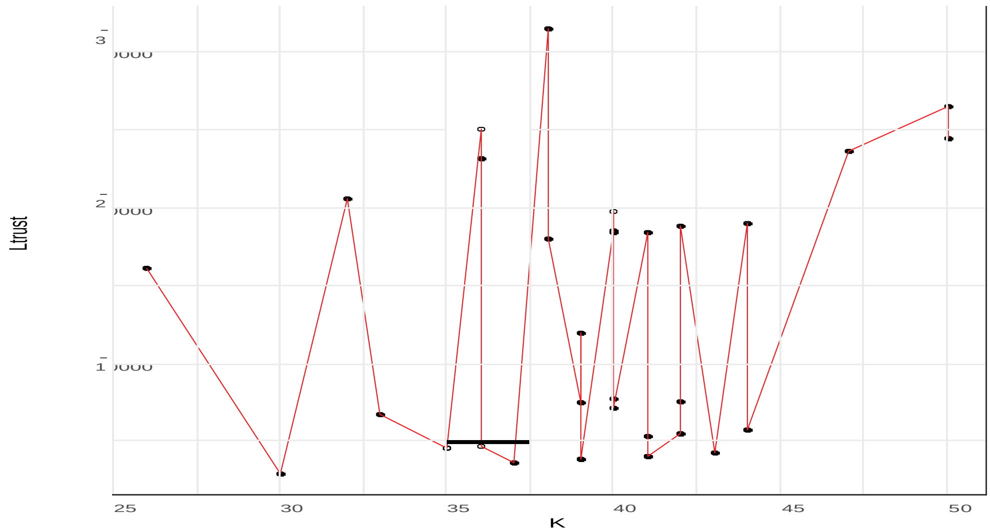

Figure 2 above shows the graph of total service liability (L_{Trust}) against number of pensionable years (k) for each member of the scheme. This shows an increase in service liability on the average moving chart that depicts the average value of benefits of the scheme members with respect to their various projected number of pensionable years.

Figure 3

Graph of L_{TRUST} *against Salaries*

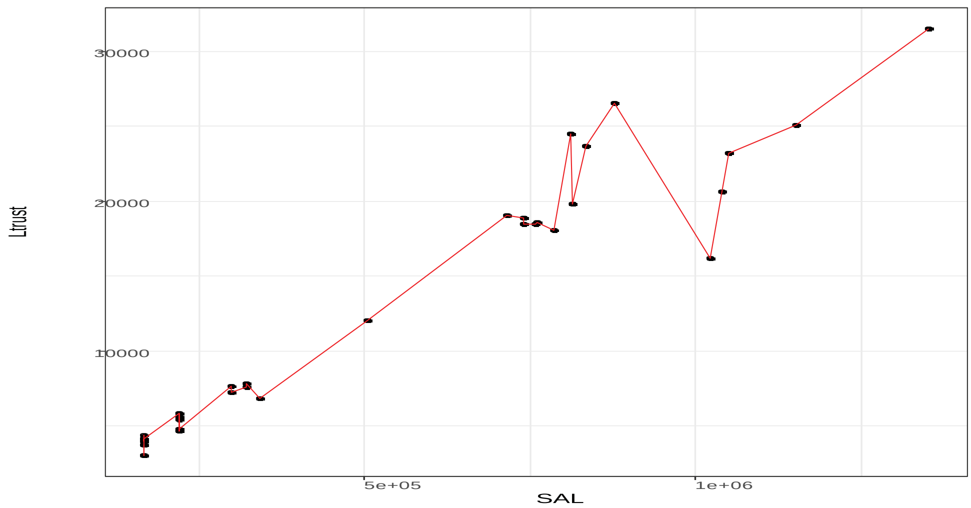

Figure 3 is a graph of the total service liabilities (L_{Trust}) against annual salary of each employee (SAL). The chart shows sudden humps at different point which is due to salary increase whenever the plan members are due for promotion to differing higher levels of service which

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attract marked increase in the scheme members salaries. This has a signicant effect on the value of their service liabilities.

Figure 4

Graph of L_{TRUST} *against x*

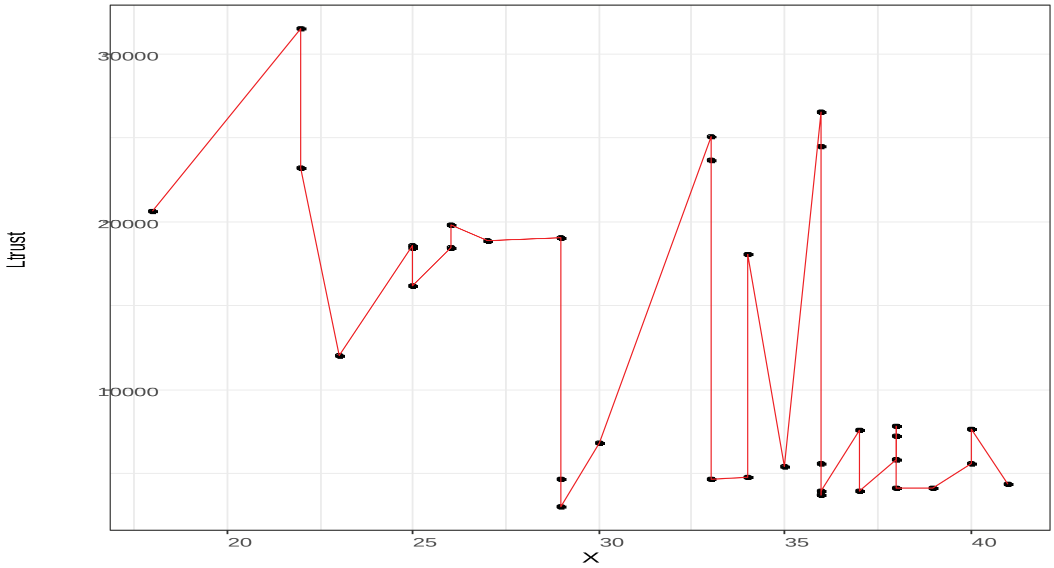

Figure 4 is a graph of total service libilities (L_{Trust}) against age x. The graph shows the estimated pension benefit expected to be paid by the scheme from the present to retirement as estimated using the trustee model.

Figure 5

Graph of L_{TRUST} *against* P_{TRUST}

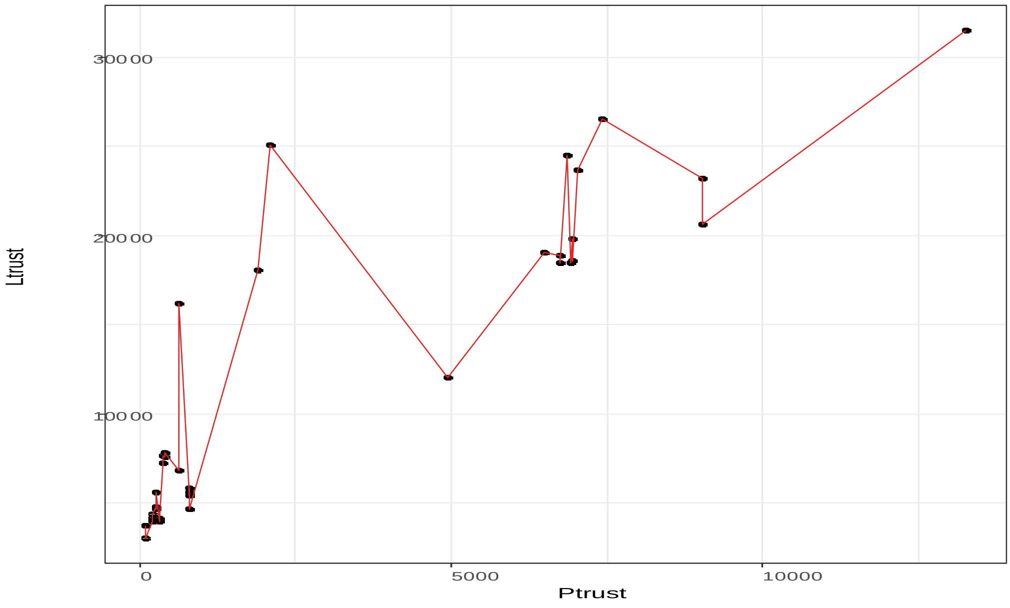

Figure 5 is the graph of total service laibilities (L_{Trust})against the respective past service

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liability of the scheme members against (P_{Trust}). The points of the liability are clustered from the origin and widens further as the trajectory progresses.

Figure 6

Graph of L_{TRUST} *against* a_x

s

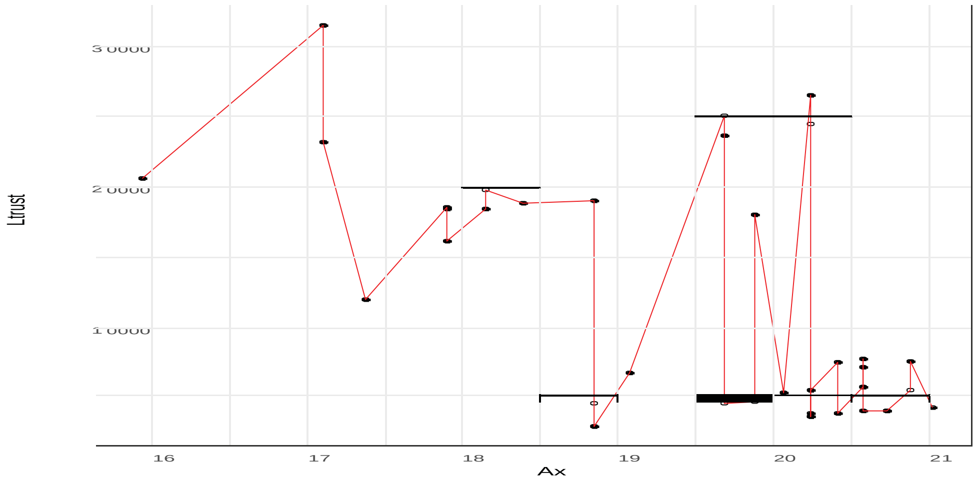

Figure 6 is a graph of service liability against the life annuity rate of each member of the scheme. The points are higher at some points above the origin and lower as the graph progress. This implies that they are projections of higher payment in the future and consequently retirement plan funding anticipates that over a long term, the contribution rate, administrative expenses and investment earnings less investment fees will be needed to cover benefit payments.

Results and Discussion

Descriptive mathematical Analysis

From the objective of the study, the column R in table 4 flows directly from the results stated below with the assumptions that:

(i) $$AR = \frac{d - p_g}{1 + p_g} \quad (45)$$

(ii) $$AF = \left[1 - \left(\frac{1 + p_g}{1 + d}\right)\right]^{\frac{N}{AR}} \quad (46)$$

where N is the expectation of life at retirement

and $L_{Trust} \rightarrow 0$

It is sufficient to prove that:

$$AF = \left[1 - \left(\frac{1 + p_g}{1 + d}\right)\right]^{\frac{N}{AR}} \rightarrow 0 \quad (47)$$

under the condition that

(i) $AF = \left[1-\left(\frac{1+p_g}{1+d}\right)\right]^{\frac{N}{AR}}$ (48)

and **(ii)** observe that $AR = \left(\frac{(1+d)}{1+p_g}\right)-1$ (49)

$$AR = \frac{1+d-\left(1+p_g\right)}{1+p_g} \quad (50)$$

$$AR = \frac{1+d-1-p_g}{1+p_g} \quad (51)$$

$$AR = \frac{d-p_g}{1+p_g} \quad (52)$$

where N is expectation of life at retirement

Computationally $\frac{1+p_g}{1+d} < 1$ (53)

Again since $N \in \mathbf{N}^+$, $AR \in \mathbf{R}^+$ and specifically $AR < 1$, then $\frac{N}{AR} \to \infty$

Apparently $1-\left(\frac{1+p_g}{1+d}\right) < 1$ (54)

Observe that if $|a| < 1$ (55)

then $\lim_{n\to\infty} a^n = 0$ (56)

Consequently, $\left[1-\left(\frac{1+p_g}{1+d}\right)\right]^{\frac{N}{AR}} \to 0$ and $AF \to 0$ (57)

Let $\frac{1}{\theta} = \sigma$ and observe that if $0 < \theta < 1$, then σ is relatively bigger than θ, that is $\sigma > \theta$

Consequently, $(\theta)^{-\frac{N}{AR}} = \frac{1}{(\theta)^{\frac{N}{AR}}} = \left[\frac{1}{\theta}\right]^{\frac{N}{AR}}$ (58)

$$(\theta)^{-\frac{N}{AR}} = \frac{1}{(\theta)^{\frac{N}{AR}}} = \frac{1}{\theta^{\infty}} = \frac{1}{\infty} = 0 \quad (59)$$

By the arguments in (58) and (59), we observe that as

$\frac{N}{AR} \to -\infty$

Since $\left(1-\frac{1+P_g}{1+d}\right) < 1$ then we observe that $\frac{1}{\left(1-\frac{1+P_g}{1+d}\right)}$ will be relatively bigger than

$\left(1-\frac{1+P_g}{1+d}\right)$ and consequently,

$$AF=\left[\left[1-\left(\frac{1+p_g}{1+d}\right)\right]^{\frac{N}{AR}}\right]_{\frac{N}{AR}=-\infty}=\left[1-\left(\frac{1+p_g}{1+d}\right)\right]^{-\infty}=\frac{1}{\left[1-\left(\frac{1+p_g}{1+d}\right)\right]^{\infty}}=\frac{1}{\infty}=0 \qquad (60)$$

Following the definitions in McNally & O'Connor (2018), the trustee valuation model is given as follows

$$L_{Trust}=\left(\frac{k}{\theta}\right)\times\left(SAL\times(1+SAL)\right)^{x}\times\left(\frac{1}{(1+d)^{x}}\right)\times\left(\frac{\left(1-\left(\frac{1}{1+A}\right)\right)^{N}}{A}\right) \qquad (61)$$

Consequently, as AF is sufficiently small. That is $AF \to 0$ then $L_{Trust}=0$ (62)

From the results obtained, the pension liability is zero and this may be surprisingly strange to the balance sheet consisting of assets and liabilities. Consequently, the detection of this phenomenon in Mc Nally and O' Connor (2018) model presents a phantom of zero liability, and this is our *main contribution* in this work.

Conclusion and Recommendations

From the results obtained, the pension liability is zero and this may be surprisingly strange to the balance sheet consisting of assets and liabilities. However, when the life table annuity replaces the annuity factor, the service liability is not vanishingly zero. Hence, this phantom effect resulting in total liability in being zero arouses an inquiry as to whether this current valuation scheme would lead to potential actuarial risk process for the pension trustees who are solely loaded with the core pension decision making responsibilities. The estimated parameters of the trustee model such as the service liability, past service liability, number of pensionable years, salary growth rate, ordinary annuity and life annuity of model was analysed. Of particular interest is the actuarial configurations in McNally and O'Connor (2018) as extended by Ogungbenle and Omede (2022) where the liability vanishes and this zero-liability detected represents the phantom in their valuation method. The annuity factor is calculated based on number of years an employee is expected to live post-retirement. If an employee retires, the number of years that he is expected to live post-retirement is projected. This is determined by mortality table which is actuarially calculated and compiled based on mortality experience and taking into account both the discount rate and expected pension increases but it may be adjusted to reflect assumptions on expected mortality experience. Retirement plan liabilities changes because of the resultant impact of income and expenses components. The modifications on pension accounting regulations which enforces employers to state the variation regularly and formally in valuation results between their defined benefit pension assets and liabilities on their balance sheet have advised investment managers of the pension uncertainties and potential risk

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dynamics which the employers sponsoring such schemes are exposed to. Consequently, based on the results, the funding strategy is recommended and advised to be actuarially modified and audited at periodic intervals to confirm if the funding strategy has actuarially complied with the scheme's funding objectives and hence the system should pay sufficient attention to the existing fund's investment strategies entrenched by the fund trustees with a goal to generating yield for the fund. As future research direction, the study can be taken from the stochastic perspectives.

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