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A Comparative Analysis of Newton-Raphson and Secant Methods Based on Convergence and Computational Efficiency for Solving Nonlinear Equations

Ramesh Khanal
Faculty of Mathematics
Balkumari College, TU, Chitwan, Nepal
Email: ramesh.khanal@bkc.edu.np

ABSTRACT

This study conducts a comparative evaluation of the Newton-Raphson and Secant methods in solving the non-linear equation f(x) =. Numerical methods are essential for solving equations where analytical methods fail or become inefficient. Among these, the Newton-Raphson and Secant methods are two widely used techniques for solving non-linear equations. This research adopts a quantitative computational approach. Both algorithms were implemented in MATLAB under consistent convergence criteria and stopping conditions, utilizing varying initial approximations. Key performance indicators such as the number of iterations, absolute error, function values at each step, and computational time were recorded. The Newton-Raphson method converged in 10 iterations, requiring approximately 0.00243 seconds, whereas the Secant method achieved convergence in only 5 iterations with a reduced computational time of 0.000329 seconds. Although both methods ultimately identified the same root, the results suggest that the Secant method offers greater computational efficiency, provided that suitable initial guesses are selected.

Keywords: Newton-Raphson method, Secant method, numerical methods, non-linear equations, convergence, computational efficiency

Introduction

The efficient and accurate resolution of non-linear equations represents a fundamental objective in numerical analysis, serving as the foundation for a wide range of scientific and engineering fields (Dianne, 2009) Among various iterative approaches, the Newton-Raphson and Secant methods are particularly notable due to their distinct characteristics concerning convergence patterns and computational demands (Romanova, 2019). The Newton-Raphson method is renowned for its quadratic convergence under appropriate conditions, requiring the evaluation of both the function and its derivative during each iteration (Chen, Jiao, & Yang, 2021). However, its dependence on derivative information can present challenges, especially when derivatives are complex to compute or analytically inaccessible, leading to the use of finite difference approximations that may introduce errors and impede convergence efficiency (Chen, Jiao, & Yang, 2021). Conversely, the Secant method eliminates the need for explicit derivative calculations by approximating derivatives through a finite difference between successive iterations, making it advantageous for applications involving complicated or black-box functions. Despite its lower computational cost per iteration, the Secant method typically achieves superliner convergence with an order of approximately 1.618, which is slower than the quadratic rate achieved by Newton-Raphson (Atkinson, 1989).

Numerical techniques are crucial when analytical methods either fail or become computationally burdensome. The Newton-Raphson and Secant methods are two prominent iterative strategies for solving non-linear equations, differing mainly in their dependency on derivative information and initial conditions. The present study seeks to analyze and compare the convergence performance and computational efficiency of these methods. Specifically, the objectives include solving the non-linear equation f(x) =+3x-2 and evaluating computational efficiency based on iteration counts and execution times using MATLAB. Key research questions include: Which method proves to be computationally more efficient for the given non-linear equation? Do both methods achieve convergence to the same root with comparable accuracy? This comparative analysis is intended to provide valuable insights for students, researchers, and professionals, facilitating an informed choice between the methods depending on the computational resources available, the desired accuracy, and the accessibility of derivative information.

Initially developed in the 17th century, the Newton-Raphson method is distinguished by its rapid quadratic convergence but requires derivative computation at each step (Burden & Faires, 2011). Alternatively, the Secant method, introduced as a derivative-free approach, achieves superliner convergence and is preferable when derivatives are difficult or impossible to compute (Quarteroni, Sacco, & Saleri, 2010). While theoretical analyses have often highlighted the superiority of Newton-Raphson under ideal circumstances, practical scenarios often favor the Secant method, particularly when derivative information is unreliable or unavailable. Despite numerous theoretical comparisons, there remains a scarcity of empirical studies that systematically examine the performance of these methods using contemporary computational tools such as MATLAB or Python. Moreover, limited research has addressed the behavior of these methods across different categories of functions, including trigonometric, exponential, and high-degree polynomial equations, especially under varying initial conditions.

Methodology

This research adopts a quantitative computational approach (Burden & Faires, 2011). The methodology includes: A diverse set of non-linear equations have been selected, including: exponential function f(x) = +3x-2. The functions exhibit different characteristics such as smoothness, oscillation, and exponential decay. Both Newton-Raphson and Secant methods have coded in MatLab using custom scripts based functions. The algorithms included iteration limits and error thresholds to manage divergence or slow convergence. Each method will be evaluated using: number of iterations to convergence, absolute error and execution time.

Data analysis

The Newton-Raphson and Secant methods was conducted to determine the root of the nonlinear equation f(x) =2^x+3x-2. The Newton-Raphson method used an initial guess of x=1 whereas the Secant method began with x=1 and x=2.

Table 1: The Iteration table of Secant Method

The Iteration table of Secant Method: f(x)= +3x-2, initial guesses: 1 and 2					
Iter	x	f(x)	Error	Time(s)	Com. Time(s)
1	0.4	0.519508		0.000146	0.000146
2	0.288883	0.088342		0.000152	0.000298
3	0.266116	0.000913		0.000016	0.000314
4	0.265878	0.000002		0.000007	0.000321
5	0.265878	0		0.000008	0.000329

Table 2: The Iteration Table of Newton's Raphson Method

The Iteration Table of Newton-Raphson Method: f(x)=2^x+3x-2, initial guess=1					
Iter	**x**	**f(x)**	**Error**	**Time (s)**	**Com. Time(s)**
1	0.480302	0.835944		0.002096	0.002096
2	0.310875	0.173085		0.000087	0.002183
3	0.274202	0.03193		0.000037	0.00222
4	0.267374	0.005737		0.000031	0.002251
5	0.266145	0.001026		0.000034	0.002285
6	0.265925	0.000183		0.000029	0.002314
7	0.265886	0.000033		0.000028	0.002342
8	0.265879	0.000006		0.000028	0.00237
9	0.265878	0.000001		0.000028	0.002398
10	0.265878	0		0.000032	0.00243

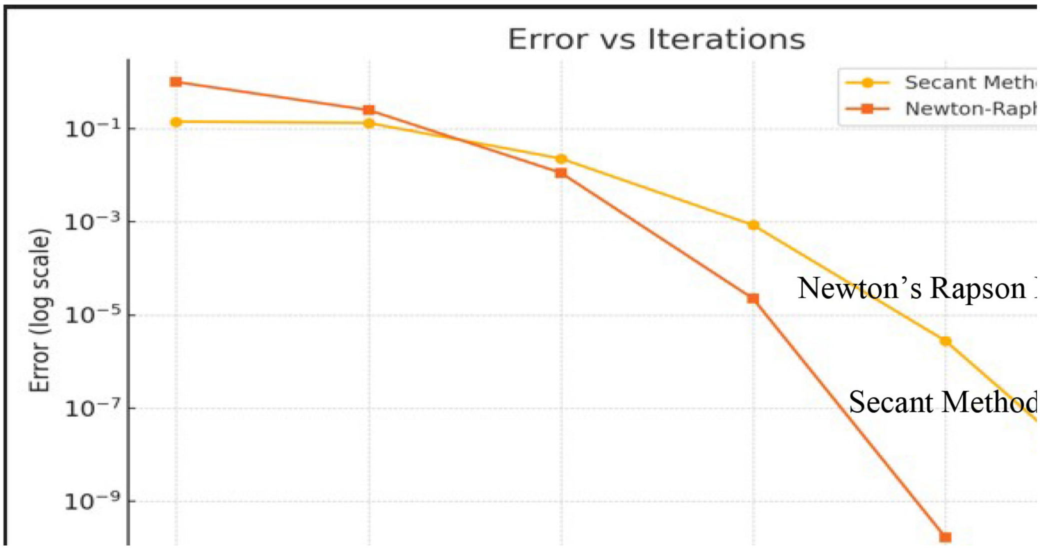

Figure 1: The Error between Newton's Raphson and Secant Methods

But differed in convergence speed and computational efficiency. The Newton-Raphson method, initiated with a guess of x=1 converged in 10 iterations with a final error of .The total computational time was approximately 0.00243 seconds. As visualized in the graph, the error decreased quadratic ally , consistent with the theoretical performance of the method when applied to differentiable functions like f(x) whose derivative f′(x)= ln(2)+3 remains positive and smooth. In contrast, the Secant method, initialized with x=1 and x=2, required only 5 iterations to reach the same root with a slightly larger final error of . Notably, the computational time was just 0.000329 seconds, indicating much higher time efficiency due to its derivative-free nature. The error reduction, though not quadratic, still followed a consistent descent, as expected from the method's order of convergence (~1.618) (Burden & Faires, 2011).

Discussion and Conclusion

Both the Newton-Raphson and Secant methods successfully approximated the root of the nonlinear equation. The Secant method proved to be more efficient in terms of computational time, despite its lower order of convergence. Thus, the choice between methods should consider the trade-off between speed and derivative availability. The findings reinforce conclusions drawn by (Burden & Faires, 2011) and (Chen, Jiao, & Yang, 2021), who emphasized the Newton-Raphson method's superior accuracy when derivatives are known, and the Secant method's suitability when function evaluations are cheap but derivatives are expensive or unavailable

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