Exploring Causes of Difficulties in Learning Linear Algebra

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Abstract: *This paper aims to explore causes of difficulties while learning linear algebra course at master level and find possible ways to mitigate these difficulties. I applied case study methods and selected two cases responded purposively. I used semi-structured interview guidelines and performance test to explore the causes of difficulties and used general inductive approach to analyze the data. The result shows that the students at master level are experiencing linear algebra course difficult for conceptual and procedural understanding because of having poor English language, unavailability of appropriate learning resources, having poor perquisites, abstract nature of concepts, unable to construct examples and counter examples, and the nature of the curriculum. These difficulties can mitigate by developing strong prerequisites before introducing new concepts, using examples of abstract concepts, choosing varieties of techniques of proofs for theorems, connecting abstraction with real-life examples, and other software and videos of the concepts in linear algebra while teaching and learning at master level.*

Keywords: *Prerequisites, abstract nature, examples and counter examples, nature of curriculum*

Introduction

Researching in learning linear algebra is an ongoing phenomenon. Linear algebra is the study of vectors and linear functions (Cherney et al., 2013). These concepts are more abstract in comparison to the mathematical concepts included in the school algebra, for example the matrix and linear equations. Understanding these very abstract concepts are the demanding tasks for both students and the teachers. At graduate and master level, the

students are not only memorizing facts, and solve numerical problems, but they need prove theoretical problems, named as theorems while learning linear algebra.

There are varieties of proofs techniques applied in higher algebra, particularly in linear algebra including formal deduction, induction, and axiomatic methods and indirect techniques such as methods of contradiction and contrapositive (Lalonde, 2013). But it is experienced that students have felt difficulty while choosing appropriate methods of proof techniques in linear algebra in addition to conceptual and procedural understanding the concepts. Also, while checking the exam papers of students at master level, it is seen that students only copied almost all the examples and proofs techniques as stated in the text books rather than their own ideas. These experiences motivated me to explore the causes of difficulties as experienced by master students in learning linear algebra course. These difficulties might be caused by various reasons. In this context, this paper intends to explore the causes of difficulties while learning abstraction in linear algebra course by master level students.

Objectives

The objectives of this study were to explore the causes that master degree students had been felt in learning linear algebra course and to reveal possible ways to mitigate difficulties in learning it.

Literature Review

Yadav (2022) explained that the language, learning environment, and culture at home and school; beliefs and support of teachers; parental involvement, and teaching methods are not supportive to learn algebraic concepts for students at school level. Because of it, students can have difficulty in learning algebraic concepts at school level. Moorthi (2018) explored that much of problems that students have in understanding linear equations at secondary school are due to lack of necessary background knowledge and students encounter great difficulties when they have to translate word problems into the symbolic mathematical form.

The student's inability to translate and solve algebraic word problems is their inability to break the questions into bits, interpret and represent words by variables (Adu, 2013,

p.27). Jupri et al. (2014) pointed that students have difficulties in algebra particularly in "translating word problem into mathematical form, understanding algebraic expressions, applying arithmetic operations in numerical and algebraic expressions, understanding the different meanings of the equal signs, and in understanding variables". Abstract and highly theoretical nature of concepts; and students' inexperience with proofs, logic and sets are the two major causes of difficulties in learning linear algebra concepts (Britton & Henderson, 2009, p.964).

The research of Isik et al. (2014) revealed the difficulties of students stemming from course structures, students, and course teaching. The reasons of difficulties stemming from course structures are the abstractness and complexity, rare relations, little knowledge about the usage area, and broadness of the topics; negative belief on transferring the learned knowledge to the future; extensive symbol usage and complexity in the symbols; high number of definition and theorems in linear algebra course. Likewise, the reasons of difficulties stemming from students are lack of priori knowledge, not internalizing of the topics, prejudice about the course, thoughts of only passing the course, not studying the topics, not asking questions and fear of examinations in learning linear algebra course. Finally, the reasons about course teaching are due to educational system, lack of additional resources and insufficient course hours.

Likewise, Ferryansyah et al. (2018) explain that students have difficulties in understanding and using symbols and notation, in using ideas of mathematics and logical reasoning, and in checking the correct applied or not for the language of linear algebra. These difficulties are because students' understanding of the material and solving linear algebra problem are quite slow in comparison to other materials, and because of less uses of media and selection of inappropriate teaching methods.

Also, the difficulties of students in linear algebra are concerning to understand definitions, procedural approach rather than a conceptual one (Stewart & Thomas, 2003).

Methodology

This research applied the constructivist research paradigm which believes that the reality is multiple, local and contextual; epistemology is subjective; and methodology is qualitative and naturalistic (Guba & Lincoln, 2005; Fard, 2012). Together with these

assumptions, I adopted the case study research design (Gillham, 2000) to conduct this study. For, I selected two students from the department of mathematics education in the central department of education, Tribhuvan University, Kirtipur, who had felt linear algebra course more difficult at master level. These two students were the case selected by using purposive sampling techniques. I used semi-structure interview guidelines to take in-depth interviews with the students and performance test to explore the cause of difficulties that master degree students had faced while learning linear algebra course. I analyzed the data and interpreted the result by using general inductive approach of Thomas (2006).

Results and Discussion

The data obtained for this study were interpreted and discussed under the separate headings as below.

Language and Resources

The educators believe that mathematical language and symbols used in mathematics are universal for most countries in the world (Xiaojuan & Kristie, 2024). They also believe that prior mathematical background is sufficient for understanding math concepts in second language. However, not proficiency in understanding English language can generate difficulties in learning mathematical concepts even in learning higher mathematical concepts. For, one selected student had expressed as:

"*I am poor in English language. All the text books of mathematics are written in English. I have difficulties in understanding the language of the theorem, how can I prove it? So, I gave emphasis on rote memorization for the definition, theorems and proof which makes me too difficult in understanding.*"

This response indicated that student having poor English language had been facing difficulties in conceptual and procedural understanding in mathematics. Likewise, the availability of learning resources in learning mathematics including linear algebra at higher level also in English language. Because of poor understanding of English language, students cannot understand what is given and what is to be proved in the statements or theorems of linear algebra. Understanding the definitions for example, the

vector space, subspaces, linear maps and others in linear algebra at master level require good skills of English language, but students had experienced that they had poor proficiency of English language which encourage master students to drill the mathematical concepts for just memorization in order to conceptual understanding. Consulting appropriate learning resources is always important for the effective implementation of the mathematics curriculum. The availability and focusing of the appropriate learning resources by the college and university help students to select authentic text books of mathematics for the course at master level. For, the selected case respondent had experienced as:

"*I completed my bachelor degree from out of Kathmandu valley. I have very difficult to get text books, syllabus, and curriculum of mathematics. We are just depending upon teachers note. No subject experts for teaching algebra in our campus at bachelor level. One teacher taught almost all content papers in each year in mathematics. I just collected some important theorems told by my teacher and read it to prepare for the exam. I have no more basic concepts of algebra required here*."

These responses reflect that there is lack of recommended course and text books of mathematics for the students who are studying mathematics at bachelor level from out of Kathmandu valley.

Likewise, they are unable to get syllabus and curriculum timely. These students are only depending upon teachers note which make difficult in conceptual understanding of the fundamental concepts for linear algebra at master level. The lack of subject teachers in teaching mathematics at bachelor level is another cause to produce difficulty for students in understanding the linear algebra concepts.

Poor Prerequisites

There is crucial role of prerequisites while learning mathematics concepts because it provides foundation and knowledge necessary for learning more advanced concepts. Students having good prerequisites are better equipped to conceptualize new ideas, solve problems and prove theorems in mathematics including in linear algebra at master level. Ningsih & Retnowati (2020) have pointed that the link of existing knowledge help to build new knowledge and the maximal utilization of priori knowledge make meaningful

learning (p.61) in mathematics. This implies that prerequisites or priori knowledge have important role to develop conceptual understanding of new concepts. The selected students had felt that:

"*We have poor prerequisites for linear algebra course at master level because we just drilled the teachers note only for memorization and to pass the exam at bachelor level. We do not understand vector spaces, subspaces, linear maps, kernel and image of linear maps and bilinear maps. So, it is difficult to understand bilinear forms, its properties and other concepts of linear algebra at master level*".

These experiences of selected students disclose that they have poor prerequisites in linear algebra concepts. So, students had experienced difficulty while learning new knowledge in linear algebra. For the conceptual clarity, it is recommended that the course teacher should develop prerequisites before entering the new advanced concepts in linear algebra.

Abstract Nature of the Concepts

Abstraction is the beauty of mathematics. The mathematical concepts become more abstract while generalizing the. For example, vector space is the generalize concepts of additive abelian group over the field K. That is if the additive abelian group satisfy the certain properties of scalar multiplication with the element of abelian group, then this algebraic structure becomes a vector space. For, the selected students replied as: "*The concepts included in the course of linear algebra are very abstract in nature. There are rare examples for the application of those concepts which makes us difficult for understanding*".

These views indicate that the abstraction of the concepts in the linear algebra for example, the vector space, modules, linear transformation, scalar product etc. have made difficulty to understand their basic features in linear algebra for students. Similarly, students did not find real life example while learning and discussing the concepts in linear algebra which have made dry and boring while learning the concepts in it. Likewise, the students replied the difficulties in the performance tests of regular classes of linear algebra at master level as:

"*The objective questions need in-depth conceptual understandings, I have poor conceptual knowledge in linear algebra, so I did mistake more objective types questions.*"

"Because of poor fundamental, I did not able to prove some statements given in the test. I just tried it but incomplete. I just rote the theorem, but I forget in exam. I required more time for the preparation of the exam."

These responses also indicate that students have poor conceptual learning which demand more time for understanding such concepts. But in our semester system, we need to take exam according to fixed time. So, there is lack of time for the preparation of the performance tests.

Unable to Construct Examples and Counter Examples

It is the fact that examples are the concrete materials to conceptualize the abstraction in higher algebraic structures, groups, rings, fields and vector spaces. But it sometime difficult to construct examples and counter examples while learning the advanced algebraic concepts. One student experienced as:

"I just focused on rote memorization for learning algebraic concepts at bachelor level, but here I came to know that if I can construct examples and counter examples of the algebraic concepts then it is very easy to understand. For example, to understand the concepts of linear maps, let us define f: $R^2 \rightarrow R^2$ by f (x, y) = (y, x), then we can test the definition of linear maps here. If it holds two properties of linear maps then it is a linear map, otherwise it is not. But I can not construct appropriate examples of every concept of linear algebra that causes difficulty".

The student responses shows that students have been practiced in rote memorization for the learning of algebraic concepts. They have rare practice of constructing of examples and counter examples of the algebraic concepts. So, students have felt difficult while learning new concepts in linear algebra.

Curriculum

Curriculum describes the nature of mathematical knowledge it includes, methods of teaching and learning, textbooks for the intended course, and assignment techniques for the evaluation of the course in the entire semester. Regarding the curriculum of linear algebra, the selected students opinioned as:

"*We have difficulty in understanding structure and operations in linear algebra, our curriculum is more theoretical than practical, we are irregular in class at bachelor level, and teachers did not give conceptual knowledge, time limitation for understanding, and lengthy course*"

These opinions indicate that the courses of linear algebra are more theoretical included abstract structures and lengthy in nature. So, students did not have sufficient time for the preparation and for conceptual understanding. Likewise, irregularity at bachelor level had made more difficult in understanding conceptual knowledge in linear algebra. Thus, they have poor fundamental knowledge which have also made difficult in learning algebra at master level.

Possible ways to Mitigate Difficulties in Learning Linear Algebra

The above sections describe the causes of difficulty that master students have experienced while learning linear algebra course. However, some possible ways to mitigate difficulties in learning linear algebra were reflected from the discussion with the selected students and from the experiences of the researcher himself. Regarding to mitigate difficulty in learning linear algebra course, the selected respondents expressed their views as:

"*The teacher has to develop prerequisites before starting new concepts, maximum use examples and counter examples to conceptualize the abstract concepts included in linear algebra.*"

These responses indicate that preparing students with strong prerequisites before starting new concepts while teaching linear algebra is necessary as experienced by students. Because they had felt that lack of prerequisite of abstraction, they have faced difficulty in learning algebra concepts. Similarly, the use of examples and counter examples while teaching linear algebra course help students to conceptualize abstract concepts. The examples and counter examples can work as concrete materials to understand the abstraction for higher mathematics.

Other ways to mitigate difficulties in learning linear algebra at higher level were experienced by respondents as:

"*The teachers only use single technique to prove the theorems/statements included in the course of linear algebra, they rarely connect algebraic concepts with real life examples, cannot manage regularity of classes and subject experts for teaching linear algebra course at university level.*"

These responses reflect that the teacher of linear algebra course need to use variety of techniques to develop conceptual understanding and need to connect abstraction in linear algebra with real-life example as much as possible. Likewise, the college which are providing mathematics courses to the graduate students need to manage subject experts and manage regularity of the classes to give quality of concepts in mathematics. Moreover, to overcome the difficulties in learning linear algebra, the selected students expressed their views as:

"*Make curriculum time and student friendly as much as possible. Use multiple resources while teaching linear algebra course. Classroom teaching should be focused on conceptual understanding of the abstraction included in the linear algebra rather than rote memorization. Manage availability of curriculum and textbooks for all students in the university.*"

These views depict that the college should manage the learning resources required for learning linear algebra course without any delay. The curriculum designer should consider the level of students and required time for the implementation of the course of linear algebra at graduate level. Finally, the classroom teaching and learning should focus to develop conceptual understanding of abstraction in linear algebra at graduate level to mitigate the learning difficulties of the students as reflected from the views of selected students.

Conclusion

Teaching and learning linear algebra course at university level is a demanding task for both teachers and students. The students at this level are experiencing linear algebra course is difficult for conceptual and procedural understanding. These difficulties are due to having poor English language, unavailability of appropriate learning resources, having poor perquisites, abstract nature of concepts, unable to construct examples and counter examples, and the nature of the curriculum. These difficulties that students are

experiencing can mitigate by developing strong prerequisites before introducing new concepts, using examples of abstract concepts, choosing varieties of techniques of proofs for theorems, connecting abstraction with real-life examples, and other software and videos of the concepts in linear algebra while teaching and learning at master level.

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