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An Overview of Mathematical Beauty in Poetry

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Abstract

It is conceived that mathematics deals with the backbreaking concept, controlled and coordinated by minds, whereas poetry deals with the interesting aspects, controlled and coordinated by hearts, and are rarely correlated. However, it is not so, in general. Mathematics works to discover where poetry tries to create, and they have bilateral connections. Mathematician, like a painter or a poet, makes beautiful patterns of colors or words harmonically to integrate eternal beauty. This work attempts a pathway between these seemingly paradoxical disciplines working with different mathematical patterns, their existence and applications, structures, and their beauties related to poetry.

Keywords: Poetry, combinatorics, Sanskrit prosody, mathematical beauty.

1. Introduction

Learning and acquisition begin through musical patterns of words generated by family, community, and society. Our thinking and perceptions of objects, events, and different abstract ideas get generated in similar ways to color combinations, pictorial patterns of a painter, cadence of musician, and the rhyme of poets. Mathematical definitions, problems, algorithms, theorems, proofs, or whatever it is, it is a rhythmic or poetic perception and expression. Poetry is the master key of all arts, and its beauty is an absurd term to define.

Songs with lyrics can be a valuable tool for learning. Some lyrics illustrate mathematical concepts with or without using mathematical terminology. Contemporary poems rarely rely on pattern and rhyme, but lyrics generally do. Melody can also reinforce different content literally. When a song's melody emerges before the lyrics, the writer often sings a dummy lyric that they later revise. Rhymes were to help small children to learn to count. The first rhyme recorded for the nursery children is as follows [10].

One, two, Buckle my shoe
Three, four, Knock at the door
Five, six, Pick up sticks
Seven, eight, Lay them straight
Nine, ten, A big fat hen
Eleven, twelve, Dig and delve
Thirteen, fourteen, Maids a-courting

Fifteen, sixteen, Maids in the kitchen
Seventeen, eighteen, Maids in waiting
Nineteen, twenty, My plate's empty.

Euclid's Elements and *Aristotle's Poetic* are the two academic works of ancient Greece. They are of equal historical importance and novelty in mathematics, poetry, and human civilization almost at the same time. The Italian mathematician Mascheroni and the French mathematician Cauchy were the famous poets of their time [13]. Bertrand Russell, philosopher, logician, mathematician, the author of the world-famous book *Principia Mathematica*, has been awarded the Sylvester Medal of the London Royal Society and Dormagen Medal of the London Mathematical Society and was also awarded the Nobel Prize in literature in 1950 AD, in recognition of his varied and significant writings in champions' humanitarian ideals and freedom of thought. Albert Einstein was a famous romantic poet of his time who used to say, *Mathematics is not simply a matter of adding or subtracting, it is the poetry of logical ideas* [11].

Poetry can contribute to the learning of mathematics in different ways. The new type of cognitions, metaphors, symbols, patterns, structures, logic, and the paradox evolved from the mathematical consequences encourages the mathematician to explore and discover the poetic imagination of poetry. The scientific language on the mathematical formulations helps the sequential development with the proper texture to poetry. Both poetry and mathematics serve a transcendental purpose in the sense that both rely on ideas, images, and metaphors and serve to say the eternal beauty of what words cannot [3]. David Hilbert noticed that poetry is the expression of imagination as the moving power of mathematical inventions. As Voltaire said, *there was more imagination in the head of Archimedes than in that of Homer* [5].

Euler's identity also known as Euler's equation established the relation between the additive identity 0, multiplicative identity 1, an imaginary unit of complex number i, Euler's constant or the Euler's number, the base of natural logarithms e, and the transcendental number, the ratio of the circumference of a circle to its diameter is taken as the most beautiful equation in mathematics as mentioned in [9] is given by,

$$e^{i\pi} + 1 = 0 \tag{1}$$

It is believed that mathematics is a tough discipline related to the exercise of the mind whereas poetry is an interesting discipline with a direct connection to the heart. However, this work tries to analyze these two seemingly paradoxical disciplines separately with possible examples and illustrations to find their intersection, if they have, concerning beauty. The rest of the paper is organized as follows. Section 2 is about the inter-connection of poetry and mathematics. Combinatorics and their properties are presented in Section 3. Sanskrit prosody in Section 4. Finally, Section 5 concludes the paper.

2. Poetry and Mathematics

Our first learning to count is to rhyme where we began to make the sense of the words through patterns by making numbers run in a sequence connecting words. Here is a piece of verse from Intro to a poem, from Nathan Zach's testimony and is about the searching for the gap between what people think, what they think to want, and what they want, from [1],

This poem is a poem of people;

What they think and what they want

And what they think and what they want.

Besides this, there aren't many things in the world

That should interest us.

A mathematical idea is said to be beautiful if it reveals some surprising inner structure. Such an idea seems to appear out of nowhere and shed light on different objects. Such a composition created by a poet demands the repeated reading of the poems with disparate meanings in various circumstances. The poetic readers can enjoy the lines and context in between. They can enjoy the written poetry as well as the hidden compositions, layers, and spaces left. Mathematicians think abstractly with the logical connections on different principles, their justifications, and further applications whereas the poets think with symbols, images, and the enlightened beauty of their feelings on the heart and brain in concrete ways. Both are the product of the imagination and are searching for hidden patterns. The mathematician, like the poet, tried to find concealed mechanisms beneath external appearances. Poetry is the language of the heart and so the result of imagination and meditation. One is obligated to use certain words in the absence of any other tool to craft a poem. It can be felt, experienced, and enjoyed, but one cannot tell others how to create. Beautiful poetry, like beautiful mathematics, is always surprising with an unexpected combination of ideas with a beautiful twist.

A painter makes patterns with shapes and colors on a canvas, whereas, a poet will do it with words. Poets have been more convergent with mathematicians as they have to pay more attention to the numerical qualities of the words [7]. One can read and understand one level immediately; a story, a logic, an event, or an image; from a poem. But beyond that level, one senses deeper truths, thoughts, and the beauty enhanced by the words expressed or is left unexpressed in the poems; which will inspire the reader to think about it. And of course, in beautiful poetry the level of thinking, knowledge, beauty, and expression will increase gradually following Pascal's triangle:

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

Poetry is the master key of all the arts. Every work of art needs a poetic flavor where a beautiful mathematical formula in a scientific paper and several brilliant lines of poetry seem symmetrical. Good and beautiful mathematics, whether formula or an elegant definition, a theorem and its proof, a counterexample whether verbal or symbolic or illustration is beautiful poetry [8]. Different poets had used mathematical metaphors, symbols, logic, equations, philosophy, etc. in their poetry. Laxmi Prasad Devkota, in his autobiographical poem The Lunatic, wears the persona of a lunatic as if it were a mask. The speaker of the poem brings out a different aspect of the intellectual aspirants of the time. The abnormality, sensitivity, aggregation, contradictory paradox, imagination, rebellion, anger, and awful majesty expressed in every stanza of the poem show the highest degree of mathematical imagination, sensitiveness, anti-parallel logical consequence, beauty, and dreams [14]. For example,

You're clever, quick with words,

Your exact equation is right forever and clever.

But in my arithmetic, take one from one-

And there's still one left.

You get along with five senses,

I with a sixth.

You have a brain, friend,

I have a heart.

Example 1: Let us have a simple magic game with a piece of simple verse.

Choose a number as you like.

Triple it and add twenty there.

Double the answer and subtract twenty-five.

Take only its one-third now.

Subtract twice the number as was chosen.

Oh! Any guess dear?

Yes! It is five.

This puzzle is set in such a way that the answer is always 5. Different computational steps demanded by the verse are to confuse the person, however, a little algebra reveals that no matter what number was chosen, the steps yield an answer of 5, i.e.,

1. Let the initial number= x.
2. Triple it and add 20 $\Rightarrow 3x + 20$.
3. Double the answer and subtract25 $\Rightarrow 6x + 15$.
4. Take only one-third $\Rightarrow 2x + 5$
5. Subtract twice the number chosen$\Rightarrow 5$.

Example 2: In the twelve century AD there lived in India a famous mathematician by the name of Bhaskara or Bhaskaracharya had great contributions to mathematics and poetry. All of his mathematical works were written in verse. His most charming book, Lilavati, contains many interesting algebraic poems. Here is one poem from Lilavati, as mentioned in [6].

Lilavati's Swarm

The fifth part of a swarm of bees came to rest

on the flower of Kadamba,

a third on the flower of Silinda.

Three times the difference between these

two numbers flew over a flower of Krutaja,

and one bee alone remained in the air,

attracted by the perfume of jasmine and a bloom.

Tell me, beautiful girl, how many bees were in the swarm.

To know the number of bees, one can proceed either by trial and error, providing a few arbitrary values and get rejected the solution, or algebraically as follows:

Algorithm 1: How many bees are in Lilawati's swarm?

Input: A prose of Lilawati's swarm, as mentioned above.

1. Let x be the unknown variable, as the solution.
2. Interpret the problem in terms of x, as an equation.
3. Solve the equation for x.
4. Check the solution.
5. Interpret the solution in words. (Verse!)

Output: Feasible solution x, as the number of bees.

It is believed that poetry and mathematics are opposite poles. The first one is linked to emotions, symbolism, and metaphor. The second one is based on rules, formulations, equations, and structural patterns. But, both have some common domains in their language skills, whether the language of verse or the language of signs and symbols. Hence, mathematics exists in poetry since both resort to images and creative thinking. Contemporarily, the poetry is written in free verse where the poets choose their patterns, styles, or beauties. But historically, most of the poetry was written in some specific mathematical pattern or some form like the sonnet, the villanelle, the rondeau, etc. Not only these, the haiku, the Fib, the ghazals, and the rubaiyat are also in some other mathematical pattern [4]. Each of these forms imposes particular constraints to some specific poetic parameters, maybe on length, meter, syllables, repetitions, etc. Moreover, Sanskrit poetry is with interesting mathematical beauty.

3. Combinatorics

Combinatorics is an area of mathematics primarily concerned with counting. It has many applications, even in poetry. For the set formulation, the order in which the elements are listed does not matter, i.e. $\{i, s\}\,\{s, i\}$.

But if the order of the elements does matter, when we use (a, b) or simply ab for what

$(a, b) \neq (b, a)$ or $ab \neq ba$.

Principle of counting. Multiplicative and additive principles are two fundamental principles of counting. The multiplicative principle says that, *if one task can be accomplished in m ways and, following this task, a second task can be accomplished in n ways, then the first task followed by the second task can be accomplished in mn ways*. But if these two tasks are mutually exclusive, then either of the tasks can be accomplished in $(m + n)$ ways, as the addition principle of counting. A similar argument for more than two nite number of tasks.

Combination. The combination is an arrangement without any regard to the order in which the objects are arranged. The number of ways r objects can be chosen from a set of n objects without regard to the order of selection is called the combination of n objects taken r at a time and is denoted by$C(n, r)$. Where,

$$C(n, r) = \frac{n!}{(n-r)!r!} \tag{2}$$

Moreover, $C(n, r) = C(n, n - r)$. If $C(n, r_1) = C(c, r_2)$ then, $r_1 + r_2 = n$.

Permutation. Permutation is an arrangement in specific regard. For example, *ab* and *ba* represent the different permutation for the elements *a* and *b*. Thus, the number of possible distinct arrangements of r objects chosen from a set of n objects in a definite order is the permutations of n objects taken r at a time and is denoted by $P\ (n;\ r)$ where

$$P(n, r) = \frac{n!}{(n-r)!},\ for\ (n \geq r) \tag{3}$$

Moreover $P\ (n, n) = n!$. And, $P\ (n, r) = C(c, r)\, r!$.

Binomial coefficients. By actual multiplication,

$$(a + x)^2 = a^2 + 2ax + x^2$$

$$(a + x)^3 = a^3 + 3a^2x + 3ax^2 + x^3$$

The coefficients in such expansion can be expressed in the form of Pascal's triangle like,

$$\begin{array}{lccccccccc}
(a+x)^0: & & & & & 1 & & & & \\
(a+x)^1: & & & & 1 & & 1 & & & \\
(a+x)^2: & & & 1 & & 2 & & 1 & & \\
(a+x)^3: & & 1 & & 3 & & 3 & & 1 & \\
(a+x)^4: & 1 & & 4 & & 6 & & 4 & & 1
\end{array}$$

The binomial theorem for any positive integer n is,

$$(a + x)^n = C(n,0)a^n + C(n,1)a^{n-1}x + \cdots + C(n,2)a^{n-r}x^r + \quad + C(n,n)x^n \qquad (4)$$

Where $C(n,0), C(n,0), \ldots\ C(n,0)$ are called the binomial coefficients and

$$C(n,0) + C(n,0) + \ \ldots + C(n,0) = 2^n \qquad (5)$$

For details, see the school mathematics like [2].

4. Sanskrit Prosody

Let us begin with a couple of holy lines, as the Shanti mantra of the Isho Upanisadha,

ॐ पूर्णमद: पूर्णमिदं पूर्णात् पूर्णमुदच्यते |

पूर्णस्य पूर्नामादाय पूर्नामेवा वशिष्यते ||

This is infinite that is infinite, from the infinite comes the infinite. If infinite is taken away from infinite, only infinite remains. It has an amazing consequence to the set continuum theory [11], i.e., *this is complete, that is complete, from the completeness comes the completeness. If completeness is taken away from completeness, only completeness remains*.

Sanskrit prosody, the discipline of *Chhanda* emerged in the 2nd-millennium BCE since the hymns of Rigveda includes the names of meters [12]. It is interesting to observe in Sanskrit poetry, where the concept of binomial coefficients, Fibonacci numbers, counting principles based on permutation and combination, and binary numeration has been in use right from the days of *Pingala*. Sanskrit poetry consists of stanzas with four quarters. Each quarter may have the same number of syllables or the same number of time units. Sanskrit poetry is categorized mainly as *Vritta* and *Jati*. In *Vritta* meter is regulated by the number and composition of syllables. In *Jati*, it is regulated by the number of time units. *Vritta* may be *samavritta* (each quarters similar), *ardhasamavritta* (alternative quarters similar), and *vishamvritta* (all the quarters dissimilar). For detail about the Sanskrit prosody and *chhanda*, we refer to [15].

Table 1: Decimal and binary representation of *gana* in Sanskrit prosody.

Name of gana	Pingala form	Decimal number	Binary form
Na	iii	0	000
Sa	iis	1	001
Ja	isi	2	010
Ya	iss	3	011
Bha	sii	4	100
Ra	sis	5	101
Taa	ssi	6	110
Maa	sss	7	111

Let us have a look at the *Pingala*, on Sanskrit poetry, where 'i' and 's' denote the short syllable and long syllable, respectively. In such poetry, the *gana* representing the collection of such syllables in the form of the triplet is based on the counting principle. Table 1 represents different *gana* of Sanskrit poetry, their respective *Pingala* formulation, decimal number representation, and the binary form in respective columns. *Pingala* had chosen syllable triplets, consisting of all short or long or a mixture of these sounds based on the principle of counting, decimal number, and binary number representation.

Table 2: Pattern analysis of *gana* in Sanskrit prosody.

$\|\xi\|$	Structure	No. of structure		Total
		No.	$B(x)$	
1	i	1	$C(1,0)$	
	s	1	$C(1,1)$	2^1
2	ii	1	$C(2,0)$	
	$is;\ si$	2	$C(2,1)$	
	ss	1	$C(2,2)$	2^2
3	iii	1	$C(3,0)$	
	$iis;\ isi;\ sii$	3	$C(3,1)$	
	$iss;\ sis;\ ssi$	3	$C(3,2)$	
	sss	1	$C(3,3)$	2^3
4	$iiii$	1	$C(4,0)$	
	$iiis;\ iisi;\ ssis;\ isii;\ siii$	4	$C(4,1)$	
	$iiss;\ isis;\ isii;\ ssii;\ sisi;\ siis$	6	$C(4,2)$	
	$isss;\ siss;\ ssis;\ sssi$	4	$C(4,3)$	
	$ssss$	1	$C(4,4)$	2^4

Let us collect different syllables denoted by $|\xi|$ based on the number of units, as a singleton, doublet, triplet, and quartet. For the singleton, the formulation is either $\{i\}$ or $\{s\}$. For the doublet, it takes either of $\{ii\}$, $\{is\}$, $\{si\}$ or $\{ss\}$.

In a similar fashion for the triplet and quartet. Their collection pattern and the number of such structures as shown in the second and third columns follow the binomial coefficients as in fourth column as mentioned in Table 2.

Let us assume that the short syllable *i* and the long syllable s denote 1 and 2, respectively. Then to represent 2, the collection of syllables be either {ii} or {s}. But for 3 it will be either of {iii} or {is} or {si}. And is similarly for 4, as in second column of Table 3. The total number of such structures to represent 1, 2, 3, and 4 can be represented in 1, 2, 3, and 5 different ways. Likewise, to represent 5 and 6, the total number of such structures are 8 and 13 respectively, as shown in Table 3, based on the principle of counting. Here, the number of such total counting's follows the sequence 1, 2, 3, 4, 5, 8, 13, which is the known Fibonacci sequence.

Table 3: Structural flows of Fibonacci sequence related to *gana* in *maatrik* verse.

No. of syllables	*Structures/patterns*	*No. of structures*
1	*i*	1
2	*ii, s*	2
3	*iii, is, si*	3
4	*iiii, iis, isi, sii, ss*	5
5	*iiii, iss, sis, ssi, iiis, iisi, isii, siii*	8
6	*iiiiii, iiss, isis, issi, siis, sisi, ssii, iiiis*	
	iiisi, iisii, isiii, siiii, sss	13

5. Conclusion

Mathematics is composed of a beautiful language, which not only records and expresses the results and ideas but also creates itself the process of thinking. Mathematics and poetry are the noble intellectual activities of human beings having a small intersection. Mathematicians have been an inspiration to writers. Mathematical structures and their different formulations provide the battle of a logical twist to the poet's innermost emotions. It is one of the expressions of the mysteries, beauty, and truth of the universe. The concept of binomial coefficients, Fibonacci numbers, counting principles based on permutation and combination, and binary numeration has been used in Sanskrit poetry.

Poetry and applied mathematics both use symbols for beautiful painting, pointing, and patterning. The pointing establishes a relationship between symbols and a world beyond the domain of symbols whereas patterning extends the relationship between the symbols and other symbols in the same domain. Their beauty is generated by imagination, hidden structures, and mysteries.

References

[1] Aharoni, R. (2013). Mathematics, poetry, and beauty. Journal of Mathematics and the Arts, https://doi.org/10.1080/17513472.2014.943490

[2] Bajracharya, D. R., Shrestha, R.M., Singh, M.B., Sthapit, Y.R., & Bajracharya, B.C. (1999). Higher Secondary Level Basic Mathematics, Sukunda Pustak Bhawan, Kathmandu.

[3] Bahls, P. (1997). Math and Metaphor: Using Poetry to Teach College Mathematics. The WAC Journal, 20, 75-90.

[4] Birken, M. & Coon, A. C. (2008). Discovering patterns in Mathematics and Poetry, Rodopi, Amsterdam, New York.

[5] Darling, D. J. (2004). The Universal Book of Mathematics. Jhon Wiley and Sons, Hoboken, NJ.

[6] Glaz, S. & Liang, S. (2009). Modeling with Poetry in an Introductory College Algebra Course and Beyond, Journal of Mathematics and the Arts, 3(3), 123-133.

[7] Hardy, G.H., (1992). A Mathematician's Apology: Cambridge University Press.

[8] Joanne, G. (2009). What poetry is found in Mathematics? Springer Science, Business media, LLC.

[9] Nahin, P. J. (2011). Dr. Euler's Fabulous Formula: Cures Many Mathematical Ills Princeton University Press.

[10] Opie, I.M.B., & Opie, P.M. (1951). The Oxford Dictionary of Nursery Rhymes, Oxford University Press, 333-334.

[11] Raymond, L. W., (1965). The Foundation of Mathematics, John Wiley and Sons, New York.

[12] Scharf, P. (2013). The Oxford Handbook of the History of Linguistics. Allan, K. (ed.), Oxford University Press, 228-234.

[13] Smith, D.E. (1958). History of mathematics: vol. I and II, Dover Publication, New York.

[14] https://nplcg.blogspot.com/2013/03/Translation-In-English-Of-THE-LUNATIC-By-Laxmi-Prasad-Devkota.html.

[15] ढुङ्गाना, गोविन्दप्रसाद शर्मा, (२०३७). छन्दोहार, साझा प्रकाशन ।

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