ISSN 2738-9898 (Print) ISSN 2738-9901 (Online)

Effect of Heat on Velocity and Heat Transfer Coefficient of the Fluid in a Pipe with a Laminar Flow

Madhav Prasad Poudel

School of Engineering, Faculty of Science and Technology, Pokhara University, Pokhara-30, Kaski, Nepal

E-mail: *pdmadav@gmail.com*

Received on: 20th Jan., 2022

Accepted for publication: 18th Feb., 2022

Abstract

The study of flow of liquids in a circular pipe has been studied for a long time. The theoretical and numerical analysis of viscous fluid is very important in the field of physics, engineering and even in medicine. In any fluid flow, the Napier- stokes equation are very important to study the nature of flow. On the basis of Reynold number, the flow is either classified as laminar or turbulent. When the heat is supplied to a circular pipe with a liquid having laminar flow, the velocity, rate of flow of volume, temperature gradient, etc. are changed. This study aims to investigate the change in velocity, pressure drop, frictional factor and temperature distribution in the thermal layer across the liquid in the laminar flow. Various boundary conditions are assumed and the conservation of energy, momentum are also considered.

Keywords: Drag- coefficient, Laminar flow, Reynold number, Specific heat, Thermal expansion

1. Introduction:

Over the past 50 years, the flow along a circular cylinder represents a classical problem in fluid mechanics and thus received considerable attention [14]. The problem of viscous incompressible flow along a circular cylinder has for a long time received much attention, both theoretically and numerically. A common situation encountered by an engineer is heat transfer to fluid flowing through a pipe/tube. This can occur in heat exchangers, boilers, condensers, evaporators, and a host of other process equipment in mechanical components of the power generation applications. The behavior of viscous flow appears in food processing, plastic manufacturing, polymer processing, biological fluids, concrete mixtures, ice and magma flows [2]. The study of heat transfer can be done either in laminar flow or in turbulent flow [19]. Both conditions show different performance characteristics of the heat transfer for several fluids ranging from viscous to incompressible fluids. Besides, viscosity is the most important characteristic of any fluid. The value and changing of viscosity have essential significance for all events in the mechanical systems. The viscosity of fluid is changed with temperature, pressure and rate of shear. However, in analyzing the mechanical system operation it is usual to use only temperature dependence of viscosity, while influence of pressure and shear rate are neglected. The changing of viscosity with temperature and pressure is important not only for the sake of theoretical analysis but also regarding the practical application to the real mechanical systems in which temperature and pressure are changing continuously. Furthermore, proper design of thermal systems (equipment, pipelines, etc.), among many other parameters, significantly dependent on using the suitable correlation for estimation of heat transfer parameters such as heat transfer coefficient or Nusselt number[13]. The viscous fluids have many applications in the field of fluid dynamics. Due to these applications, many research activities are conducted in this field.

Various studies have been conducted to examine the flow of fluid over a stretching surface in two dimensions. Sakiadis [17] investigated the analysis of viscous fluid flow in 3-Dimension. The two cases of heat transfer on heat flux, and surface temperature with unsteady fluid flow over a continuous moving surface temperature, have been investigated by Tsou *et al*.[20]. They concluded that the velocity changes with the change in stretching parameter. In case of uniform or non-uniform heat fluxes, the numerical study of heat transfer has been analyzed by Ishak *et al.* [7]. They claimed that at the Micro polar fluid has a higher coefficient of convective heat transfer than viscous fluid flow.

The flow of a fluid in a pipe may be laminar or turbulent. If the streak lines remain as a well-defined line with only slight blurring due to molecular diffusion, then it is a laminar flow and if streak fluctuates in time and space, then it is a turbulent flow [6].

For the pipe flow, Reynolds number, $R_e = V_D/\upsilon$ is the most important quantity. It is the ratio of the inertia to viscous effects in the flow. Hence, the term flow rate should be replaced by Reynolds number, where V is the average velocity of the fluid in the pipe. The distinction between laminar and turbulent pipe flow and its dependence on an appropriate dimensionless quantity was first pointed out by Osborne Reynolds in 1883 A.D.. The flow in a round pipe is laminar if the Reynolds number R_e is less than 2100 and is turbulent if it is greater than 4000. [4].

2. Problem Formulation:

2.1. Laminar Flow:

Consider a two-dimensional steady flow of an incompressible liquid flowing with the uniform velocity (U_0) and temperature (T_0) over an infinite long circular cylinder oriented with its long axis normal to the flow as shown in the Fig. 1. The longitudinal flow relationship between the pipe and the fluid is also shown in the Fig. 1. It shows the actual velocity profile of the fluid. The surface of the cylinder initially is maintained at a constant wall temperature T_w. In order to keep the level of complexity at a considerable level the viscous dissipation effects are assumed to be negligible and thermal physical properties (heat capacity, thermal conductivity, viscosity and density) are assumed to be independent of temperature. These two assumptions lead to decoupling of momentum and thermal energy equations.

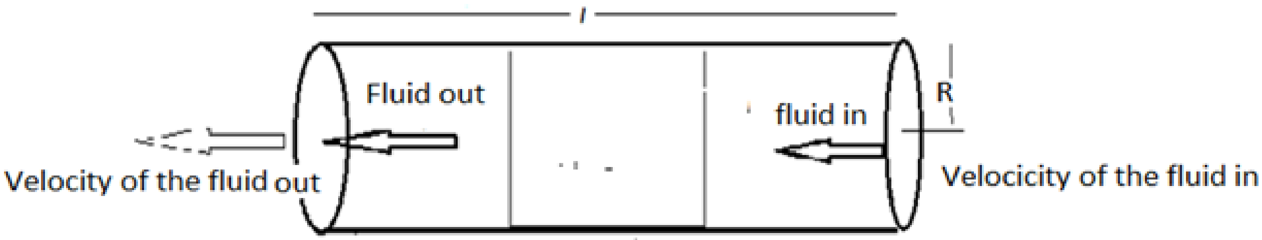

Figure 1: Motion of the fluid in a pipe

The balance principle of momentum for viscous fluid is given by Navier -Stokes equations [16]

$$\frac{\partial \boldsymbol{V}}{\partial \boldsymbol{t}} + \boldsymbol{V}.\nabla \boldsymbol{V} = -\frac{1}{\rho}\nabla \boldsymbol{P} + \Delta \boldsymbol{V} \qquad (1)$$

The equation of continuity is

$$\nabla.\boldsymbol{V} = \boldsymbol{0} \qquad (2)$$

The momentum equation is

$$\rho\left(\frac{\partial U}{\partial t} + U.\nabla U - f\right) - \nabla.\sigma = 0 \qquad (3)$$

And the energy equation is

$$\rho C_p\left(\frac{\partial U}{\partial t} + U.\nabla T\right) - k\nabla^2 T = 0 \qquad (4)$$

where P is the pressure, ρ is the density, V is the volume of the liquid, U is the velocity vector, T is temperature, f is body force, Cp is heat capacity, k is specific heat capacity and σ is stress tensor.

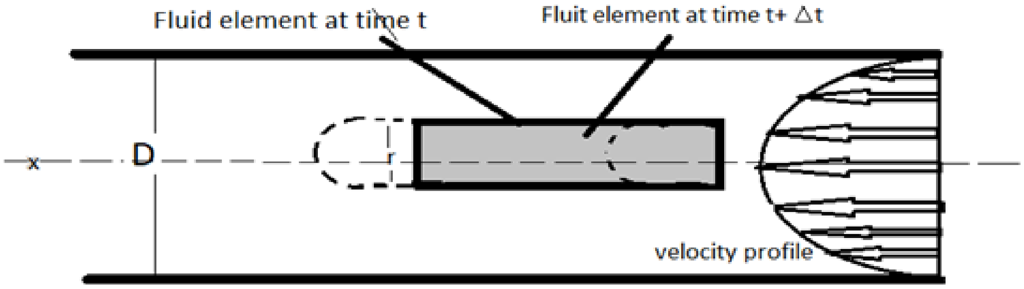

Figure 2: The actual velocity profile diagram in a laminar flow

Kudela [12] assumed that the gravitational effect is neglected. The flow is merely a balance between pressure and the difference between the viscous forces and the pressure difference acting on the end of the cylinder of area πr^2. The shear stress acting on the lateral surface of the cylinder is of area *2prl*. This force balance can be written as

$$P_1\pi R^2 - (P_1 - \Delta P) \pi R^2 - 2\pi R l\tau = 0$$

or equivalently

$$\frac{\Delta P}{l} = \frac{2\tau}{R} \tag{5}$$

Equation (3) represents the basic balance in forces needed to drive each fluid particle along the pipe with constant velocity. Here $\frac{2\tau}{R}$ must be independent of r. That is, $t = CR$ where C is a constant. At the centerline of the pipe, the shear stress is a maximum, denoted by τw the wall shear stress. Hence, $C = \frac{2t}{D}$ and the shear stress distribution throughout the pipe is a linear function of the radial coordinate.

$$\tau = \frac{2\tau_w R}{D} \tag{6}$$

The linear dependence of t on r is a result of the pressure force being proportional to R^2 (the pressure acts on the end of the fluid cylinder; area $A= \pi R^2$) and the shear force being proportional to r (the shear stress acts on the lateral sides of the cylinder; [5]. If the viscosity were zero there would be no shear stress, and the pressure would be constant throughout the horizontal pipe. The pressure drop and wall shear stress are related by

$$\Delta P = \frac{4l\tau_w}{D} \tag{7}$$

For laminar flow of a fluid, the shear stress is simply proportional to the velocity gradient, $\tau = \mu\frac{du}{dy}$ in the notation associated with pipe flow, this becomes

$$\tau = - \mu\frac{du}{dR} \tag{8}$$

The negative sign is included to give $t > 0$ with $\frac{du}{dr} < 0$ (the velocity decreases from the pipe centerline to the pipe wall). Hence from equations (3) and (6) we get

$$\frac{du}{dR} = - (\frac{\Delta P}{2\mu l}) \tag{9}$$

This can be integrated to give the velocity profile:

$$u = -(\frac{\Delta P}{2\mu l})^2 + C_1 \tag{10}$$

where C_1 is a constant. Because the fluid is viscous it sticks to the pipe wall so that $u = 0$ at $R = \frac{D}{2}$. Thus, $C_1 = 4R^2\frac{\Delta P}{16\mu l}$ [13].

Hence, the velocity profile of the fluid during the flow can be written as

$$\mu\ (r) = (\frac{\Delta P R^2}{4\mu l})[1 - \frac{r}{R})^2]$$

$$= V\ max\left[(1 - \frac{r}{R})^2\right] \tag{11}$$

where $V_{max} = (\frac{\Delta P R^2}{4\mu l})$ is the centreline velocity. This is the velocity profile of the fluid in the pipe.

Now let us consider the heat being supplied to the pipe as shown in the figure. Saleem *et al*. [18] has studied the heat transfer analysis of viscous incompressible fluid by combined natural convection and radiation in an open cavity. Also, Derby *et al*. [4] examined the performance of the approximations for modeling a representative problem of heat transfer and buoyant flow in optically thick fluid.

The volume flow rate through the pipe can be obtained by integrating the velocity profile across the pipe [11], [21]. Since the flow is asymmetrical about the center, the velocity is constant on small area element consisting of rings of radius *r* and change in thickness (*dr*) thus

$$q_v = \int u dA = \int_o^R u(r) 2\pi r dr$$

$$= 2\pi V_{max}\left[1-\left(\frac{r}{R}\right)^2\right] rdr$$

$$= 2\pi V_{max}\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right] \quad \text{from r = 0 to R}$$

By definition, the average velocity is the flow rate divided by the cross- sectional area

$$V\ av = \pi R^2 \frac{V_{max}}{2} \tag{12}$$

$$V = \frac{V_{max}}{2} = \frac{\Delta P R^2}{8\mu l} \tag{13}$$

$$\text{and } q_v = \frac{\pi R^4 \Delta P}{8\mu l} \tag{14}$$

2.2. Frictional Factor for Laminar Flow:

For the frictional factor in a laminar flow, Darey Weisbach [1] equation is

$$hL = \frac{\Delta P}{\rho g} = f \frac{L}{D}\frac{V^2}{2g} \tag{15}$$

And we have $f = \frac{\Delta P}{\frac{2}{2}\rho V^2}\frac{D}{L}$ (16)

Equation (11) can be rearranged to obtain

$$\Delta P = \frac{8\mu V l}{R^2} = \frac{32\mu V}{D}\frac{L}{D} \tag{17}$$

Inserting (14) to (13) we obtain

$$f = \frac{64}{Re} \tag{18}$$

2.3. Coefficient of Thermal Expansion:

The volume coefficient of thermal expansion is given by

$$\alpha_v = \frac{1}{V}\left(\frac{dV}{dT}\right)_p \tag{19}$$

where *V* is the volume, *T* is the temperature and *P* is the pressure of the fluid.

The vertical section of the flow is shown by Fig. 4. During the flow of the fluid in the pipe after the heating process, as shown in Fig. 3. The following equations should be considered.

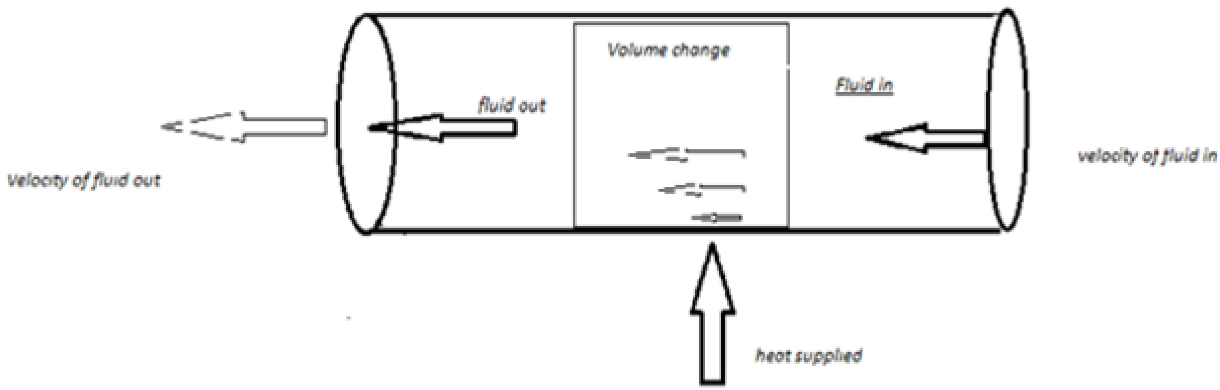

Figure 3: Horizontal section of the flow

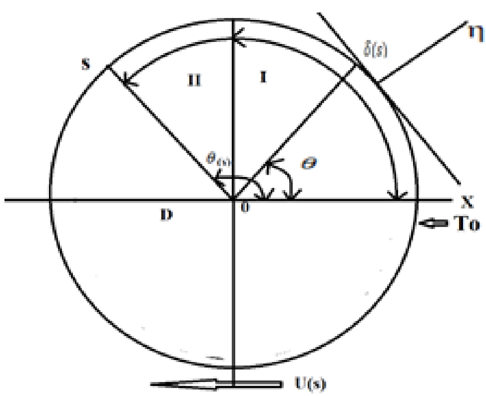

Figure 4: Vertical section of the flow

The equation of continuity

$$\frac{\partial u}{\partial s}\ \frac{\partial v}{\partial \eta}=0 \tag{20}$$

The equation of s- momentum is

$$u\frac{\partial u}{\partial s}+v\frac{\partial u}{\partial \eta}=-\frac{1}{\rho}\frac{dP}{ds}+v\frac{\partial^2 u}{\partial \eta^2} \tag{21}$$

n- momentum

$$\frac{\partial P}{\partial \eta}=\ 0 \tag{22}$$

Bernoulli equation:

$$\frac{1}{\rho}\frac{dP}{ds}=\ U(s)\ v\frac{dUs}{ds} \tag{23}$$

The energy equation is

$$\frac{\partial u}{\partial s}+v\frac{\partial u}{\partial \eta}=\ \alpha\frac{\partial^2 T}{\partial n^2} \tag{24}$$

The stress tensor is the sum of the isothermal pressure and the deviation stress tensor

Therefore

$$\sigma=-pl+\tau \tag{25}$$

For the incompressible fluids the extra stress tensor is related to the rate of deformation tensor as

$$\tau=2\eta\epsilon \tag{26}$$

Where ϵ is te component of the rate of strain tensor, given by

$$\epsilon=\frac{1}{2}[(\nabla U)+(\nabla U)^2] \tag{27}$$

For the fluid the viscosity is given by

$$\eta=m\left(\frac{l_2}{2}\right)^{\frac{n-1}{2}} \tag{28}$$

2.4. The Boundary Conditions:

The boundary conditions defined by Khan *et al.* (9) are as follows

The Inlet Boundary Conditions

The uniform flow in the *x* direction and uniform fluid temperature are imposed at the inlet as

$$U_x=U_o,\quad U_y=0 \text{ and } T=T_0 \tag{29}$$

On the surface of the cylinder

The standard no slip condition is used and the cylinder is heated so that its surface is maintained at constant temperature T_w

$$U_x=0,\ U_y=0,\ T=T_w \tag{30}$$

At the Exit Boundary

The default outflow boundary conditions, a zero-diffusion flux is considered. When there is no change in area at the outflow boundary, the gradients in the cross-stream directions may still exists at the outflow boundary. This is similar to the Neumann conditions. i. e.

$$\frac{\partial \emptyset}{\partial t}+U_0\frac{\partial \emptyset}{\partial t}=0 \text{ where } \emptyset=U_{x,}\,U_y \text{ and } T \tag{31}$$

The hydrodynamic Boundary Conditions [9]

At the cylinder surface, η =0

$$\text{U}\ =\ 0\ \text{ and }\ \frac{\partial^2 u}{\partial \eta^2}=\frac{1}{\mu}\frac{\partial P}{\partial s} \tag{32}$$

At the edge of the boundary layer i.e. at $\eta=\delta(s)$

$$\text{U} = \text{U(s)},\ \frac{\partial u}{\partial \eta}=0\ and\ \frac{\partial^2 u}{\partial \eta^2}=0 \tag{33}$$

The Thermal Boundary conditions

The boundary conditions for the uniform wall temperature (UWT) and the uniform wall flux UWF) are;

$\eta=0,$

$$\begin{bmatrix} T=T_w\, for & UWT \\ \dfrac{\partial T}{\partial \eta}=-\dfrac{q}{k_f}\ for & UWF \end{bmatrix} \tag{34}$$

$$\eta=\delta(s), T=T_\alpha, and\frac{\partial T}{\partial \eta}=0 \tag{35}$$

Velocity Distribution

Assuming a thin boundary layer around the cylinder, the velocity distribution in the boundary layer can be approximated by a fourth order polynomial as suggested by Pohlhausen [15]

$$\frac{u}{U(s)}=\left(2\eta_H-3\eta^2{}_H+3\eta^4{}_H\right)+\frac{\lambda}{6}\ (\eta_H-3\eta^2{}_H+3\eta^3{}_H-4\eta^4{}_H) \tag{36}$$

Where $0\le\eta_H=\frac{\eta}{\delta(s)}\le 1$ and λ is the pressure gradient parameter given by

$$\lambda=\frac{\delta^2}{v}\frac{dU(s)}{ds} \tag{37}$$

Temperature distribution

Assuming a thin thermal boundary layer around the cylinder, the temperature distribution in the thermal boundary layer can be approximated by a third order polynomial

$$\frac{T-T_a}{T_w-T_a} = 1 - \frac{3}{2}\eta_T + \frac{1}{2}\eta^3{}_T \quad (38)$$

for the isothermal boundary conditions

The Reynolds number(Re) and Prandtls number (Pr)

The Reynolds number and Prandtls number for power flow fluids are defined as

$$Re = \frac{\rho D^n U_0{}^{2-n}}{m} \; and \; Pr = \frac{mc_p}{k}\left\{\frac{U_0}{D}\right\}^{n-1} \quad (39)$$

Fluid flow

The first parameter of interest is fluid friction which manifests itself in the form of the drag force FD, where FD is the sum of the skin friction drag Df and pressure drag Dp. Skin friction drag is due to viscous shear forces produced at the cylinder surface, predominantly in those regions where the component of shear force in the flow direction is given by

$$D_f = \int \tau_w \frac{D}{2} \sin\theta d\theta \quad (40)$$

Where τ_w is the shearing stress along the cylinder wall, which can be determined by the Newton's law of viscosity

Then $\quad \tau_w = \left\{\mu \frac{\partial u}{dy}\right\}$ at y = 0 (41)

In dimensionless form it can be written as

$$C_f = \frac{\tau_w}{1/2}\rho U^2 \quad (42)$$

The friction drag coefficient can be defined as

$$C_{df} = \int_0^{\pi} C_f sin\theta d\theta = \int_0^{\theta(s)} C_f sin\theta d\theta + \int_{\theta(s)}^{\pi} C_f sin\theta d\theta \quad (43)$$

Since the shear stress on the cylinder surface after the boundary layer separation is negligible. The angle of separation depends upon the velocity distribution outside the boundary layer. Khan et al. [8] have shown that for infinite flow condition the separation occurs at $\theta(s) = 107.71$ degrees. By applying the boundary conditions (29-35). By applying the boundary conditions from 32-38, the value of drag coefficient be simplified. The second integral will be zero and the frictional drag coefficient [9] can be written as

$$C_{df} = \int_0^{\pi} C_f \sin\theta d\theta = \frac{5.786}{\sqrt{\text{Reynold number}}} \quad (44)$$

Pressure drag is due to the unbalanced pressure which exists between the relatively high pressures and the upstream surfaces and the lower pressures on the downstream surfaces. In dimensionless form it can be written as

$$C_{Dp} = \int_0^{\pi} C_p \, cos\theta d\theta \quad (45)$$

The pressure difference ΔP is obtained by integrating θ-momentum equation with respect to θ. In dimensionless form it is written as

$$\frac{\Delta P}{\frac{1}{2}\rho U^2} = 2(1 - cos\theta) + \frac{8}{reynold\ no}(1 - cos\theta) \quad (46)$$

Heat Transfer

According to Khan [10] The second parameter of interest in this study is the dimensionless average heat transfer coefficient NuD, for large Prandtl numbers ≥ 0.71. This parameter is determined by integrating (24) from the cylinder surface to the thermal boundary layer edge. Assuming the presence of a thin thermal boundary layer T along the cylinder surface, the energy integral equation for the isothermal boundary condition can be written as

$$\frac{d}{ds}\int_0^{\delta_T}(T - T_a)u d\eta = -\alpha \frac{\partial T}{\partial \eta} \; at \; \eta = 0 \quad (47)$$

The local surface temperatures [9] for the two regions can be obtained from the temperature distribution (38) as

$$\Delta T_1(\theta) = \frac{2q\delta r_1}{3k_f} \text{ and } \Delta T_2(\theta) = \frac{2q\delta r_2}{3k_f} \quad (48)$$

The local heat transfer coefficient can now be obtained from (48) i. e.

$$h_1(\theta) = \frac{q}{\Delta T_{1(\theta)}} \text{ and } h_2(\theta) = \frac{q}{\Delta T_{2(\theta)}} \quad (49)$$

Assumption for energy equation:

By the law of conservation of energy, It is neither created nor destroyed. But it is conserved in the system of flow. The heat energy supplied from the source is partially converted into kinetic energy and in overcoming friction. However, the following assumptions are done in energy equation.

a. Axial conduction is neglected.

b. Entry temperature profile is assumed constant over the flow cross-section.

c. Constant wall flux and constant wall temperature boundary conditions are considered.

d. Natural convection effects are neglected.

e. Viscous dissipation is considered.

f. Fluid properties other than viscosity are assumed to remain constant.

Assumption for Momentum Equation:

By the law of conservation of momentum, the total momentum remains conserved in the flow. In case of momentum following assumptions are done.

a. Steady laminar flow is assumed.

b. The entry flow is assumed fully developed.

c. Radial pressure variations are neglected.

d. Temperature varying viscosity is included.

e. Radial velocities are assumed significant and two-dimensional flows is considered.

3. Results and Discussion:

The present study aims to study the various dimensions when a cylindrical pipe with a laminar flow is heated from all sides. The results are obtained as follow;

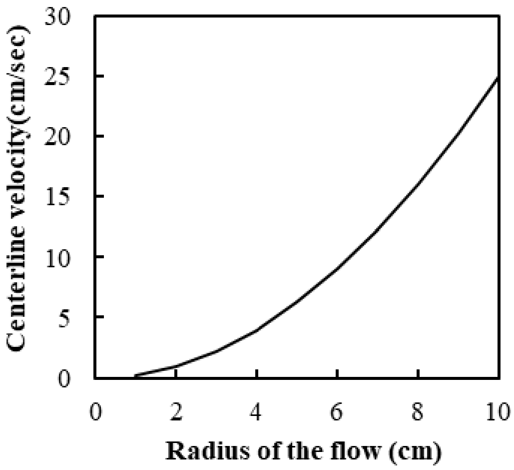

Figure 5: Velocity profile of the liquid

The velocity of the fluid along the center of the pipe is given by equation (11). This is the maximum velocity and as the length of the radius increases, the velocity is reduced. The relationship between the distance of the lamina from the center and its velocity is given in Fig. 5. This graph is based in assuming the pressure drop, coefficient of viscosity to be constant for unit length. The significant change also appears in the rate of volume flow. This is expressed in Fig. 6. Governed by the equation (12). Obviously, the rate of flow is directly proportional to the square of the distance from the center of the pipe.

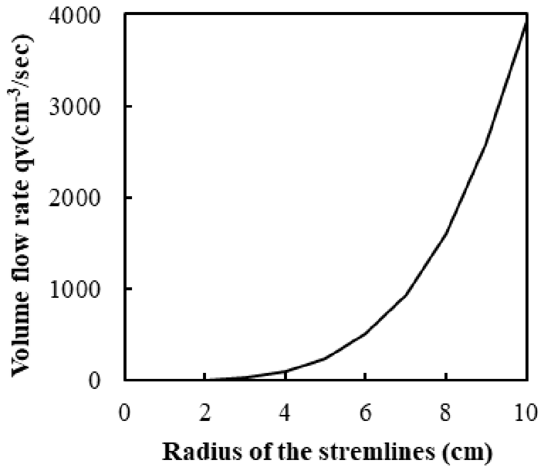

Figure 6: Volume flow analysis of the liquid

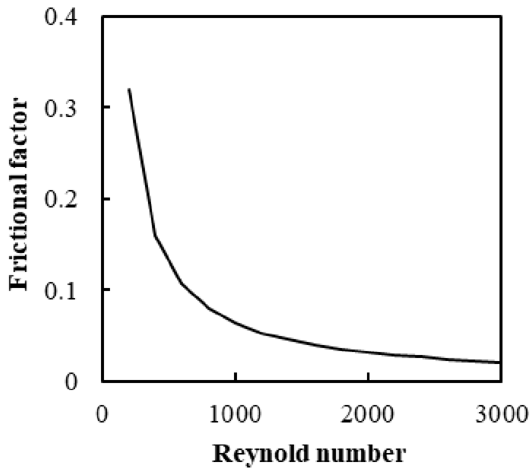

Figure 7: Relationship between Reynold number and frictional factor

Another influencing factor for the laminar flow of the pipe is the friction. The relationship between Reynold number and frictional factor is given by equation (18) and is expressed in Fig. 7. In laminar flow, the effect of frictional factor is high whereas in turbulent it is less effected. When the Reynold number approaches to zero, the frictional factor is nearly infinity.

Frictional factor directly affects the Frictional drag coefficient. The relationship between frictional drag coefficient and Reynolds number is given by equation (44) and pictorially expressed in Fig. 8. The similarity is directly seen among the two figures.

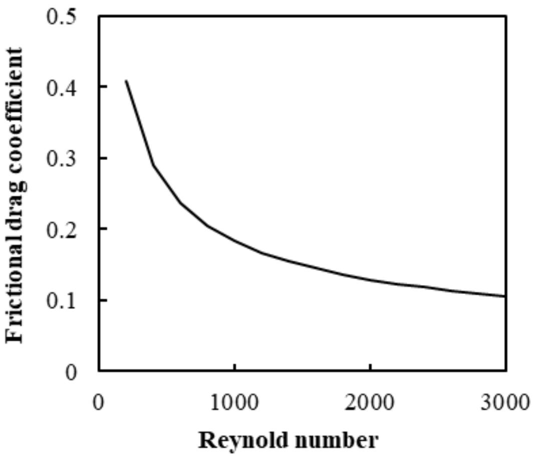

Figure 8: Relationship between Reynolds number and frictional drag coefficient

The temperature distribution along the radius of the pipe for the flow of the pipe is given by equation (38) and is expressed graphically in Fig. 9. The relationship between the coefficient of viscosity and temperature distribution is not a linear one but it can be deduced that the coefficient of viscosity inversely affects the temperature distribution in a pipe.

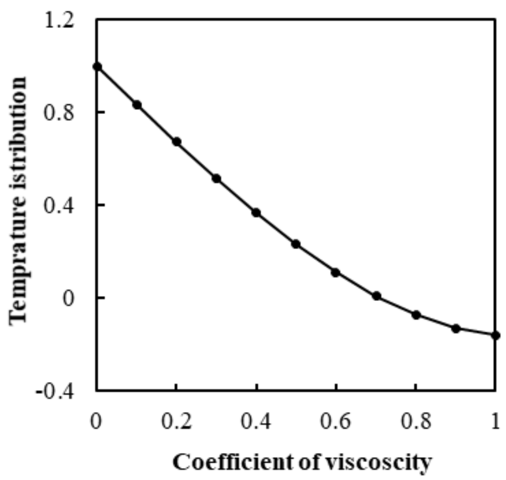

Figure 9: Relationship between coefficient of viscosity and temperature distribution

Apart from the above results, the entire study can be summarized as follows;

a. The velocity profile is directly proportional to the change in pressure gradient and inversely proportional to the length.
b. The centerline velocity of the fluid is directly proportional to the radius of the pipe and inversely proportional to the length of the pipe.
c. In a horizontal pipe, the flow rate is directly proportional to the pressure drop, inversely proportional to the viscosity, inversely to the pipe length, and proportional to the pipe radius to the fourth power. Further, the rate of transfer of heat from the source to the fluid is directly proportional to the heat transfer coefficient of the materials, and the specific heat capacity of the materials. The Reynold number and the Nusselt number and Prandtl's numbers cannot be ignored.

4. Limitation of Study:

This research is totally based in the laminar flow of the viscous fluid. Under this study the following factors are not considered:

a. The turbulent flow
b. The flow in the rectangular pipe
c. If the soluble substances are mixed in the fluid, then their effect in the flow is also ignored.

If the pipe is not uniformly cylindrical then the effect caused by the irregularity is not considered

5. Limitation of Study:

The problem of temperature-varying properties of the fluid is more complex than that of constant properties. The different property ratio correlations of different fluids increase the complexity of the variable-temperature properties problem. It is also difficult to give a correction for the temperature-dependent behavior of all viscous liquids in all tubes. The viscosity varies more markedly than the other thermo-physical properties for most liquids, so little work appears to have been done for the pipe flows involving temperature-dependent viscosity and velocity in the field of heat transfer. Therefore, the proposed study has significant importance on fluid flow and heat transfer for temperature-dependent thermo-physical properties in straight tubes with circular cross section for finding the correlation among those properties.

Furthermore, this study is best fitted in the following cases:

a. Expansion of Mercury and Alcohol in the thermometer to minimize the error due to constriction of clinical thermometer.

b. Effect of heat in water pipes in summer system in water distribution system.

c. Delivery of petroleum substances in districts transportation though pipelines outlets.

Acknowledgements:

The author gratefully acknowledges the financial support of Nepal Mathematical Council of Gandaki province for providing the grant to this article.

References:

[1] Brown, G. O. “The history of the Darcy-Weisbach equation for pipe flow resistance. In Environmental and water resources history” pp. 34-43, 2003

[2] Chhabra, R. P. “Non-Newtonian fluids: an introduction. In Rheology of complex fluids” (pp. 3-34). Springer, New York, NY, 2010.

[3] Derby, J. J., Brandon, S., & Salinger, A. G. “The diffusion and P1 approximations for modeling buoyant flow of an optically thick fluid”. International journal of heat and mass transfer, 41(11), 1405-1415, 1998.

[4] Eckhardt, B., Schneider, T. M., Hof, B., & Westerweel, J. “Turbulence transition in pipe flow”. Annu. Rev. Fluid Mech., 39, 447-468, 2007.

[5] F. M. White, 1999. Fluid Mechanics, McGraw-Hill.

[6] Goldstein, D., Handler, R., & Sirovich, L. “Modeling a no-slip flow boundary with an external force field”. Journal of computational physics, 105(2), 354-366, 1993.

[7] Ishak, A.; Nazar, R.; Pop, I. “Heat transfer over a stretching surface with variable heat flux in viscous fluid”. Phys. Lett. A , 372, 559–561, 2008.

[8] Khan, W. A., Culham, J. R., and Yovanovich, M. M., “Fluid Flow and Heat Transfer from a Pin-Fin: Analytical Approach,” AIAA Paper 2003-163, Jan. 2003.

[9] Khan, W. A., J. R. Culham, and M. M. Yovanovich. "Fluid flow and heat transfer from a cylinder between parallel planes." Journal of thermophysics and heat transfer 18.3: 395-403, 2004

[10] Khan, W. A., J. R. Culham, and M. M. Yovanovich. "Fluid flow around and heat transfer from an infinite circular cylinder." Journal of thermophysics and heat transfer : 785-790, 2005.

[11] Kudela, H. “Hydraulic losses in pipes”. Wroclaw University of Science and Technology 2012.

[12] Kudela, H. “Viscous flow in pipe.” 2016.

[13] Munson, B. R., Young, D. F., Okiishi, T. H., & Huebsch, W. W. “Fundamentals of Fluid Mechanics”, John Wiley & Sons. Inc., USA, 2006.

[14] Patnana, Vijaya K., Ram P. Bharti, and Raj P. Chhabra. "Two-dimensional unsteady forced convection heat transfer in power-law fluids from a cylinder." International Journal of Heat and Mass Transfer 53.19-20 4152-4167, (2010

[15] Pohlhausen, K., “Zur Näherungsweise Integration der Differential Gleichung der Laminaren Reibungschicht,” Z. Angew. Math. Mech., 1, pp. 252– 268, 1921

[16] Qian, Y. H., d' Humières, D., & Lallemand, P. “Lattice BGK models for Navier-Stokes equation”. EPL (Europhysics Letters), 17(6), 479, 1992.

[17] Sakiadis, B.C. “Boundary layer behavior on continuous solid surfaces and boundary layer on a continuous flat surface”. J. Am. Inst. Chem. Eng. 221–225, 1961.

[18] Saleem, M., Hossain, M. A., Saha, S. C., & Gu, Y. “Heat transfer analysis of viscous incompressible fluid by combined natural convection and radiation in an open cavity”. Mathematical Problems in Engineering, 2014.

[19] Subramanian, R. S. "Heat transfer in flow through conduits". Department of Chemical and Biomolecular Engineering, Clarkson University Project 2015.

[20] Tsou, F.K.; Sparrow, E.M.; Goldstein, R.J "Flow and heat transfer in the boundary layer on a continuous moving surface". Int. J. Heat Mass Transf. 10, 219–235, 1967.

[21] Y. Nakayama and R.F. Boucher, "Introduction to Fluid Mechanics", Butterworth Heinemann, 1999.