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Factors Behind Graduates Students' Difficulties in Abstract Algebra: Evidence From University Students

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Article Info **Abstract**

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The main aims of this study are to identify the noteworthy factors that contribute to graduate students' difficulties in learning abstract algebra. It pays a qualitative method using an ethnographic research design. The participants included five graduate students from the Department of Mathematics Education at Mahendra Ratna Campus, Tribhuvan University, who were completed the first semester of the master's degree examination. The required data were collected through in-depth interviews and focus group discussions among the participants. We found from the interview data, on the basis of analysis four major factors which were main causes of students' learning difficulties in abstract algebra. These were, prior knowledge, and lecturer performance, along with teaching methods and the nature of assignments and assessments. Previous understanding was found to be decisive for supporting student interest and participation, whereas current knowledge assisted as a substance for grasping new abstract ideas. Furthermore, the effectiveness of instructional strategies, the role of the teachers, and the value of assignments and assessments played a noteworthy role in shaping students' comprehension and academic success in abstract algebra of graduate level.

Introduction

The abstract algebra course, considered one of the most challenging subjects in mathematics, encompasses topics such as group theory, rings, fields, and field extensions (Subedi, 2020; Wasserman, 2017). Abstract algebra offers a transformative approach that deepens understanding by revealing the relationships among algebraic entities within a broader structural framework (He & Kim, 2023).The process of constructing new mathematical concepts from previously learned concepts represents a key aspect of mathematical abstraction (Israel, 2007). Several studies have also highlighted that courses in group theory, rings, fields, and field extensions particularly enhance students' thinking skills, with abstraction being the most prominent ability developed (Ticknor, 2012). Abstract algebra generally agreed that the axiomatic method is one of the most effective and systematic approaches to the learning process (Pramesti & Retnawati, 2019). The axiomatic principles used in abstract algebra encourage students to engage with concepts in a highly theoretical manner (Gnawali, 2024); however, these principles can be quite different from other mathematical approaches and often prove challenging to apply in practical or implication contexts (Judson, 2020). Mathematical systems can be viewed as specific instances of a common abstract structure.

Cognitive and Instructional Factors Influencing Learning Difficulties of Abstract Algebra

Cognitive and instructional factors play a crucial role in shaping students' learning experiences in abstract algebra, as difficulties often arise from limited prior knowledge, challenges in abstract reasoning, and ineffective teaching

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approaches that hinder conceptual understanding (Agustyaningrum et al., 2021; Subroto & Suryadi, 2018). Previous researches have exposed many reasons contributing to students' learning complications in mathematics. The findings of Astuti and Sari (2018), the main difficulties of learning abstract lack of exemplified instruction, difficulty in understanding given problems (Hwang et al., 2019) which leads to poor problem-solving approaches, and inadequate preparation in solving problems (Hazzan, 1999; Herscovics, 2018). Students' struggles further dependent by little knowledge of prerequisite concepts, making it harder to grasp new topic and concepts. Moreover, lecture-based teaching strategies may diminish learners' motivation, while a lack of lecturers' devotion and low teaching skill also hamper effective learning (Herscovics, 2018). Consequently, these factors create important difficulties to students' understanding and success in abstract algebra. Therefore, these findings suggest that students' weak problem-solving tactics and insufficient understanding of prior ideas signify internal factors, as they originate within the students themselves (Astuti & Sari 2018; Chow, 2011). On the contrary, the external factors comprise limited learning content, teacher center instructional methods, low lecturer engagement and collaboration, and unsatisfactory teacher competency, all of which branch as of the students' learning classroom practices environment (Astuti & Sari 2018).

Furthermore, students' learning difficulties in abstract algebra are predisposed by both internal and external factors. Internally, learners' individual attitudes, enthusiasm, and prior knowledge such as set, functions, number theory, and different components of abstract algebra and its properties (Agustyaningrum et al., 2021). Consequently, externally, lecturer performance, teaching methods, and the availability of learning resources play a crucial role in facilitating comprehension of complex concepts such as proving the long theorem step by steps thoroughly (Agustyaningrum et al., 2021). Consequently, these factors shape how effectively students learn abstract algebra and taking long term retention for the new concepts.

In this concept, abstract algebra courses support learners identifying and extract common structures shared various mathematical systems which have previously come upon in subjects like calculus, analytic geometry (Cetin & Dekici, 2021). The notions of abstract algebra also offer valued opportunities to develop learners' affective traits and overall mathematical attitudes. In general, abstract algebra tends to generalize fundamental properties, including closure, associativity, identity, and inverse (Veith et al., 2022). And for the ring we also added two properties such as distributives as well both additive and multiplicative. For the field, it exists multiplicative inverse. These courses also run students with the chance to gain a more profound understanding of fundamental concepts such as identities, inverses, equivalence relations, and functions.

In the same context, the advancements of abstract algebra apply in science and computing have expanded the applications to fields of computer science, physics, chemistry (Faizah et al., 2020). As a consequence, it is now widely recognized by professional organizations and educational institutions as a crucial component of the mathematics content for future teachers. Despite its importance, many students struggle with learning abstract algebra. Research by Weber (2002) views that difficulties of abstract algebra in theorem proving, particularly with homomorphisms, isomorphism, automorphism, centralizer, normalizer, Sylow theorem, solvable and nilpotent group, while Arikan, Ozkan, and Ozkan (2015) found that students often struggle to understand group theory, which is essential for progressing to more advanced topics like rings and fields, extension of field and Galois theory.

Abstract algebra is an essential zone of innovative mathematics that plays a critical role in several fields of science and technology (Faizah et al., 2020). Though, many students at Tribhuvan University encounter significant complications in understanding its abstract concepts and applying them effectively in problem-solving context in the classroom teaching and learning (Dubinsky & Harel, 1992; Sfard, 1991). Such challenges not only affect students' academic achievement but also hinder their confidence and interest in higher-level mathematics (Hutchinson, 1993). The basic difficulty of abstract algebra, categorized by symbolic reasoning and abstract structures, demands strong cognitive engagement, which can overwhelm students without adequate support of the teachers (Sweller, 1988).

Several factors contribute to these learning difficulties. Insufficient prior knowledge in foundational topics such as set theory and logic exacerbates cognitive load and impedes understanding (Mason, 2002). Additionally, teaching methods and lecturer performance significantly influence how well students grasp abstract concepts; ineffective instructional strategies and unclear explanations can increase extraneous cognitive load, thereby limiting meaningful learning (Chandler & Sweller, 1991; Hattie, 2009). Moreover, the nature and design of assignments and assessments play a crucial role in either reinforcing or hindering conceptual development. Assignments that focus excessively on rote procedures rather than conceptual understanding fail to build the necessary mental schemas (Black & Wiliam, 1998; Sweller et al., 2011). Abstract Algebra courses bridge undergraduate and graduate studies, focusing on abstract structures and their properties.

Thus, abstract algebra has emerged as a dynamic and fertile discipline, revitalizing the study of mathematics and creating new avenues of inquiry that continue to enrich the field (Alam, & Mohanty, 2024). Abstract algebra is widely acknowledged as a vital part of the mathematics curriculum in the first semester of the M.Ed. program in faculty of education(FOE). Despite these known challenges, such as abstract nature of curriculum, lack of prerequisite knowledge in students, teachers instructional style, low performance of students, there is limited empirical research exploring the specific factors contributing to students' struggles with abstract algebra in the context of Tribhuvan University. Understanding these factors is essential for developing targeted instructional improvements that can reduce cognitive overload and foster deeper learning. Therefore, this study aims to fill this gap by investigating the key factors behind students' difficulties, thereby contributing to enhanced teaching practices and improved student outcomes in abstract algebra.

We have applied this study as a grounded in Cognitive Load Theory (Sweller, 1988), this study explores the issues behind students' difficulties in abstract algebra at Tribhuvan University by considering how the essential difficulty of the subject interacts with the limitations of working memory. Abstract algebra such as group theory, ring structures, and field extensions demands a high level of concept and symbolic use, which can impose a weighty inherent cognitive load. After instructional materials and concrete concepts are not clearly structured or when irrelevant details are encompassed, needless cognitive load rises, additional hindering understanding. Additionally, insufficient opportunities for practice and conceptual reinforcement may limit the relevant cognitive load, preventing students from forming robust mental schemas necessary for deep understanding. By applying this theoretical lens, the study pursues to explore how the balance or disparity of these cognitive load pays to learning difficulties, offering insights into instructional strategies that can condense cognitive load and improve command in abstract algebra.

Methods

We used ethnographic research design as a qualitative research method and five students participants of Mahendra Ratna Campus were selected as the sample through convenience sampling grounded on their importance to the research topic. Before beginning data collection, we visited each participant personally and visibly clarified the purpose and ethical considerations of the study. We found their informed consent and confirmed that their participation was entirely voluntary. The autonomy of the participants was strictly appreciated during the study. Their personalities were kept confidential to continue privacy, and assumed name were used during analysis and reporting even final reporting.

Semi-structured, in-depth interviews were used as the primary data collection tool. This format allowed for flexibility, enabling participants to freely express their views while still addressing the core research questions. Each interview was audio-recorded with the participants' permission to ensure accuracy and depth in data analysis. The interviews were then transcribed verbatim to retain the authenticity of the participants' responses. Field notes and reflective memos were also maintained to capture non-verbal cues and the context of interactions, contributing to a richer understanding of the data.

The analysis was conducted rigorously using a thematic approach. We first transcribed the recorded data. The process involved various stages of coding: initial coding to categorize basic themes, then grouping these into organizing themes, and finally synthesizing them into overarching global themes. This hierarchical structure allowed for the development of meaningful patterns and insights within the data. Triangulation was guaranteed through cross-verification of interview data, data-based notes, and replications, which helped improve the trustworthiness and credibility of the findings. Over this process, the analysis took the lived experiences when learning the abstract algebra class in M Ed first semester, perceptions, and contextual factors influencing the participants.

Results and Discussion

The study's findings were organized around key themes, including students' prior knowledge, lecturer performance, teaching methods, and the nature of assignments and assessments.

Prior Knowledge

Throughout their in-depth interviews, participants emphasized that learning activities in abstract algebra become effective only when they build on a variety of prior knowledge. Prior knowledge in abstract algebra teaching states to the foundational mathematical notions and skills that have developed before engaging with the abstract structures of algebra, particularly group theory at first time. This includes familiarity with set theory and its 'operations, functions and their properties, and logical reasoning of homomorphism which are essential for understanding the core components of group theory. Moreover, generators of cyclic groups, permutation group and group actions on sets enables students to recognize how group elements can be generated, while knowledge of homomorphisms, isomorphisms, and automorphisms helps them explore structural similarities and mappings between groups. Consequently, concepts like internal and external direct products allow learners to shape and analyze complex groups from simpler ones, and understanding the center, centralizer, and normalizer of a group that gives insight into the internal symmetry and subgroup behavior within algebraic structures.

Furthermore, ability in reasonable perceptive and logical comprehensive proof (such as direct proofs, contradiction, and induction) is vigorous, as abstract algebra heavily relies on formal argument. Without such prior knowledge, students may struggle to make sense of the abstract and formal nature of group theory. Therefore, developing these foundational skills and understandings is necessary to facilitate meaningful learning and deeper comprehension of abstract algebra.

"One participant shared that having strong prior knowledge is essential for understanding abstract algebra, especially concepts like sets, functions, and logical reasoning that form the base of group theory. He explained that I have no pre knowledge, in first class present, without this foundation, it becomes very difficult to follow and understand abstract ideas such as homomorphisms or direct products. Reflecting on my experience, that I admitted that missing classes during undergraduate studies left gaps in my understanding, forcing them to rely on rote memorization rather than true comprehension. As a result, I still found abstract algebra challenging due to the weak conceptual base developed earlier".

He explained that their difficulty with abstract algebra began during undergraduate studies, when work obligations prevented them from attending classes regularly. Missing these sessions meant they could not grasp the fundamental rules, principles, and concepts of the subject. Consequently, they resorted to memorizing sets of past exam questions to pass, scoring only 35 marks. This lack of a strong conceptual foundation continues to make the subject challenging for them.

The pre-knowledge of abstract algebra for students depends on the following concepts.

Concept	**Z**	**S_3**
Group Types	Infinite Cyclic and Abelian Group	Finite form of abelian group of order6
Cyclic Group	Z_4={0,1,2,3}, Entire group = <1>	Subgroup <(123)> = {e, (123), (132), order 3
Homomorphism	Structure-preserving map between two groups f(n) = n, mod 5, from $Z \longrightarrow Z_5$	Sign map: $(\sigma): S_3 \rightarrow Z_2$, even/odd permutation
Isomorphism	Z_n<k> □ Z_n	<(123)>Z_3
Automorphism	F(n) = -n, inversion is a group automorphism	Inner automorphism $\phi() = \tau\ \sigma\ \tau^{-1}$
Internal Direct Product	Z is not the internal direct product of any two non-trivial subgroups	Not possible: No two normal subgroups A, B □ S_3 with AB =S_3 and A∩B= {e}
External Direct Product	$Z \times Z_2$: abelian group of ordered pairs	$Z_3 \times Z_2 \cong S_3$ (as a group structure)
Normalizer $N_G(H)$	Whole group for any subgroup (since Z is abelian)	For H = (12) $NS_{3\,=}\,S_3$ (as all elements conjugate transpositions)
Centralizer CG(a)	All of Z (since group is abelian)	For a = (12) , $C_{S3}(a)$ ={ e, (12) }
Centre Z(G)	Whole group Z(Z) = Z	$Z(S_3)$ = {e} only identity
Generators	1 and -1	(123), (12) and (13) together generate whole group
Orders	Infinite	6
Abelian	Yes	No

Furthermore, another second participant explained that in abstract algebra, developing a strong foundation begins with understanding key group-theoretic structures like the center, centralizer, normal subgroup, and normalizer. These concepts help describe how elements or subgroups behave within a larger group, especially in terms of symmetry and internal structure. The center of a group, denoted as Z(G), includes all elements in a group G that commute with every other element of G. That is, $Z(G) = \{z \in G: zg=gz \text{ for all } g \in G\}$. The center is always a normal subgroup and represents the most commutative part of the group.

Third participant said that the centralizer of an element $a \in G$, written as $C_G(a)$, refers to the set of all elements in G that commute specifically with a, i.e., $C_G(a) = \{g \in G: ga=ag\}$. Unlike the center, which is concerned with commuting with all elements, the centralizer focuses on just one. These structures help identify symmetry within the group and play a role in simplifying computations and understanding the group's internal actions. Similarly, a normal subgroup $N \trianglelefteq G$ is a subgroup that remains invariant under conjugation by any element of the group. This means for all $g \in G$ and $n \in N$, the element gng^{-1} also lies N. Normal subgroups are essential in defining quotient groups and understanding group homomorphisms.

Forth participant said that the normalizer of a subgroup $H \subseteq G$ is the set of all elements $g \in G$ such that $gHg^{-1} = H$. In other words, it consists of elements that stabilize the subgroup under conjugation. Understanding the relationship among these concepts is essential pre-knowledge for abstract algebra because they form the basis for more advanced ideas such as group actions, Sylow theorems, and classification of groups. Prior knowledge of these structural features equips learners with the logical reasoning and abstraction skills necessary to engage deeply with the formal theories and proofs in group theory and beyond.

Finally, fifth participants put his view, about the difficulties and previous experiences when learning ring mathematical structure with addition and multiplication. Teacher taught rings but I did not understand easily. After 15 days I got few concepts some rings allow a division algorithm, which expresses any number as $a = bq+r$ and are called Euclidean domains. During my regular classroom participation, I gained knowledge about several important algebraic concepts. The Euclidean algorithm is used in various domains to compute the greatest common divisor (GCD) efficiently. In Unique Factorization Domains (UFDs), every element can be uniquely factored into irreducible elements, similar to the prime factorization of integers. To determine whether a polynomial is irreducible, especially over the field of rational numbers
Q, Eisenstein's Criterion is often applied. Additionally, a field is defined as a ring in which every non-zero element has a multiplicative inverse, and field extensions involve expanding a field to include new elements beyond those in the original field.

Lecturer Performance

Lecturer performance in classroom of abstract algebra refers to the effectiveness of the lecturer in simplifying students' understanding of complex and theoretical mathematical concepts such as concept of groups, rings and field, and related structures. It encompasses command of the subject matter, clarity of appearance, and the capability to break down abstract ideas into practicable steps using clear explanations and diverse, relevant examples.

During the in-depth interview one participant emphasized that abstract algebra, despite its highly theoretical and complex nature, can be made accessible if the professor's presentation is simple, clear, and focused on the subject matter. She stressed that clarity does not mean removing the depth of the content but rather presenting it in a way that connects with students' prior knowledge and builds their understanding step by step. A well-structured lecture that stays aligned with the learning objectives helps students navigate difficult concepts without becoming overwhelmed.

Learning becomes truly long-lasting when teachers complement theoretical explanations with practical examples, especially in abstract topics like ring polynomials. Here, one participant said his experience after a classroom lesson on Eisenstein's Criterion, a theorem used to determine the irreducibility of polynomials. In that class, the teacher explained only the theoretical statement of the criterion but did not provide worked examples. However, in the exam, students were asked to apply the criterion to specific polynomials. This created a challenge for those who had memorized the theory but lacked exposure to its practical application.

As a result, although I understood the theorem in words, I could not demonstrate the process for checking polynomial irreducibility through examples, which led to a lower score. This experience highlighted the importance of linking theory with concrete illustrations to strengthen understanding and improve retention. Example: Consider the polynomial $f(x) = x^5+10x^3+25x+15$ over the integers Z. Let's take prime (p) = 5. Now we check the Eisenstein's Criterion such as p divides all coefficients except the leading one the coefficients 1, 0, 10, 0, 25, 15 and 5 divides 0, 10, such as $5|0$, $5|10$, $5|0$, $5|25$, $5|15$. Here p does not divide the leading coefficient 1, i. e. $5\nmid 1$, p^2 does not divide the constant term 15 i. e. $25\nmid 15$. By Eisenstein's Criterion (with p = 5), f(x) is irreducible over Q.

He concluded that the context of polynomial ring, Eistein Criterion is very important for both students and teachers as well as educators.

Moreover, the participant highlighted that the effectiveness of the lecturer performance and directly impacts the simplicity of the learning process for students. Even in abstract topics such as fields, a professor who uses accessible explanations, relatable analogies, and structured examples can transform seemingly complicated material into something understandable. In his view, the difficulty of abstract algebra is often more a matter of *how* it is taught than the inherent nature of the subject itself.

> *One participant expressed that his view the lecturer's performance plays a crucial role in understanding abstract algebra. He explained that even though the subject is highly theoretical, it becomes easier to grasp when the lecturer presents concepts clearly, connects them with conceptual understanding, and uses practical examples. Recalling his experience, he mentioned that when the teacher only explained the theory of Eisenstein's Criterion without examples, students struggled to apply it in classroom learning. He realized that effective teaching involves linking theory with practice, using clear explanations and relevant illustrations, which not only simplifies complex ideas but also enhances long-term understanding.*

He says that, instead of teaching only theorems, it is more effective to teach by providing examples, such as illustrating Galois groups through concrete cases. For example, the polynomial x^2-2 over Q has the splitting field Q(2) whose only nontrivial automorphism sends to giving the Galois group Z_2. Similarly, x^3-2 over Q has the splitting field Q(,ω) is a primitive cube root of unity, and its Galois group is S_3. In the cyclotomic case, x^4-Q has the splitting field Q(i) with Galois group Z_2 and the quartic x^4-2 has the splitting field Q(, i) with Galois group D_8. Such worked examples make abstract concepts easier to grasp and help students connect theory with application, leading to deeper and longer-lasting learning. Using examples in teaching not only aids student learning but also demonstrates a teacher's skill, preparation, and ability to connect theory with practice, reflecting positively on their overall teaching performance.

One participant also emphasized that, when teaching, it is important to connect concepts to the construction of finite fields and give simple, clear examples such as Trace, Norm, and Hilbert's Theorem 90. For instance, over $F_2=\{0,1\}$, the polynomial x^2+x+1 is irreducible, and using it we can construct $F_4 = F_2[x]/(x^2+x+1)$ with elements $\{0, 1, \alpha, \alpha+1\}$, where $\alpha^2 = \alpha + 1$. In this field, the Trace of α to F_2 is $\alpha + \alpha^2 = 1$, and the Norm of α is $\alpha \cdot \alpha^2 = 1$. Hilbert's Theorem 90 can then be illustrated by noting that if an element has Norm 1, it can be expressed in the form $a/\sigma(a)$, where σ is the Frobenius automorphism $\sigma(x) = x^2$. Such examples make abstract algebraic concepts easier to understand and remember.

In abstract algebra, a field becomes an extension field by expanding its elements beyond those of the original field. A lecturer's presentation is effective only when clear and meaningful examples of such extensions like the rational numbers (Q), the real numbers(R), and the complex numbers (C) are provided to help students understand how one field extends another. The construction of finite fields, together with the concepts of Trace, Norm, and Hilbert's Theorem 90, offers a clear framework for grasping the structure and relationships within field extensions. These tools not only deepen theoretical knowledge but also provide practical methods for solving problems in algebra and related areas.

Teaching Methods

Effective teaching of abstract algebra at the graduate level involves a combination of diverse methods to address its abstract and challenging nature. Clear, well-structured lectures paired with interactive discussions allow students to engage with definitions, theorems, and proofs while clarifying doubts in real time. First participant said that we need to use Problem-Based Learning (PBL) which encourages critical thinking by assigning challenging problems that require applying theoretical concepts. Using concrete examples and visualizations, such as internal and external products, students can actively engage by constructing examples themselves. For instance, they might work in groups to build the external direct product $Z_2 \times Z_3$ by listing all ordered pairs and defining addition component wise. This hands-on activity helps students understand how combining smaller groups creates a new group structure. Later, they can explore whether this external product corresponds to an internal product inside a larger group, deepening their grasp of group composition., Collaborative learning through group work and peer discussions promotes deeper understanding by facilitating the exchange of ideas and resolving misunderstandings.

In this context, second participants explored about the difficulties of nilpotent and solvable groups, students did not benefit without guided inquiry by investigating concrete examples. A practical case is the group of upper-triangular 2×2 matrices with entries in Zp. By examining the commutator subgroups, students discover that this group is nilpotent. This direct analysis helps them connect abstract definitions to concrete algebraic objects, improving conceptual clarity and appreciation of group properties.

When studying polynomial roots, problem-based learning engages students in applying theory to find solutions. For instance, given the polynomial $x^3 - 2$, learners can find its roots in the complex numbers and visualize them as points on the complex plane. This geometric understanding not only explains the landscape of complex roots but also links algebraic and visual conceptual understanding, making the learning experience comfortable.

Third participant clarified that guided proof-writing workshops provide step-by-step instruction and feedback, strengthening students' logical reasoning and mathematical writing skills. In the topic of finite field construction, visualization and modeling assist students in comprehending abstract ideas. By constructing F_4 as $F_2[x]/(x^2+x+1)$, students openly list the field's elements and express operations like addition and multiplication. This actual action transforms the theoretic process of establishing finite fields into an interactive learning task that nurtures profounder understanding for us.

> *He expressed that the teaching method plays a dynamic role in learning abstract algebra, as the subject's complexity entails clear explanation, guidance, and active engagement. He also shared that when teachers use methods like problem-based learning, group discussions, and guided discovery-writing, students can better link theory with practice. Practical examples such as constructing groups, exploring nilpotent structures, or building finite fields make abstract ideas more understandable and meaningful. He emphasized that interactive and example-based teaching helps transform difficult topics, like radical or cyclotomic extensions, into learning experiences that are easier to grasp and more enjoyable, enabling deeper conceptual understanding.*

Fourth participant viewed that the radical extension is very complex in abstract algebra it needs to guide from the teacher side. For radical extensions, teachers need to concrete the examples of solving the quadratic equation $x^2 - 3 = 0$ by adjoining to Q which illustrates how radical extensions work. Students reflect on how extending fields by roots leads to new elements and solutions, helping us appreciate the role of radicals in solving polynomial equations.

Finally, the fifth participant explained that they felt very sad when the teacher taught about cyclotomic extensions in class because they were unable to understand the topic. However, such complex concepts can be taught more effectively through interactive lectures supported by concrete examples. For instance, students can actively construct the field Q(ζ3) by adjoining a primitive cube root of unity $\zeta_3 = e^{2\pi i/3}$ to the rational numbers Q. I could then study its minimal polynomial and explore its Galois group.

Assignment and Assessment

Assignments and assessments in abstract algebra means to the numerous methods used to assess learners' conceptual understanding of the subject content. In this context, assignment includes regular attendance, classroom learning and participation, the first internal assignment, the second internal assignment, and the final internal assignment, together making whole 40% of the entire score. Moreover, the assessment of these assignments often depends on factors such as regular attendance and active participation in class practices, which inspire students to engage reliably with the course material. These assessments are designed to evaluate not only rote memorization but also conceptual understanding, problem-solving skills with collaboration, and the ability to apply abstract algebraic structures. The remaining 60% of the marks arise from examinations conducted and evaluated by the Faculty of Education (FOE). This division ensures that both ongoing learning behaviors and formal testing contribute to the final grade, providing a balanced assessment of student performance in abstract algebra.

The first participant shared that regular classroom attendance helped them understand the concept of assignments. They regularly and actively participated in classroom learning, and the teacher encouraged both them and the entire class throughout the sessions. After two months of classes, the participant attended the first terminal assignment and scored 4 marks out of 10. During this assessment, the questions focused mainly on group theory, particularly on nilpotent groups and solvable groups. A few questions were also asked from ring theory, such as those related to Eisenstein's Criterion and irreducible polynomials. The participant admitted that they were unable to solve all the questions because of the abstract nature of the course, although they acknowledged that the teacher taught with full dedication and effort.

During the in-depth interview, the second participant stated that her experiences in classroom learning were mixed: some theorems were easy to understand, while others were very difficult. She explained that the internal assignments played an important role in preparing for the final examination. In her view, the final internal assignment served as a valuable aid, making the preparation process easier. She recalled that she practiced all the theorems related to groups, rings, fields, and extensions of fields during this period, which helped her reinforce the concepts, even though some remained challenging. She further noted that the abstract and rigorous nature of the subject made it difficult to fully understand the concepts and solve related problems during the examination. For example, topics such as the Fundamental Theorem of Algebra, the Galois group of a polynomial, separable polynomials, and cyclotomic extensions were particularly abstract and challenging for her to grasp.

The third participant also reflected that my undergraduate experience with learning mathematics was not very helpful. At the community college in my district, there were no teachers available to teach mathematics, which meant I had to study the subject entirely on my own. This made it extremely challenging to understand worked-out examples through self-study alone. Although I was unable to attend all regular classes, I still managed to maintain 80% attendance, which met the semester system's requirement. Classroom discussions were regarded as highly beneficial for the final examination, as most of the objective questions were based on topics discussed in class. Over time, particularly after my first terminal examination at the graduate level, I gradually developed more effective study strategies and learned to prepare thoroughly for examinations despite limited instructional support. Additionally, the first assignment played a significant role in enabling me to engage with and study the entire course in a systematic and comprehensive manner.

I consistently studied abstract algebra and took the final exam administered by the Faculty of Education (FOE). I managed to solve all the questions asked in the exam, demonstrating my strong effort. Although I hope to receive a good result, I am still unsure about the actual outcome of my performance.

When I collected data from the fourth participation, he said, at the beginning, I found the internal assignment quite challenging due to the unfamiliarity with the content and the level of effort required. However, as I continued to study, my understanding improved, and the process became more enjoyable. Through regular practice, I developed the habit of submitting assignments on time, which enhanced my sense of responsibility and time management skills. The internal assessment also played a crucial role in preparing me for the final examination conducted by the Faculty of Education (FOE), enabling me to perform well in that test. From this experience, I conclude that regular assignments and assessments not only foster continuous learning but also enhance students' preparedness, ultimately contributing to the achievement of higher academic results.

> *"In my view, assignments were very effective for learning, but at first, I found them quite difficult because I didn't know how to complete the first one properly. Later, with the support of teachers and friends, I was able to complete the assignments successfully during my studies, and my scores improved gradually. However, I believe our assessment system still needs further improvement by incorporating more project-based, seminar-based, and workshop-based learning approaches".*

Finally, the fifth participant stated that internal assignments are a key factor in developing knowledge in abstract algebra, as it provides opportunities to work with examples, enhance skills in presenting assignments in the classroom, and encourage searching for worked-out examples related to theorems. However, at the graduate level, students often have multiple courses, research responsibilities, and sometimes jobs or internships. Frequent internal assignments can therefore create an excessive workload, leading to stress and burnout. Sometimes, the emphasis may shift toward finalizing the assignments hurriedly to meet deadlines rather than completing deeply it, which can reduce the quality of learning and studying. Furthermore, if assignments are too rigid or repetitive, it might bound originality, critical thinking, and autonomous study skills that are indispensable at the graduate level where deeper exploration is expected. Also, an overload of internal assignments can reduce the time available for more substantial and independent scholarly actions.

Moreover, internal assignments and assessments play a crucial role in supporting students' learning of abstract algebra by encouraging consistent engagement with complex concepts such as groups, rings, and fields. Regular assignments encourage students to search for relevant examples and counter examples, apply theorems into examples, and connect abstract ideas to practical situations in classroom practices, thereby deepening their understanding. Regular and timely assessments provide valuable feedback on their improvement in conceptual understanding, and helping them identify strengths, weakness and areas for improvement while reinforcing long-term retention of knowledge. This continuous process of practice and evaluation not only strengthens problem-solving skills but also nurtures self-regulating learning behaviors, finally enhancing students' command of abstract algebras' classroom practices.

Discussion

The findings of this study designate that learners' complications in abstract algebra at Tribhuvan University are strongly influenced by prior knowledge, lecturer performance, teaching methods, and the nature of assignments and assessments, all of which can be understood over the lens of Cognitive Load Theory (Sweller, 1988). The students through lacking prior mathematical basics mostly in set theory and their properties, functions and it's types, number theory and basic concepts of algebra are more likely to practice high inherent cognitive load, as they must concurrently cope with unfamiliar symbols, definitions, and theorems (Paas & van Merriënboer, 1994).

Moreover, lecturer performance plays a key role in handling this load; when explanations are unclear or examples are disconnected from students' experiences, the unnecessary cognitive load increases in classroom practices, distracting mental resources from meaningful learning (Chandler & Sweller, 1991). It indicates that lecturer performance is essential to remove the difficulty of abstract algebra classroom practices. In particular, when an abstract algebra teacher conducts a lesson with examples, learners can simply grasp and retain concepts only algebraic if the teacher clarifies them efficiently and visibly, while also linking new ideas to their prior knowledge. Furthermore, teaching methods that depend on seriously on abstract formalism without support or visual representation may overcome working memory, while collaborative and problem-based methods can improve relevant cognitive load by supporting schematic construction (Kirschner, Sweller, & Clark, 2006). Moreover, the nature of assignments and assessments also effects learning consequences; tasks that highlight rote procedures over conceptual understanding of abstract might fail to strengthen the mental structures needed for higher-order reasoning, whereas well-designed assignments can serve as spaced classroom practice, encouraging long-term retention (Sweller, Ayres, & Kalyuga, 2011). In this context, these results suggest that improving lecturer clarity and simplicity for instructional strategies, adopting scaffolding, aligning assignments with theoretical and practical goals, and addressing gaps in prerequisite knowledge are indispensable for reducing cognitive overload and supporting for abstract algebra learning.

Conclusion

The study explored the factors behind students' difficulties in abstract algebra: evidence from university students. An ethnographic research design was employed, using semi-structured, in-depth interviews as the primary tool for data collection. Based on the analysis of the interview data, four major factors were identified as the main causes of students' learning difficulties in abstract algebra: prior knowledge, lecturer performance, teaching methods, and the nature of assignments and assessments.

Prior knowledge emerged as the main conclusion of this research. It includes familiarity with set theory, operations on sets, functions, and logical reasoning, all of which are essential for understanding the core components of group theory. Without such foundational knowledge, students struggle to comprehend the abstract and formal nature of the subject.

Regarding lecturer performance, the study found that although abstract algebra is inherently theoretical and complex, many lecturers are unable to simplify teaching techniques or provide relevant examples and counterexamples related to theorems. The participants emphasized that the effectiveness of a lecturer's performance directly influences the ease and simplicity of students' learning processes.

In terms of teaching methods, the study revealed that collaborative learning through group work and peer discussions promotes a deeper understanding by facilitating the exchange of ideas and resolving misconceptions. Participants also explained that guided proof-writing workshops offering step-by-step instruction and feedback strengthen students' logical reasoning and mathematical writing skills. Abstract concepts can be taught more effectively through interactive lectures supported by concrete examples.

Finally, the assignment and assessment system was found to play a vital role in students' learning. The system includes internal assessments such as assignments, presentations, class participation, and periodic tests, along with final examinations that measure deeper conceptual and problem-solving abilities. The evaluation of assignments often depends on students' regular attendance and active participation, encouraging consistent engagement with the course material.

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