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Research Article

Principal Curvature: Modeling With Mathematica and JavaScript

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Abstract

This is participatory action research (PAR) explored master level students' concept image and their understanding of principal curvature, in the course 'Differential Geometry'. The steps used in PAR were P-plan, A-Action, O-Observation, and R-reflection (PAOR). This study focused on answering the question "How do students understand the concept of principal curvature while using Mathematica and JavaScript modeling?". To answer this question, the study is based on University Campus, Central Department of Mathematics Education, Tribhuvan University during the 2024 academic year. The participants of this study were thirteen master's level students (five girls and eight boys). The study followed the iterative cycles of PAR to understand how Mathematica and JavaScript visualization help students to build a concept image of mathematical understanding. In this study, observation checklist, students' interactions and notes, and concept maps as artifacts were used as the data collection tools. The collected data were transcribed, and thematic coding was used to analyze the collected information. Based on the data analysis, the findings of this study showed that Mathematica and JavaScript-based interactive visualizations provided a bridge to connect 'concept image and concept definition' through engaged and interactive participation while learning Differential Geometry.

Keywords: JavaScript, mathematica, participatory action research, principal curvature, virtual learning environments

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Introduction

The teaching learning practice of mathematics education is based on a tendency that students focus on procedural understanding but lacking conceptual image. As a university teacher, I have a consistent observation of this tendency, in a geometry course I teach, most of the content are abstract in nature and students frequently rely on procedural understanding only.

In mathematics, when concepts are difficult to visualize, students often memorize rules and algorithms without a deep understanding. This kinds of rote memorization is also discussed in a literature (Erol & Saygı, 2024; Sulastri et al., 2021). This creates a self-reinforcing cycle where the abstract nature of the content promotes rote learning. It is also mentioned in literature that, "dynamic visualization can enhance formation of concept image, support for better understanding, and guide for learning motivation"(Bos & Wigmans, 2025). However, it is also mentioned that, 'if visuals are not aligned with pedagogical thoughtfulness, or if not with clear learning goals, it can also hinder learning outcomes'(Bos & Wigmans, 2025).

Modern students exhibit a strong preference for visual learning styles that involve direct interaction, preferring to see, to touch. This fundamental shift in learning preferences is due to growing use of internet resources, GenAI and use of mathematical software. This shift highlights the need to move beyond traditional, static and text only teaching methods in mathematics to more visual and simulation-based teaching. In a research it is mentioned that, "a good understanding of mathematics can be achieved through use of manipulatives" (Caniglia & Meadows, 2025).

Mathematica and JavaScript, as a programming language, is widely applicable in creating "interactive visual content" and is supported by a rich ecosystem of libraries, supporting the development of dynamic tools in web platform. These

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Mathematica and JavaScript, as a programming language, is widely applicable in creating "interactive visual content" and is supported by a rich ecosystem of libraries, supporting the development of dynamic tools in web platform. These libraries are equally beneficial for the course 'Differential Geometry'.

In addition, Virtual Learning Environments (VLE) provide additional platform for integrating interactive digital tools, offering a structured and flexible learning environment to deliver contents mathematics education. VLEs offer numerous advantages, including enhanced engagement, increased flexibility, personalized learning pathways, improved accessibility, and the development of essential technology skills (Navarro-Ibarra et al., 2017).

Digital interactive tools embedded within VLE, such as virtual manipulative, digital simulation can be instrumental in facilitating students activities for deeper exploration of mathematical concepts (Dhakal, 2023). In a research, it is mentioned that "digital game can be an effective tool to enhance conceptual schema development" (Alkan & Ada, 2024). In similar fashion, this study can be useful to visualize abstract nature of content in Differential Geometry, as I perceived, as researcher.

Geometry in higher education is aimed to develop 'spatial awareness, geometrical intuition and the ability to visualize', as I experienced. If students struggle to form accurate mental images through text only based resources, their internal representations may incomplete, because these new generations, also called generation-z are heavily rely on digital visuals. This "visualization gap" in academic environment an compels them to rely on inaccurate definitions, which can leads to the formation of common misconceptions, such kinds of discussions are part of number of literatures (Dhakal, 2019; Hernández-Delgado et al., 2022). The same type of problems is seen in my classroom experience too.

Hence, the fundamental problem of mathematics education is not just the abstractness itself, but the absence of effective, interactive tools or pedagogical methods to transform the abstract concept into something visually and tangibly comprehensible digital and interactive resources, as explained by Dhakal(Dhakal, 2023).

Therefore, I prioritized using interactive visual mediators that directly bridge this 'visualization gap', thereby laying a stronger and better foundation to help student to build better conceptual understanding.

Methods

This paper is based on participatory action research (PAR) aimed to improve conceptual understanding in mathematics through JavaScript integrated digital simulation-based learning resources. The research question was "How do students understand the concept of principal curvature while using Mathematica and JavaScript modeling"?

PAR is used in this study, as it emphasizes practitioner involvement, iterative improvement, and a direct focus on content (Denzin & Lincoln, 2018; Lawson et al., 2015; McNiff & Whitehead, 2012). This approach aligns with the goal of improving teaching practices using TPACK based learning content to enhance student's learning outcomes.

PAR is fundamentally a form of inquiry where practitioners, including teachers and students, together they investigate and evaluate their own professional practice to improve intended learning outcomes. The Jean McNiff model for this action research provides a concrete, 4-step cyclical process for implementing PAR(Denzin & Lincoln, 2018; Lawson et al., 2015; McNiff & Whitehead, 2012).

PAR is characterized as 'bottom up' and 'inside out' research, fostering a partnership between practitioners, and other stakeholders(Creswell et al., 2018; Denzin & Lincoln, 2018; Lawson et al., 2015; McNiff & Whitehead, 2012), if applicable. Crucially, it defines all participants as 'experts with important knowledge and perspectives'. This emphasis on collaboration, empowerment, and inclusivity is essential for effectively involving both students and teachers in the research process.

This PAR study also used Technological Pedagogical Content Knowledge (TPACK) (Mishra et al., 2007) framework for technology integration based "participatory-action-reflection cycle" as its theoretical framework(McNiff & Whitehead, 2012). The steps used are plan, action, observation, and reflection (PAOR) based on TPACK framework.

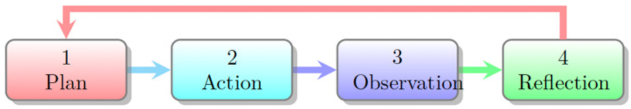


Figure 1: *TPACK based PAOR cycle*

The participants of this study were thirteen masters' degree students (five girls and eight boys) studying at University Campus. The academic session was 2024. In this study, data were collected through observation checklist, students interactions and notes, and concept maps as artifacts (Creswell et al., 2018; Denzin & Lincoln, 2018). These concepts maps are identified as a tool for students to 'show their understanding through visual representation, help them to connect concept image to the concept definition'. The collected data were analyzed based on meaningful codes and verbatim.

Results

In Differential Geometry, the study of surfaces involves understanding of intrinsic and extrinsic properties. These properties are about surface's shape, curvature, and topology. In number of text book, it is defined that "a surface is 'two-dimensional manifold', which explains that, surface resembles a flat plane locally around each point, but globally it may have more complex structure and curvature" (Pundir et al., 2021).

As mentioned in number of textbooks, "a surface is defined as locus of points (x,y,z) whose Cartesian coordinates x,y,z are function of two independent parameter, say u and v" (Dhakal & Koirala, 2024).

It is written as

"x= f(u,v), y= g(u,v), z= h(u,v)" (Dhakal & Koirala, 2024).

In vector form, "surface is explained as locus of point (x,y,z) whose position with respect to origin O is function of two independent parameter u and v"(Pundir et al., 2021).

It is written as

$\vec{r} = \vec{r}\,(u,v)$

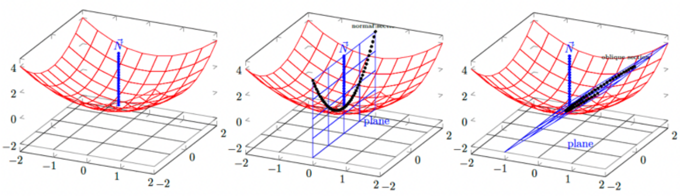
In implicit form, "surface is defined as locus of points (x,y,z) whose Cartesian coordinate satisfy an equation of the form

F(x,y,z)=0.

Based on the definitions given above, intrinsic property of a surface is defined as "invariant, inherent or unchanging property of surface" (Pundir et al., 2021) . Some common example of non-intrinsic properties of a surface are given as "(a) Normal Vector: the orientation of the normal vector is determined by the surface's embedding in space (b) Curvature of sections" (Pundir et al., 2021). These are the "measure of how the surface bends in the surrounding space"(Dhakal & Koirala, 2024).

Cycle 1: normal section

The first cycle of this participatory action research involved the design and initial implementation of interactive visualizations of section of the surface. In reading text, it is written that, "give a surface S: = (u,v) and be a plane at P on the surface. Then SP describes a curve C on the surface, called section of the surface"(Dhakal & Koirala, 2024; Pundir et al., 2021). In this definition, if the plane P is spanned by and , then C is called normal section. Otherwise, C is called oblique section of the surface.



(a) Normal to surface $\vec{N}$ (b) Normal section (c) Oblique section

Figure 2: *Section of the surface*

The JavaScript code of this visualization is as below.

```
board.create('view3d', [ [-6, -3], [8, 8], [[-5, 5], [-5, 5], [-5, 5]]],
view.create('parametricsurface3d', [ (u, v) => u, (u, v) => v, (u, v) =>-4+0.2*u**2+0.2*v**2,
[-4,4], [-4, 4]], {strokeColor: 'green'
view.create('parametricsurface3d', [ (u, v) =>u, (u, v) => v, (u, v) =>-4+u, [-2,4], [-4, 4]], {});
view.create('curve3d', [(t) =>2.5 + 2.5*Math.cos(t), (t) => 2.5*Math.sin(t), (t) =>2.5*Math.cos(t)-1.5, [1,5.25]], {});
```

the visualization is available at

www.bedprasaddhakal.com.np/2024/08/local-non-intrinsic-property-of-surface.html.

The visualization given in Figure 2 illustrated the different ways a surface can be sectioned by a plane. When student used this content through JavaScript visualization, one student reported that "...Looking at the figure, I came to know that there are two types of sections, some a normal line, and some do not contain a normal line ".

It is mentioned in textbook that, a normal vector is perpendicular to the tangent plane of the surface at that point. This visualization has provided concept image of a normal to the surface. Further, another student said that "... intersection of the plane and surface form a curve is called 'normal section' if the plane is containing the surface normal. I also learned about 'oblique section', the section which does not contain the normal line".

In summary, based on both quotations, it can be said that

JavaScript visualization helped student to distinguishes between normal and oblique sections of a surface based on whether the cutting plane contains the surface's normal vector at the point of interest.

Using the visualization, student also became able to understand that there are infinitely many normal sections on the surface. In a normal section, they reported that

$\vec{N} = \vec{n}$

Cycle 2: Normal curvature

In this cycle, students were asked to explain normal curvature. In the textbook, normal curvature is defined as, "given a surface $S:\vec{r}=\vec{r}(u,v)$ and C be a normal section on it, then curvature of C is called normal curvature, which is denoted by n"(Dhakal & Koirala, 2024; Pundir et al., 2021).

It is measure of how the surface bends in a particular direction. Also, the expression of normal curvature is given by

=

Using the Mathematica code, the researcher computed variants of normal curvature during the class, then student became aware that, as section varies, the curvature varies. The mathematics code is given below.

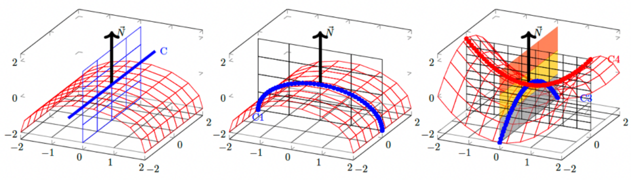
```
r[u_, v_] := {u, v, u^2 - v^2};
Manipulate[
Module[{ru, rv, ruu, ruv, rvv, E, F, G, L, M, N, du, dv, kappa, n},
ru = D[r[u, v], u] /. {u -> 0, v -> 0};
rv = D[r[u, v], v] /. {u -> 0, v -> 0};
ruu = D[r[u, v], {u, 2}] /. {u -> 0, v -> 0};
ruv = D[r[u, v], u, v] /. {u -> 0, v -> 0};
rvv = D[r[u, v], {v, 2}] /. {u -> 0, v -> 0};
E = ru.ru; F = ru.rv; G = rv.rv;
n = Normalize[Cross[ru, rv]];
L = ruu.n; M = ruv.n; N = rvv.n;
du = Cos[\[Theta]]; dv = Sin[\[Theta]];
kappa =(Ldu^2+2 M du dv+N dv^2)/(E du^2+2 Fdu dv+G dv^2);
Row[{"\[Kappa]n(\[Theta]) = ",
NumberForm[kappa, {5, 3}]}]], {{\[Theta], 0,
"Direction Angle (\[Theta])"}, 0, 2 Pi}]
```

Since Mathematica code licensing software, the code is converted to JavaScript code to visualize normal section and normal curvature in webpage for dynamic simulation. The JavaScript is open source and compatible with HTML for webpage deployment.

Cycle 3: Principal section

In the third cycle, the focus of the PAR deployment is to visualize principal sections. In textbook (Pundir et al., 2021), principal section is define as "given a surface S:=(u,v) and P be a point on it, then there are infinitely many normal sections at P, the two sections, which give maximum and minimum curvature at P, are called principal sections at P"(Dhakal & Koirala, 2024; Hernández-Delgado et al., 2022).

These principal sections are always orthogonal.



(a) Principal section (b) Principal section (c) principal sections

Figure 3: *Principal section of the surface*

```
r[u_, v_] := {u, v, 0.5 u^2 - 0.5 v^2};
p0 = r[0, 0];
ru = D[r[u, v], u];
rv = D[r[u, v], v];
nVec = Cross[ru, rv] /. {u -> 0, v -> 0};
t1 = Normalize[ru /. {u -> 0, v -> 0}];
t2 = Normalize[rv /. {u -> 0, v -> 0}];
sectionDirection[\[Theta]_] := Cos[\[Theta]] t1 + Sin[\[Theta]] t2;
normalSectionCurve[\[Theta]_] := Module[{dir, gamma},
dir = sectionDirection[\[Theta]];
gamma[s_] := r @@ (s*dir[[1 ;; 2]]);
ParametricPlot3D[gamma[s], {s, -1.5, 1.5}]];
rotatingPlane[\[Theta]_] :=
Module[{dir = sectionDirection[\[Theta]]},
ParametricPlot3D[p0 + s*dir + t*nVec, {s, -2, 2}, {t, -2, 2}]];
surfacePlot = ParametricPlot3D[r[u, v], {u, -2, 2}, {v, -2, 2}];
normalArrow =
Graphics3D[{Blue, Thick, Arrow[{p0, p0 + Normalize[nVec]}]}];
Manipulate[
Show[{surfacePlot, normalSectionCurve[\[Theta]],
rotatingPlane[\[Theta]], normalArrow}], {{\[Theta], 0,
"Rotate Plane (\[Theta])"}, 0, 2 Pi}]
```

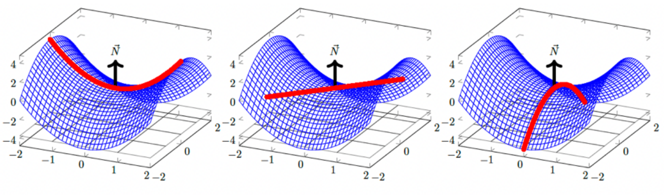


Figure 4: *Normal section at difference rotation*

While student used these visualization, given in Figure 4, it helped them to understand the concept definition, "two sections, which have maximum and minimum curvature at P, are called principal sections at P"(Dhakal & Koirala, 2024; Hernández-Delgado et al., 2022; Pundir et al., 2021). This phenomenon was discussed with students during interview. Then a student reported that 'Oh! I get it now, curves that bend the most and the least, those are the principal sections! So, normal section with the largest and smallest curvature are called the principal sections, ...ummmm, ..right?'

Based on the quotation, it can be said that Mathematica and JavaScript visualization supported students to explain the principal sections effectively. This helped student to understand about the equation of principal section too, which is explained as

(EM-FL)du^2+(EN-GL)dudv+(FN-GM)dv^2=0 .

In discussion, it is also concerned that principal directions are always orthogonal, student were able to verify the relation of orthogonality, saying that principal directions are always orthogonal

ER-QF+GP=0

Using the responses given by students, the study resulted that Mathematica and JavaScript coded visualization gave better idea to the students that the tangents to principal sections are principal directions. Since 'two principal sections (in general) exist, there are two principal directions (in general) that exist at every point on a surface and it will be always orthogonal', which is also explained in textbooks(Dhakal & Koirala, 2024; Pundir et al., 2021).

Cycle 4: Principal curvature

In the fourth cycle, the focus is given to the principal curvature. In textbook, it is given that, "given a surface S: $\vec{r}=\vec{r}(u,v)$ and P be a point on it. Then there are infinitely many normal curvatures at P. Now, two curvatures, which are maximum and minimum at P, are called principal curvatures at P"(Dhakal & Koirala, 2024; Pundir et al., 2021).

These two curvatures are denoted by

k_a and k_b

These maximum and minimum curvatures always occur at right angles to one another. The corresponding radii of principal curvatures are called principal radii and are denoted by

P_a and P_b

The visualization given in Figure 3 and Figure 4, it helped students to understand the dynamics of principal curvature. They experienced that these maximum and minimum curvatures always occur at right angles to one another. Also, it is discussed the differential equation of principal curvature, which is reported as

$k_n^2(EG-F2)-k_n(EN-2FM+GL)+(LN-M^2)=0$

In this definition, the first curvature is

$$\kappa_a+\kappa_b=\frac{EN-2FM+GL}{EG-F^2}$$

The mean curvature is

$$\frac{\kappa_a+\kappa_b}{2}=\frac{EN-2FM+GL}{2(EG-F^2)}$$

The gaussian curvature is

$$\kappa_a\kappa_b=\frac{LN-M^2}{EG-F^2}$$

Findings and Discussion

The findings of this PAR indicated that interactive JavaScript-based visualizations enhance students' conceptual understanding. For example, normal sections and its normal curvature. Similar results are also discussed in a literature, which has mentioned that, "dynamic visuals can support visual imagery as concept image, ir support for better understanding, and can be a guide for learning motivation"(Bos & Wigmans, 2025).

The PAR was found instrumental to this research because it helped in the student's mathematical progression by treating each iterative loop of Plan, Action, Observation, and Reflection (PAOR) to scaffold the "concept image" of principal curvature. This iterative process helped students to distinguish the four fundamental characteristics (a) normal and oblique sections (b) normal curvature (c) principal sections and (d) principal curvature. The loop was based on PAOR framework in different directions through dynamic Mathematica and JavaScript simulations.

In this research, the essence of 'learning motivation' has also been justified. The visualization has given an opportunity to the students to analyze how a plane intersects a given surface so that they were able to differentiate between normal and oblique sections as discussed above. The visualizations helped to form a thoughtful 'concept image' of the 'concept definition'. There are similar findings in literature which argue that "visual helps to form concept image"(Alkan & Ada, 2024; Erol & Saygı, 2024; Sulastri et al., 2021).

Using this PAR, and interactive visualizations using JavaScript and Mathematica, students were able to explore and understand the concept of normal curvature and principal sections through engaged and interactive participation. As explained by TPACK theory(Herring et al., 2014; Mishra et al., 2007), in this research students observed how rotating planes intersect the surface to produce different curvatures. These kinds of simulation helped them to identify the principal sections by drag and drop hands on experience. Therefore, it is argued that "pedagogical and thoughtful integration of technology use can support students' conceptual understanding", which is explained by number of research works(Becerra-Romero et al., 2019; Caniglia & Meadows, 2025; Dhakal, 2023). The JavaScript visual dynamics conveyed a message to the students that 'tangents to principal sections are called principal directions, and that these directions are orthogonal'. These dynamics helped students to reinforcing textbook definitions and enhanced mathematical intuition.

Based on iterative use of PAR cycles, students were able to edify the concept of principal curvature, which as explained by them, refers to the maximum and minimum values of normal curvature at a point P on a surface S: $\vec{r}=\vec{r}(u,v)$. They conceptualized that, these curvatures, denoted by and , occur in orthogonal directions and their reciprocals are known as the principal radii and , which aligned with textbook definition, for example Dhakal & Koirala; Pundit et all(Dhakal & Koirala, 2024; Pundir et al., 2021).

The Mathematica and JavaScript visualizations enabled students to dynamically explore how these curvatures vary with direction and to identify the directions in which they attain their extreme values. Students were engaged with interactive simulations and formalize the differential equation of principal curvature, which is

$$2(EG - F2) - (EN - 2FM + GL) + (LN - M2) = 0$$

Based on the equation, students were able to derive key curvature measures. They also learned that the sum of principal curvatures given by

$$\kappa_a + \kappa_b = \frac{EN-2FM+GL}{EG-F^2}$$

the mean curvature is

$$\frac{\kappa_a+\kappa_b}{2} = \frac{EN-2FM+GL}{2(EG-F^2)}$$

and the Gaussian curvature is

$$\kappa_a\kappa_b = \frac{LN-M^2}{EG-F^2}$$

These explorations were achieved through aided dynamic visualizations. So, the result supported that student was able to connect geometric intuition with analytical expressions, while using dynamic visualizations. However, the study acknowledge the essence of technology with TPACK thought, which argue that, "visuals should be used aligning with clear curricular goals"(Bos & Wigmans, 2025).

Based on results and discussions, this research found that student better understand the concept of principal curvature while using Mathematica and JavaScript modeling through the concept image and connecting it with analytical expressions, as concept definition.

Conclusion and Recommendations

This PAR, conducted through four iterative cycles, demonstrated the effectiveness of interactive visualizations in enhancing students' understanding on 'principal curvature'. Based on results and discussions, it is concluded that, Mathematica and JavaScript visualization helped students to distinguish normal and oblique sections, to identify principal sections and directions, and to comprehend the analytical expressions of principal curvature.

Based on the findings, it is recommended that curricular content should include technology-enhanced learning modules that allow students to manipulate and explore algebraic and geometric concepts dynamically. Further research could be conducted to evaluate the long-term impact of such visual tools on students' mathematical thinking and problem-solving abilities.

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Conflict of Interest

The author declares that there are no conflicts of interest associated with the content of this article.

Data Availability

The visual materials and interactive resources that support the findings of this study are publicly accessible through www.bedprasaddhakal.com.np/2024/08/local-non-intrinsic-property-of-surface.html.

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