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Lexicographic Static FlowLoc with/without Excess Storage

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Abstract. Maximizing the flow value in a multi-terminal (multiple sources and multiple sinks) network using lexicographic supply from sources to sinks, reduce conflicts in the movement of flows toward the destinations. However, installing facilities on arcs reduce the arc capacities, which may hinder the maximum flow value. Nevertheless, such installations are necessary to supply facilities to the commodity. On the other hand, if intermediate nodes have storage capacity, the excess flow which cannot reach the sinks, can be stored at these nodes. As a result, the overall flow value can be improved.

In this paper, we integrate the concepts of facility allocation togather with intermediate storage and introduce lexicographic maximum static flow problems with and without intermediate storage. We investigate their corresponding flow models. The multi-facility problems are solved using heuristic approaches, whereas polynomial-time algorithms are presented for the single-facility problem.

Keywords: Lexicographic flow, FlowLoc, intermediate storage, $\mathcal{NP}-$completeness, heuristic

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1. Introduction

Motivation. The lexicographic maximum static flow (LMSF) problem in a transportation network topology involves maximizing the simultaneous movement of a commodity flow, like people, goods, etc., from the fixed origin (sources) to the prescribed destination (sinks) without violating the capacity constraints. In the lexicographic maximum flow framework, flows are optimized according to the predefined priority patterns at the sources and sinks, such that the higher-priority terminals are satisfied first. Also, the flow location (FlowLoc) problem involves the installation of facilities at optimal locations. The main objective is to facilitate the commodity and minimizing the reduction in the network's maximum flow value. On the other hand, due to the minimum cut capacity, it is often not possible to fully utilize the outgoing arc capacities from the source. However, if intermediate nodes have storage capacity, the excess flow that cannot reach the sinks can be stored there. As a result the full arc capacity can be utilized. The integration of the models is very useful in the transportation network for optimizing the movement of the flow from the source to the sink by supplying the facility to the commodity.

Literature review. The maximum flow problem deals with the largest possible flow that can be transshipped from the source to the sink. The maximum static flow problems was first introduced and solved by Ford and Fulkerson [1]. They proved that the maximum amount of static flow in a network is equal to the bottleneck capacity. The maximum flow problem in abstract network was solved in [2]. The various maximum flow problems applied in evacuation planning are presented in [3, 4, 5].

The lexicographic maximum flow (LMF) problem concerns with the maximization of the flow value by a fixed leaving pattern at the sources and reaching pattern at the sinks. The lexicographic maximum static flow (LMSF) problem was solved by presenting a polynomial-time algorithm in [6, 7]. The lexicographic multi-objective network flow problems are found in Calvete and Mateo [8]. The application of lexicographic max flow in the context of evacuation planning can be found in [9, 10, 11]. The different network flow problem and models applicable in evacuation planning are found in survey paper of Dhamala [12].

In a network, if the out going arc capacity from the sources is greater than the minimum cut capacity, then the total capacity cannot be used. Pyakurel and Dempe [13] solved this problem by storing the excess flow at the intermediate nodes. The lexicographic contraflow problems with excess flow storage were solved by Pyakurel et al. [14]. The mathematical model about the fuzzy flow with the excess storage was investigated in [15, 16]. The different network flow problem with excess storage can be found in [2, 17, 18, 19, 20, 21, 22, 23, 24].

The flow location (FlowLoc) problem concerns with the determination of the exact locations for the facilities and optimization of the flow value. The first industrial facility location theory was introduced by Weber [25]. The different discrete facility location models were investigated by Daskin [26]. Hamacher et al. [27] combined the network flows theory with locational analysis and introduced the flow location (FlowLoc) problem. They developed the models and presented polynomial time algorithm and heuristic for single and multi-facility cases, respectively. Dhungana and Dhamala [28] solved maximum static and maximum dynamic ContraFlowLoc problems in polynomial time. The single and multi-quickest FLowLoc problems were solved by Nath et al. [29]. The maximum static and dynamic FlowLoc problems with the excess flow storage were introduced by Dhamala et al. [30]. They solved the single and multi-facility cases by polynomial time algorithms and heuristics, respectively. The lexicographic maximum flow location problem in a single source, multi-sink network was solved by Wagle et al. [31]. Using the temporally repeated approach, the maximum dynamic FlowLoc, earliest arrival FlowLoc and ContraFlowLoc problems were solved by Wagle et al. [32].

Research gap. In the existing literature, mathematical models for the lexicographic maximum static flow (LMSF) problem with and without excess-flow storage as well as the maximum FlowLoc problem have been studied in two terminal network. An integrated approach that deals with these problems with and without excess flow storage in a multi-terminal networks is a major research gap.

Our contribution. For the single as well as multi-facility, the lexicographic maximum static FlowLoc problems with and without excess flow storage are introduced in multi-terminal network for the first time. Polynomial time algorithms and heuristics are presented for the case of single and multi-facilities, respectively.
Organization of the paper. The basic notation used through out the paper are provided in Section 2. The mathematical models of the problems: lexicographic maximum static flow about multiple facility (LMSqFL) and single facility (LMS1FL) are provided in Section 3. This section provides the polynomial time heuristic and algorithm for multiple and single facility cases, respectively. Section 4 provides the results with excess flow storage in a single source and multi-sink network. Section 5 concludes the paper.

2. Preliminaries

Let, G be a static network having the set of vertices V with $|V| = n$, arcs A with $|A| = m$, sources $S = \{s_k, k = 1, 2, \dots, h\}$ with priority order $s_1 \succ s_2 \succ \cdots \succ s_h$ (i.e. s_1 has the first priority and so on), intermediate nodes I and sinks $D = \{d_k, k = 1, 2, \dots, l\}$ with order $d_1 \succ d_2 \succ \cdots \succ d_l$. Each arc $a = (u, v) \in A$ has an attribute, a nonnegative capacity $b_a : A \to \mathbb{Z}^+$ that bounds flow on the arc. Suppose that the set of incoming arc to and outgoing arc from $u \in V$ are denoted by $A^-(u)$ and $A^+(u)$, respectively. If $L \subseteq A$ be the set of possible locations for the given set of facilities $\mathbb{P}$, then the network is denoted by $G = (V, A, L, b_a, S, I, D)$. The functions $r : \mathbb{P} \to \mathbb{N}$ and $\mu : \mathbb{P} \to L$ are the size and allocation of the facilities on $a \in L$, respectively. Assume that the function $\eta : L \to \mathbb{N}$ bound the number of facilities on $a \in L$. The network is reduced to $G^R = (V, A, b_a^R, S, I, D)$ by installing facilities on L, where $b_a^R = b_a - \max\{r_p : \mu(p) = a\}$. Thus, if multiple facilities are installed on the same arc, its capacity is reduced by the largest facility size among them. We want to maximize the flow value in G^R.

3. Lexicographic FlowLoc without intermediate storage

In this section the LMSqFL problem, its integer programming formulation are introduced and solved by heuristic. Further more the single facility problem (i.e. $q = 1$) is solved by polynomial time algorithm.

Problem 1. *Let $G = (V, A, L, b_a, S, I, D)$ be a given network. The LMSqFL problem maximizes the prioritized $S - D$ flow value by locating the q facilities on $a \in L$ where each arc $a \in L$ can hold at most $\eta(a)$ facilities.*

Mathematical formulation. Let G be the given network with static flow function $x_a : A \to \mathbb{R}^+$. Let us define a decision variable

$$y_{ap} = \begin{cases} 1 & \text{if the facility } p \text{ is placed on } a \in L \\ 0 & \text{otherwise} \end{cases}$$

then the mathematical programming formulation of Problem 1 is to maximize Objective 1 subject to Constraints (2)–(7).

$$\max \sum_{a \in A^-(D)} x_a = \sum_{a \in A^+(S)} x_a \tag{1}$$

subject to

$$\begin{aligned}
\sum_{a\in A^+(v)} x_a \quad - \quad \sum_{a\in A^-(v)} x_a = 0, \forall v \in V \setminus \{S,D\} && (2)\\
\sum_{p\in\mathbb{P}} y_{ap} \quad \leq \quad \eta(a), \forall a \in L && (3)\\
\sum_{a\in L} y_{ap} \quad = \quad 1, \forall p \in \mathbb{P} && (4)\\
0 \quad \leq \quad x_a + y_{ap} r_p \leq b_a, \forall p \in \mathbb{P}, a \in L && (5)\\
0 \quad \leq \quad x_a \leq b_a, \forall a \in A && (6)\\
y_{ap} \quad \in \quad \{0,1\}, \forall a \in L, p \in \mathbb{P}. && (7)
\end{aligned}$$

Here, Constraints (2) show the flow conservation to each intermediate vertices. The number of facilities on any location are bounded by Constraints (3). According to Constraints (4), each facility has to be fixed on exactly one location. Constraints (5) and (6) bound the flow on each arc.

Hamacher et al. [27] reduced the q-FlowLoc problem in two terminal network to 3 SAT and proved that the problem is $\mathcal{NP}-$complete. Also the q-FlowLoc in multi-terminal is more complex so is $\mathcal{NP}-$complete (c.f. Theorem 1).

Theorem 1. *The LMSqFL problem is $\mathcal{NP}-$complete*

Note that taking $q = 1$ the problem is reduced to single facility (LMS1FL) problem which is to maximize Objective 1 subject to Constraints (2), (5)–(7).

3.1. **Solution procedure.** In this subsection, the LMSqFL and LMS1FL problems are solved.

3.1.1. *q-FlowLoc.*

. First of all, we compute the lexicographic maximum static flow with fixed priority ordering without placing any facility on the given locations as follows. Let $S_1 = \{s_1\}, S_2 = \{s_1, s_2\}, \ldots, S_h = \{s_k, k = 1, 2, \ldots, h\} = S$ and $D_1 = \{d_1\}, D_2 = \{d_1, d_2\}, \ldots, D_l = \{d_k, k = 1, 2, \ldots, l\} = D$ be the set of sources and sinks, respectively. Then we get the set inclusion, $S_1 \subset S_2 \subset \cdots \subset S_l = S$ and $D_1 \subset D_2 \subset \cdots \subset D_j = D$. According to [7], if $X(D)$ is the maximum flow value that enters the sinks in D, then $X(D_i), i = 1, 2, \ldots, l$ represents the maximum flow value to each D_i, which is a lexicographic maximum static flow on sinks. Similarly, $X(S_i)$ delivers the lexicographic maximum static flow value out from each source $S_i, i = 1, 2, \ldots, h$. We maximize out flow from S, which is pushed to D under a fixed priority ordering. The residual capacities of each location are then calculated and sorted them according to the capacities. Also, the facilities are sorted using their size. Now, the location $e_1 \in L$ with the largest residual capacity is selected and fixed $\eta(e_1)$ facilities there and continue until all the facilities are fixed on L. Note that for any location a_i, if the largest facility to be fixed is $p_k \in \mathbb{P}$ such that $r_{p_k} > b_{a_i}$, then $p_{k_1}, p_{k_2}, \ldots, p_{\eta(a_{i+1})}$ are fixed on the next location $a_{i+1} \in L$, and so on. After placing the given facilities, the network is then reduced to $G^R = (V, A, b^R_a, S, I, D)$. The LMSF is then computed on G^R. Now present a polynomial time heuristic, in Algorithm 1.

Example 1. *Let G be the given network with the set of locations $L = \{(s_2, y_1), (s_2, y_3), (s_1, y_2), (s_1, y_3), (y_1, y_2), (y_2, d_3)\}$ (c.f. Fig. 1(a)). Suppose that $S = \{s_1,\, s_2\}$ and $D = \{d_1,\, d_2,\, d_3\}$ are the set of sources and sinks with order $s_1 \succ s_2$ and $d_1 \succ d_2 \succ d_3$, respectively. Also we have $S \succ D$. Let $\eta(a) = 3$, $\forall a \in L$ and $\mathbb{P} = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$ with size $r_{p_1} = 5$, $r_{p_2} = 4$, $r_{p_3} = 2$, $r_{p_4} = 6$, $r_{p_5} = 1$, $r_{p_6} = 3$, $r_{p_7} = 2$, $r_{p_8} = 1, r_{p_9} = 3, r_{p_{10}} = 4$.*

First of all using the solution procedure of [7], the LMSF is computed in G without placing any facility on L by introducing the super terminal nodes D_1 and D_2 (c.f Fig. 1(b)–1(f)). Using the paths: $D_2 - s_1 - y_1 - d_1 - D_1$, $D_2 - s_1 - y_3 - d_1 - D_1$, $D_2 - s_1 - y_1 - y_2 - d_2 - D_1$, $D_2 - s_1 - y_1 - d_2 - D_1$, $D_2 - s_1 - y_2 - d_2 - D_1$, $D_2 - s_1 - y_2 - d_3 - D_1$, $D_2 - s_2 - y_1 - d_2 - D_1$, $D_2 - s_2 - y_1 - d_3 - D_1$ and $D_2 - s_2 - y_3 - d_3 - D_1$, we can push 27 units of static flow. Of these, 21 and 6 units of flow

Algorithm 1: LMSqFL algorithm

Input : Network G , facilities $\mathbb{P}$, size $r : \mathbb{P} \to \mathbb{N}$.

Output: The LMSqFL, locations.

(1) Obtain the LMSF in G without fixing the facilities on any $a \in L$.
(2) Sort the facilities with their size: $r_{p_1} \succ r_{p_2} \succ \cdots \succ r_{p_q}$.
(3) Sort the locations according to their residual capacities: $a_1 \succ a_2 \succ \cdots \succ a_{|L|}$.
(4) Place the facilities $p_1, p_2, \dots, p_{\alpha(a_1)}$ on $a_1 \in L$ and so on.
(5) When placing facilities at a location $a_i \in L$, if the largest facility that should be fixed on it is $p_k \in \mathbb{P}$. Also if $r_{p_k} > b_{a_i}$, then interchange a_i and a_{i+1} and so on. The network is reduced to $G^R = (V, A, b^R_a, b_a, S, I, D)$.
(6) Compute the maximum static flow with priority ordering in G^R.

are sent from s_1 and s_2, respectively. Consequently the sinks d_1, d_2 and d_3, receive 7,10 and 10 units of flow, respectively. The residual capacities $b^r_a = b_a - x_a$ for $a \in L$ are $b^r_{(s_2,y_1)} = 5$, $b^r_{(s_2,y_3)} = 3$, $b^r_{(s_1,y_2)} = 0$, $b^r_{(s_1,y_3)} = 0$, $b^r_{(y_1,y_2)} = 0$ and $b^r_{(y_2,d_3)} = 4$. Using the residual capacity the locations in L are sorted as: $(s_2, y_1) \succ (y_2, d_3) \succ (s_2, y_3) \succ (s_1, y_2) \succ (s_1, y_3) \succ (y_1, y_2)$. Also the facilities in $\mathbb{P}$ are sorted as there size: $p_4 \succ p_1 \succ p_2 \succ p_{10} \succ p_6 \succ p_9 \succ p_3 \succ p_7 \succ p_5 \succ p_8$.
Using Algorithm 1, (s_2, y_1) gets p_4, p_1, p_2, (y_2, d_3) gets p_{10}, p_6, p_9, (s_2, y_3) gets p_3, p_7, p_5 and (s_1, y_1) gets p_8. After fixing these facilities, the network is reduced to G^R (c.f. Fig. 2(a)). The LMSF is computed in G^R, (c.f.Fig. 2(b))

3.1.2. *1-FlowLoc.* To solve 1-FlowLoc problem, at first, LMSF is computed in G without placing the given facility p on any location $a \in L$ and the residual capacity of each $a \in L$ is calculated. We check, whether there is any $a \in L$ whose residual capacity is enough to host the facility. If such a location exists, p is fixed there. In this case LMSF is not reduced. If we cannot get such location, p is placed on one of the location $a_1 \in L$ and LMSF is computed in G^R. After this, p is removed from a_1, fix it on another $a_2 \in L$ and repeat the process. This process is continued to every remaining $a_i \in L$. Each time we get the lexicographic maximum flow value. Finally, we select the reduced network G^R in which overall the LMSF is maximized. Here, we present Algorithm 2 to obtain the LMS1FL.

Algorithm 2: LMS1FL algorithm

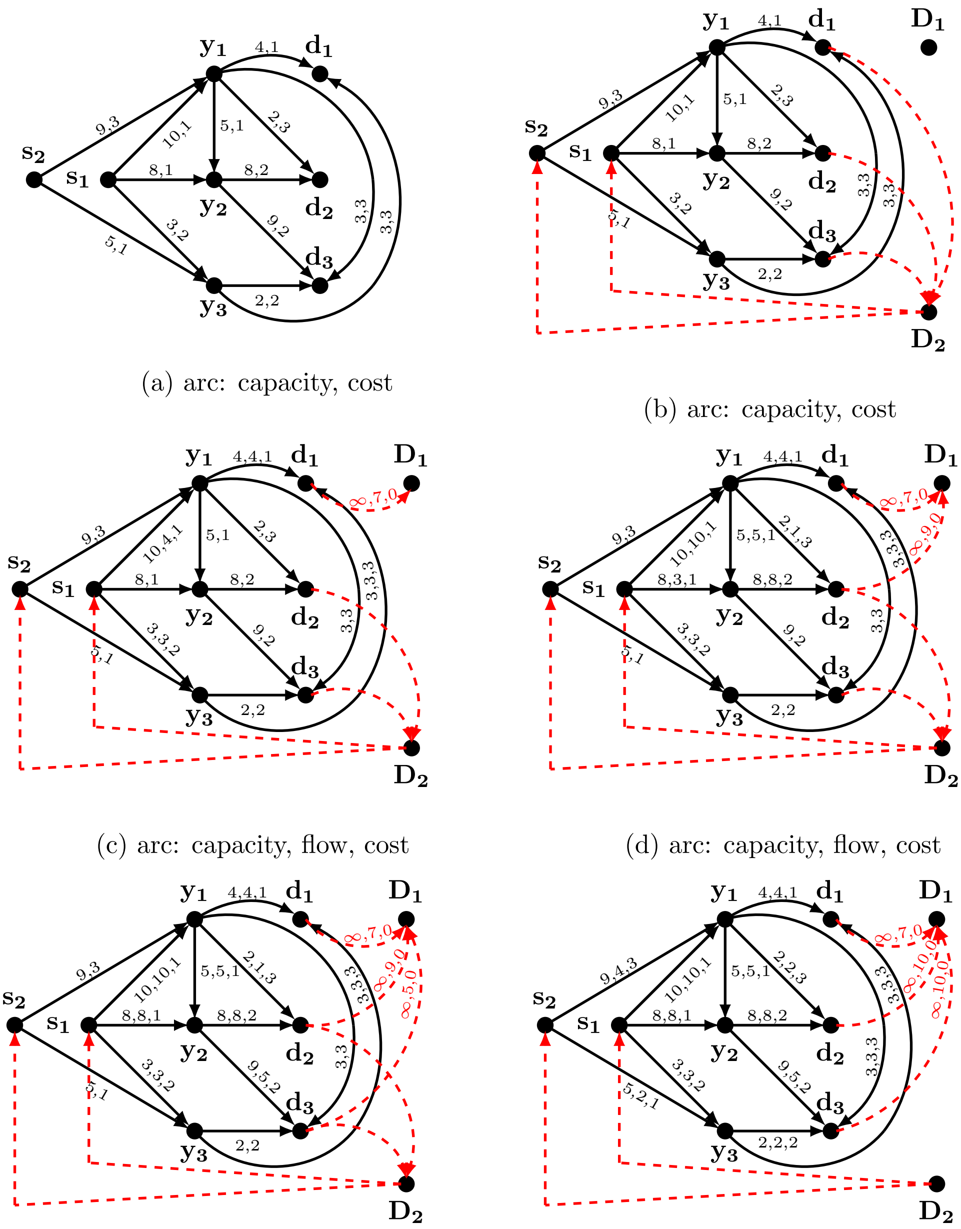
Input : Given network G, location L, facility p with size r_p

Output: The LMS1FL and location.

(1) Compute Steps (1)–(2) to obtain LMSF and residual capacity b^r_a of each $a \in L$.
(2) If there exists an $a \in L$ with $b^r_a \geq r_p$, place p on a. The LMSF obtained from Step 1 is the LMS1FL.
(3) If $b^r_a < r_p \forall a \in L$ proceed as follows.
    (a) Fix the facility p on an $a_1 \in L$, the network is reduced to $G^R = (V, A, b^R_a, S, I, D)$.
    (b) Obtain the LMSF in the network G^R.
    (c) Remove p from a_1, fix it on another $a_2 \in L$, repeat the Steps (3a)– (3b) and continue to each remaining $a \in L$.
    (d) Neglect $a \in L$ if $b_a < r_p$.
    (e) Pick up the location $a \in L$ overall maximum flow value.

Theorem 2. *The LMS1FL problem can be solved optimally in a polynomial time.*

Proof. First of all, we prove the the solution obtained from Algorithm 2 is feasible. Here, locating the facility, constructing reduced network and obtaining the LMSF in the reduced network are feasible.



(a) arc: capacity, cost

(b) arc: capacity, cost

(c) arc: capacity, flow, cost

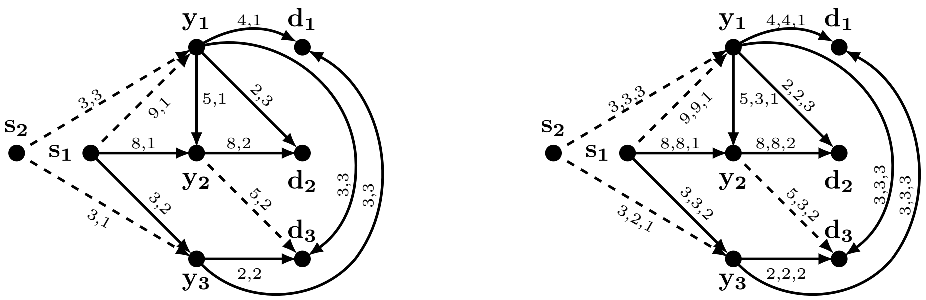
(d) arc: capacity, flow, cost

(e) arc: capacity, flow, cost

(f) arc: capacity, flow, cost

Figure 1. (a) Given network G, (b)–(f) process of obtaining LMSF in G

Now, we prove optimality. Using Algorithm 2, LMSF is computed in G without fixing the facility p, residual capacity b^r_a of each $a \in L$ is calculated and checked whether there exists an $a \in L$ with $b^r_a > r_p$. If such location finds, p is fixed there. The LMSF is LMS1FL. If $b^r_a < r_p$ is found for each $a \in L$, then p is fixed on a randomly selected location $a_1 \in L$, LMSF is computed in G^R as proceeding above and continue to all remaining $a_i \in L$. Each time we get an optimal solution. Finally, we select maximum flow value among these solutions which is the MS1FL solution.
Here, the Lex-maximum flow problem in G can be solved in $O(\delta mn)$ time, where δ is the number of terminals. Since the reduced network can be obtained in linear time, Algorithm 2 solves LMS1FL problem in $O(|L|\delta mn)$ time. $\square$



(a) arc: capacity, cost

(b) arc: capacity, flow, cost

FIGURE 2. (a) Reduced network, (b) LMSqFL solution

Example 2. *Let G be a given network having set of the sources $S = \{s_1, s_2\}$ with order $s_1 \succ s_2$, set of sinks $D = \{d_1, d_2, d_3\}$ with order $d_1 \succ d_2 \succ d_3$ as in Fig. 1(a). Suppose that a facility p with size $r_p = 5$ has to be fixed on the exact location so that the maximum flow can be computed in G. For this, as in Example 1, LMSF is computed in G (which is 27 units) and the residual capacity b_a^r of each $a \in L$ is calculated. Since $b^r_{(s_2,y_1)=5}$, so p is fixed on (s_2, y_1). So there is no loss in the maximum flow value. Hence, the LMS1FL is 27 with the optimal location (s_2, y_1)*

Example 3. *If we consider $r_p = 6$ in Example 2, then $b_a^r < r_p$ for each $a \in L$. So we compute LMSF by fixing the facility p one at a time for each $a \in L$. Also arcs $(s_2, y_3), (s_1, y_3)$ and (y_1, y_2) have capacity less than r_p so are neglected form L. The LMSF values obtained by fixing p on the remaining locations are presented in Table 1.It is observed that the LMS1FL value equals 26, with the optimal location at (s_2, y_1).*

TABLE 1. LMS1FL solution

S.N.	*Location*	*Lex maximum flow vlue*					
		from s_1	*from s_2*	*at d_1*	*at d_2*	*at d_3*	*Total flow*
1	(s_2, y_1)	*21*	*5*	*7*	*10*	*9*	*26*
2	(s_1, y_2)	*15*	*6*	*7*	*9*	*5*	*21*
3	(y_2, d_3)	*21*	*4*	*7*	*10*	*8*	*25*

4. Lexicographic FlowLoc with Storage

In this section, we focus on storing the excess flow at the intermediate nodes. For this, the nodes are prioritized according to their longest distance from the source. In a multi-source and multi-sink network, the farthest node from a source may not be farthest from other sources, the intermediate node may not always be prioritized. So we restrict the network to a single source s while retaining multiple sinks D and extend the results of Sections 3 to the case where excess flow may be stored at intermediate nodes. For this, it is assumed that each intermediate node u has a storing capacity b_u such that $b_u \geq \sum_{a \in B(u)} b_a$. If $x_u : I \to \mathbb{R}^+$ be the static storage flow functions at $u \in I$, then the LMSqFL problem with excess flow storage is defined as follows.

Problem 2. *For the given $s - D$ network G, the LMSqFL problem with excess flow storage maximizes the amount of feasible flow leaving from s that is to be sent towards D plus the excess flow at I, by locating at most $\eta(a)$ facilities on the appropriate location $a \in L$.*

Mathematical formulation. Let G be a given network with fixed priority ordering on the sinks D. The LMSqFL problem with intermediate storage maximizes Objective (8) with respect to the constraints (3)-(7), (9)-(10).

$$\max \sum_{a\in A^+(s)} x_a = \sum_{a\in A^-(D)} x_a + \sum_{u\in I, b_u>0} x_u \quad (8)$$

$$x_u = \sum_{a\in A^-(u)} x_e - \sum_{a\in A^+(u)} x_a \geq 0, \forall u \in I \quad (9)$$

$$0 \leq x_u \leq b_u, \forall u \in I \quad (10)$$

$$\text{together with Constraints} \quad (3)-(7)$$

where Constraints (9) represent the excess flow and Constraints (10) bound the flow at each $u \in I$.

Theorem 3. *The LMSqFL problem with excess flow storage is $\mathcal{NP}$-complete.*

Note that, by taking $q = 1$, the problem is reduced to single facility (LMS1FL) problem with excess flow storage which is to maximize Objective 8 subject to Constraints (5)–(7), (9)–(10).

4.1. **Solution procedure.** In this subsection, we solve LMSqFL and LMS1FL problems with intermediate storage.

4.1.1. *q-FlowLoc.* To solve Problem 2, first we obtain the reduced network G^R as described in Subsection 3.1.1. To obtain the LMSqFL with intermediate storage, the solution procedure of [13] is used. We prioritize I according to the distance dist(s, u) of $u \in I$, create the virtual sink u' of $u \in I$ having dist$(u, u') = 0$ and $b_u = b_{u'}$. With this, the modified network $G^R_M = (V_M, A_M, b^R_a, b_u, s, I_M, D_M)$ is constructed, where I_M is the set of virtual sinks, $V_M = V \cup I_M$, $A_M = A \cup \{(u, u')\}$ and $D_M = D \cup I_M$. Here, G^R_M is a single source and multi-sink network without excess flow storage, where the sinks have priority order: $d_1 \succ d_2 \succ \cdots \succ d_l \succ u'_1 \succ u'_2 > \cdots \succ u'_i$. Note that $u_1 \in I$ farthest from s. Now, we compute the LMSF in G^R_M. Finally, the solution of G^R_M is transformed to G^R by eliminating I_M. Here, Algorithm 3 is presented to solve the problem.

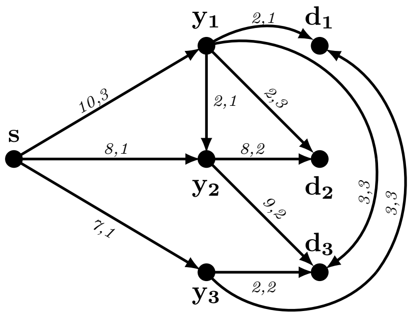
Algorithm 3: LMSqFL with excess flow storage

Input : Given network G, facilities $\mathbb{P}$, size $r : \mathbb{P} \to \mathbb{N}$

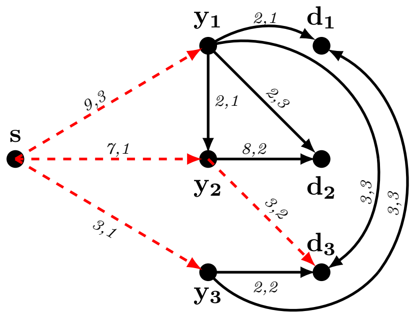
Output: The LMSqFL solution with excess flow storage, and the locations.

(1) Compute Steps (1)- (5) of Algorithm 1 to obtain the reduced network $G^R = (V, A, b^R_a, b_u, s, I, D)$.
(2) Apply the shortest path algorithm to calculate the shortest distance dist$(s, u), \forall u \in I$.
(3) Give the first priority to the prioritized sinks D and next to the prioritized nodes I.
(4) Construct the modified network $G^R_M = (V_M, A_M, b^R_a, b_u, s, I_M, D_M)$ by creating the virtual sinks.
(5) Compute the LMSF in G^R_M, where max flow values at D are already obtained from Step (1).
(6) The solution of G^R_M is transformed to G^R by eliminating the virtual sinks.

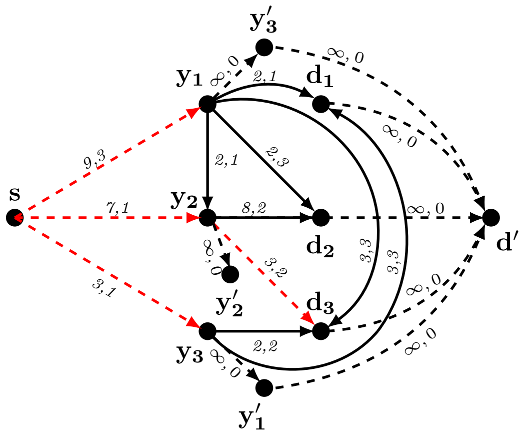
Example 4. *Let G be a given network with source s, set of sinks $D = \{d_1, d_2, d_3\}$ with priority order $d_1 \succ d_2 \succ d_3$ and $L = \{(s, y_1), (s, y_2), (s, y_3), (y_1, y_2), (y_2, d_3)\ (y_3, d_3)\}$ as in Fig. 3(a). Suppose each $a \in L$ can hold 3 facilities. Let the intermediate nodes y_1, y_2 and y_3 have capacities: 10,8 and 9 respectively. Assume that, a set of facilities $\mathbb{P} = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$ with size $r_{p_1} = 5$, $r_{p_2} = 4$, $r_{p_3} = 2$, $r_{p_4} = 6$, $r_{p_5} = 1$, $r_{p_6} = 3$, $r_{p_7} = 2$, $r_{p_8} = 1, r_{p_9} = 3, r_{p_{10}} = 4$ has to fix on L. As in Example 1, LMSF is computed in G and residual capacity b^r_a, $a \in L$ is calculated. It is found that 1,0,2,0,4 and 0 are the residual capacity of (s, y_1), (s, y_2), (s, y_3), (y_1, y_2) and (y_2, d_3), respectively. Hence the priority order of the facilities and locations are: $p_4 \succ p_1 \succ p_2 \succ p_{10} \succ p_6 \succ p_9 \succ p_3 \succ p_7 \succ p_5 \succ p_8$ and $(y_2, d_3) \succ (s, y_3) \succ (s, y_1) \succ (s, y_2) \succ (y_1, y_2) \succ (y_3, d_3)$, respectively. Using Algorithm 3, the facilities: $\{p_4, p_1, p_2\}, \{p_{10}, p_6, p_9\}, \{p_3, p_7, p_5\}$ and p_8 are placed on $(y_2, d_3), (s, y_3), (s, y_1)$ and (s, y_2), respectively. The reduced network G^R is given in Fig. 3(b). The network is then modified to G^R_M (c.f. Fig. 3(c)). Finally the LMSF is computed in G^R_M. The LMSqFL with excess flow storage is 19 (c.f. Fig 3(d)).*



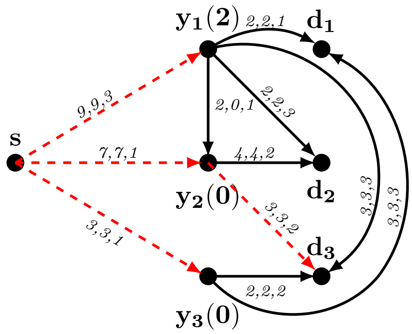
(a) arc: capacity, cost



(b) arc: capacity, cost



(c) arc: capacity, cost



(d) arc: capacity,flow, cost

FIGURE 3. (a) Given network, (b) reduced network, (c) modified network, (d) LMSqFL solution

4.1.2. *1-FlowLoc.* To solve the LMS1FL problem with excess flow storage, as in Subsection 3.1.2, the optimal reduced network G^R is constructed and modified to G^R_M (see Subsection 4.1.1). As in Subsection 4.1.1, LMSF computed in G^R_M. Here, we present Algorithm 4 to solve the problem.

Algorithm 4: LMS1FL with excess flow storage

Input : Given network G, facilities p, size r_p

Output: The LMS1FL with excess flow storage, and location.

(1) Compute Steps (1)- (3) of Algorithm 2 to obtain the reduced network $G^R = (V, A, b^R_a, b_u, s, I, D)$.

(2) Compute Steps (2)-(6) of Algorithm 3 to obtain the solution.

Theorem 4. *The LMS1FL problem with excess flow storage can be solved optimally in a polynomial time.*

Proof. To obtain the LMS1FL with excess flow storage, the reduced network G^R and modified network G^R_M are constructed in polynomial time. Here, G^R_M is a single source multi-sink network without storage at intermediate nodes. Using Theorem 2, the LMSF can be computed in G^R_M in polynomial time. □

5. Conclusions

Maximizing the flow value under a prescribed leaving pattern at the sources and/or a reaching pattern at the sinks (i.e. using lexicographic configuration) reduces conflicts in arc usage and helps ensure smoother flow in the network. The arc capacity is reduced by the size of the facility if we fix it at that arc. This may affect the maximum flow value in the network. However, it is very important to supply facilities to the commodity in the network. Also due to the bottleneck capacity the full arc capacity out from the source may not be used. If the intermediate nodes have storage capacity, then the excess flow can be stored there. This may improve the maximum flow value.

In this paper, integrating these concept, we have introduced the LMSqFL and LMS1FL problems with and without excess flow storage. We have investigated FlowLoc models of these problems. The multi-facility cases are solved by heuristics where as the single facility problem have solved in polynomial time. To the best of our knowledge, we have introduced and solved these problems for the first time. We aim to solve the problem using the installation charge of the facility on the location.

Limitations. The limitations of this study are:

- Priorities of sources and sinks are pre-determined.
- Facility establishment costs are not considered.
- All intermediate nodes are assumed to have sufficient storage capacity.

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