

Simulation of Fundamentals of Space Curves using JavaScript¹

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Abstract

This qualitative case study explores pre-serve teachers understanding on fundamentals of space curve in three-dimensional differential geometry, while studying at master level at Tribhuvan University in Nepal. The Technological-Pedagogical-Content-Knowledge (TPCAK) theoretical model was used to explore their understanding particularly on three fundamental vectors: tangent, principal normal and binormal vectors and three fundamental planes: osculating plane, normal plane and rectifying plane. In this study, there were thirteen participants, who were studying the course “Differential Geometry” using teacher made JavaScript based 3D visualizations. After the analysis of participants understanding on learning content, it is found that they gain enhanced conceptual understanding through interactive-concept-image of the concept definition.

Keywords: Case study, fundamental vector, fundamental plane, JavaScript, TPACK,

Introduction

The information communication technology (ICT) use in mathematics education has been an instrumental tool since decades. Recently, with high number of growing digital tools and mathematical software, it is now possible to visualize higher mathematical abstract concepts into tangible forms. It is said that these kinds of concrete representation of learning content can support students learning and helps to develop concrete to abstract thinking (Ayd, 2005). However, during my teaching that students are struggling to master the contents in differential geometry, which is also discussed in the papers (Dhakal, 2019, 2023).

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In differential geometry, one of the key concepts is “fundamentals of space curve, which includes three-unit vectors: tangent, principal normal and binormal vector and three orthogonal planes: osculating plane, normal plane and rectifying plane. Very often student feel difficulty to uncover the dynamic nature of these fundamentals, for example while point move along the curves in space, what would be the nature of these fundamentals: static or dynamic.

As a teacher of the course “differential geometry”, we feel a research gap between symbolic representation of text and its geometric visualization, which may help learner to understand the concept visually. Next, the students of this course are pre-service teachers, so these kinds of visual and interactive learning experience ultimately help them to become a better teacher and help them to cascade these kinds of visualization in their teaching journey, which ultimately benefit large number of school students in the nation.

Number of literatures have also shown evidence that visualization can support students learning through concept image. For example, Emin (2005) have noted that “ICT based simulations can create realistic scenarios for experimentation, which may be impractical in traditional physical settings”. Ayd (2005) have emphasized to use ICT based tools to materialize the concept of three-dimensional reasoning.

Number of researchers have already shown that digital tools and computer software like Mathematica, MATLAB, Maple, which are related to mathematics education, are instrumental in mathematics learning (Ayd, 2005). But these all need to be used with pedagogical thoughtfulness. So, in this study, a computer program called JavaScript, and its visualization capacity has been used to understand how effectively it can support learner to build mathematical conceptual understanding. In literature it is also said that the visualization and working with algorithm, together it can support to build higher order thinking skills like “critical thinking, decision making, problem solving and metacognition” (Sugilar et al., 2023).

Beyond several digital software, some of which are paid versions, for example MATLAB, Maple, Mathematica. In this study an open-source software which is cost free, which is also universally accessible in computer program, the JavaScript is used. This JavaScript computer program is compatible with any web browser and can visualize three dimensional geometrical concepts through engaged and interactive manner. In literature, it is said that these kinds of interactivity can foster conceptual understanding through experimentation and help to learn on their own pace (Lahteenmaki et al., 2024).

Problem statement

The growing innovation in artificial intelligence in education has reset the students learning culture. The students prefer to navigate digital visualizations to understand concepts. This practice is even growing in the field of mathematics education where most of the concepts are abstract in nature. As a course facilitator, it has been experienced that that master students who are learning differential geometry, are struggling to understand the basic concepts of fundamentals of space curve. While using text definitions from the textbook, concept definitions only, it is being difficult for them to apply the learned concept for further mathematical proofs. Without having a strong foundation and the visualization of fundamentals on the space curve, pre-service math teacher may lack spatial reasoning, even in their future teaching. Therefore, this study explores research-based evidence on “how JavaScript visualization helps students to understand the basics of fundamental of space curves through interactive simulations?”

Research question

This study aims to address a research question:

How do pre-service teachers perceive and visualize fundamental vectors and fundamental planes in the context of space curves, while using JavaScript simulations?

Research objectives

The research objective of this study is:

To explore pre-service teachers’ conceptual understanding of vectors, planes, and space curves before and after using Java Script-based simulations on fundamental vectors and fundamental planes.

Methods

This study is based on qualitative case study design. As Yin (2014) has mentioned that case study helps to uncover evidence grounded in phenomenon. In this study student learn interacting with digital tool like JavaScript simulation. Using this simulation, student can actively engage with learning resources and activities at their own learning pace and pace. So, this study investigates students learning culture using digital tools. The study is about students (pre-service teacher’s) learning perception, interaction and experience while using digital tools, as a case. Therefore researcher has used case study

research design, as suggested in number of research books. (Creswell et al., 2018; Denzin & Lincoln, 2018; Yin, 2014).

In particular, this study explores the pre-service teacher's conceptual meaning making process on fundamental vectors, tangent vector, principal normal vectors and binormal vectors; and three fundamental planes: the osculating plane, normal plane and rectifying plane. The study will be focusing on students learning experience on natural learning environment, but using digital tools, particularly JavaScript simulations, so as Yin (2014) mentioned, case study is applied.

The participant in this study was thirteen pre-service teacher's (eight male and five female), we were studying at master level program in mathematics education. These students were participated form 2024 cohort studying third semester at central department of education, university campus, Tribhuvan University, Nepal.

As a course tutor researcher have provided digital learning resources and learning activities based on JavaScript simulations, which were publicly accessible through researcher's personal webpage <https://www.bedprasaddhakal.com.np/2024/06/fundamentals-on-space-curve.html>. The learning content on the webpage is fully based on the prescribed syllabus including YouTube based videos, dynamic text and JavaScript simulations. The participants of this study used these resources online during the study.

The data collection tools used in this study were researchers note, classroom interaction, participants responses from open ended questions and classroom observations. These tools are parts of the data collection process as prescribed by number of books (Creswell et al., 2018; Denzin & Lincoln, 2018; Yin, 2014). Then collected data were transcribed, and the codes were generated based on the research questions to understand how Java Script simulation contributes on student's understanding on "fundamentals of space curve".

Theoretical framework

This study is about how technology contributes on students learning in higher mathematics. Therefore, technology and pedagogy related theory is considered as theoretical framework, which is TPACK. TPACK is about Technological **Pedagogical Content Knowledge and how it integrates in teaching learning process. This framework was developed by** Mishra and Koehler (Herring et al., 2014; Mishra et al., 2007) to highlight the interplay between three domains of knowledge: Technological

Knowledge (T), Pedagogical Knowledge (P), Content Knowledge (CK). Therefore, this study used TPACK as a lens to analyze the respondents' responses on how they understand the mathematical concepts using technology as learning tool.

Results

This section provides information, findings and results of this case based qualitative study. The course contents of "Differential Geometry Math Ed 537" were developed in web-based learning contents including you tube videos and java script simulations. The essence of this learning resource was to help students to learn the concepts in more engaged and better interactive manner. To support the student's easy access to the digital materials, as a researcher and web content developer, I have guided them to explore "what and where" so that can find the related information in the need. It is informed to the students that they can use it at their own learning pace, at their own time and their own interaction level.

During the course learning period, participants' classroom interactions, informal talks, classroom observations and their reflections on learning were collected as data and based on these data major results are extracted and discussed. Based on the prescribed textbook, it is mentioned that, "let $t \in [a; b]$ be a real parameter then a curve in 3D space is defined as the locus of a point $P(x; y; z)$ whose Cartesian coordinates x, y, z are a function of t " (Dhakal & Koirala, 2024; Pundir et al., 2021), which is given by

$$x = f(t); y = g(t); z = h(t)$$

In the vector form, this space curve is defined as

$$\vec{r} = \vec{r}(t)$$

In implicit form, space curve is defined as intersection of two surfaces of the form

$$f_1(x; y; z) = 0; f_2(x; y; z) = 0$$

In the space curve in three-dimensional geometry, fundamental vectors describe how physical objects on the path changes along different directions such as velocity and acceleration etc. These are tangent, principal normal and binormal vector. For example, a helix or spiral-like path in 3D space is a space curve given by

$$\vec{r} = (t \cos t, t \sin t, t)$$

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the Mathematica coded tangents to this curve are given below.

$$r[t_]:= \{3 \text{ Cos}[t], 3 \text{ Sin}[t], t\}$$

$$uT[r_][t_]:= r'[t]/\text{Sqrt}[r'[t].r'[t]]$$

These codes are further visualized in webpage by JavaScript as below-

```
view.create('curve3d', [(t) => 3 * Math.cos(t), (t) => 3 * Math.sin(t), (t) => t, [-5, 5]);
view.create('curve3d', [(t) => 3 * Math.cos(2) - 3 * t * Math.sin(2), (t) => 3 * Math.sin(2) +
3 * t * Math.cos(2), (t) => 2 + t, [-1, 3]]);
```

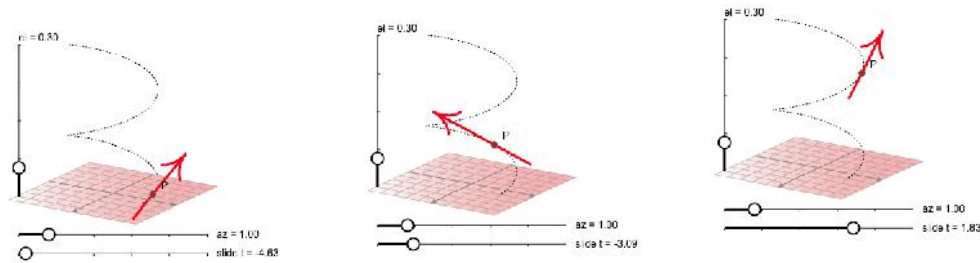


Figure 1: tangent is a local properties, see webpage url

The figure above illustrated students concept image of a tangent vector to a curve. At each point on the curve, the direction in which the curve is heading is given by this tangent vector, which is calculated as

$$\vec{T} = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$$

where $\vec{r}'(t)$ is the derivative of the position vector with respect to the parameter t , and $\|\vec{r}'(t)\|$ is its magnitude, which is explained in prescribed *references* (Dhakal & Koirala, 2024; Pundir et al., 2021).

During the informal talk while students are manipulating the visualization (shown in Figure 1), a student responded that

“... the tangent vector shows the instantaneous direction of the curve at a specific point, as the point moves along the curve, the direction of the tangent vector also changes, so the tangent vector is not static, it is dynamic”.

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These responses demonstrated that tangent vector is a local property, as it depends only on the behavior of the curve at that specific point, which is experienced by the students while learning with JavaScript simulations. In textbook it is mentioned that “let $C: \vec{r}=\vec{r}(t)$ be a space curve in which P and Q be neighboring point. Then tangent at P is defined as limiting position of a secant PQ as $Q \rightarrow P$ ” (Dhakal & Koirala, 2024; Pundir et al., 2021), and such tangent is unique at each of its points, it is found that this concept definition is extended to core concept image through the Java Script visualizations.

Based on classroom discussion, student also explored about three fundamental vectors \vec{T} , \vec{N} , and \vec{B} , which are three mutually perpendicular unit vectors, so are also called orthogonal triads, these vectors move in a space curve along positive directions and satisfy the relations given as

$$\vec{T} \cdot \vec{T} = 1; \vec{N} \cdot \vec{N} = 1; \vec{B} \cdot \vec{B} = 1$$

$$\vec{T} \cdot \vec{N} = 0; \vec{N} \cdot \vec{B} = 0; \vec{B} \cdot \vec{T} = 0$$

$$\vec{T}_x \vec{T} = 0; \vec{N}_x \vec{N} = 0; \vec{B}_x \vec{B} = 0$$

$$\vec{T}_x \vec{N} = \vec{B}; \vec{N}_x \vec{B} = \vec{T}; \vec{B}_x \vec{T} = \vec{N}$$

Similarly, to ensure the concept image of normal and principal normal, following Mathematica code is converted in Java Script to visualize it as dynamic applet, The snapshot the 3D JavaScript visualization of normals and principal normal as given in Figure 2.

$$uN[r_][t_]:=uT[r]'[t]/\text{Sqrt}[uT[r]'[t].uT[r]'[t]]$$

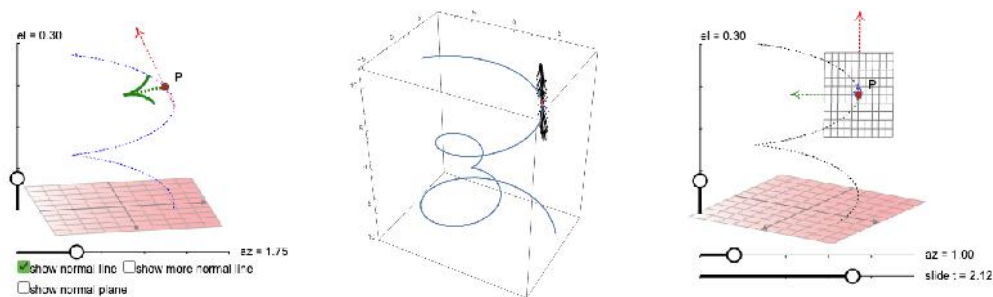


Figure 2: principal normal lies in osculating plane, see web url

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The Figure 2 illustrated that the concept of normal lines to a space curve and highlights the special role of the principal normal line. As mentioned in a literature, tangent vector $\vec{T}(t)$ gives the direction of the curve, where as normal line is any line that passes through P and is perpendicular to the tangent vector at that point P (Herring et al., 2014; Mishra et al., 2007). While students are using this dynamic JavaScript applet, a student replied that

“There are infinitely many normal lines at point P because infinitely many directions are perpendicular to the tangent vector in three-dimensional space. Among these, the principal normal line is special one, which lies in the osculating plane”.

Based on the visualization, student explained that “let $C: \vec{r}=\vec{r}(t)$ be a space curve and P be a point on it, then normal at P is a vector perpendicular to the tangent at P. For three-dimensional curves in space, there are infinitely many normal vectors at a given point. Now, principal normal at P is a normal lying in the osculating plane at P” which directly sync with the textbook definitions as well, for example Dhakal & Koirala (2024) and Pundir et al. (2021). We denote the unit vector along this principle normal by \vec{n} .

Similarly, to ensure the concept image of binormal, following Mathematica code is converted in JavaScript to visualize binormal as dynamic applet, the screenshot of this interactive simulation is given in Figure 3.

$$uB[r_][t_]:=uT[r][t]\text{ \textbackslash [Cross] }uN[r][t]$$

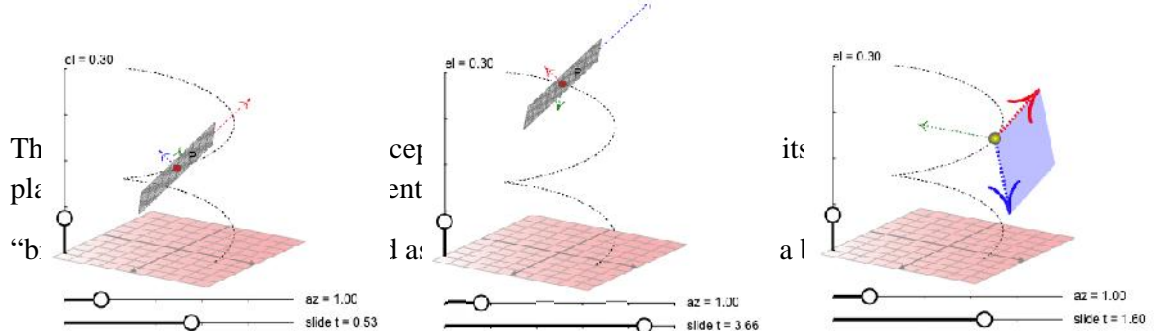


Figure 3: binormal is perpendicular to osculating plane

so,

While students are using the dynamic JavaScript applet as shown in Figure 3, a student mentioned that

“...the osculating plane is formed by the tangent and normal vectors. The binormal vector is perpendicular to tangent vector, which is also perpendicular to normal vectors, and hence, binomial vector is perpendicular to the osculating plane”.

This geometric relationship, as student experience through dynamic visualization, is fundamental in the Frenet–Serret frame, which also described the local behavior of space curves. It is found that JavaScript visualization helped students to understand the symbolic representation of a relation

$$(\mathbf{R} - \vec{r}) \cdot \vec{B} = 0$$

In this equation, \mathbf{R} vector represents position of arbitrary point on the osculating plane.

Based on classroom discussion, student became able to give a precise definition of fundamental unit vectors, one of the responses is

“let $C: \vec{r} = \vec{r}(t)$ be a space curve and P be a point on it, then binormal at P is a normal perpendicular to the osculating plane at P , and the unit vector along the binormal line is denoted by \vec{b} ”.

With the responses, it resulted that student could explain the equation of fundamental vectors in line with textbook definitions, as they mentioned that

equation of tangent is

$$\mathbf{R} = \vec{r} + \lambda \vec{T}$$

equation of principal normal is

$$\mathbf{R} = \vec{r} + \lambda \vec{N}$$

equation of binormal is

$$\mathbf{R} = \vec{r} + \lambda \vec{B}$$

In these equations, \mathbf{R} vector, as general variable, represents position of arbitrary point on the tangent, principal normal and binormal respectively.

The further discussion led to a result that common fundamental planes are osculating plane, normal plane, and rectifying plane, and these fundamental planes are also local properties. These conception help students understanding on how the curve twists in space curve and so it determines the dynamics of fundamental planes, which is also explained in references (Dhakal & Koirala, 2024; Pundir et al., 2021).

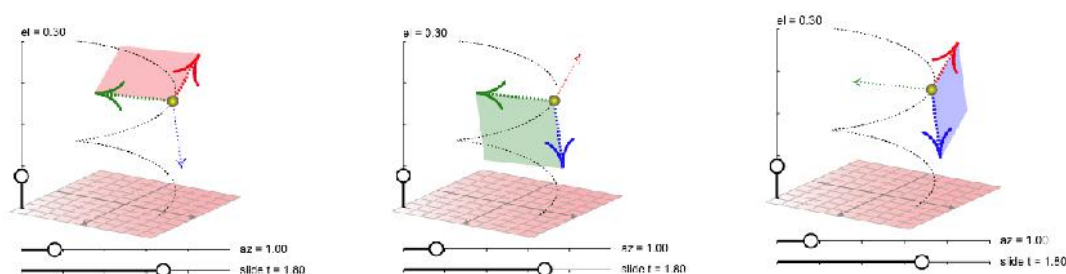


Figure 4: fundamental planes

The Figure 4 illustrates a screenshot of Java Script simulation, which shows three fundamental planes associated with a space curve, such as osculating plane normal plane, and rectifying plane. These planes are defined fundamental vectors: tangent vector $\vec{T}(t)$, principal normal vector $\vec{N}(t)$, and binormal vector $\vec{B}(t)$.

While students are using the dynamic JavaScript simulation, as given in Figure 4, a student replied that

“... the osculating plane is defined by the tangent vector $\vec{T}(t)$ and the principal normal vector $\vec{N}(t)$, the normal plane is determined by the principal normal vector $\vec{N}(t)$ and the binormal vector $\vec{B}(t)$, and the rectifying plane”.

Also, another student noted that

“..seeing the curve rotate in real-time in this visualization helped me understand how

the tangent and normal vectors define the osculating plane, which is formed by the tangent vector $\vec{T}(t)$ and the binormal vector $\vec{B}(t)$ ".

Based on the quoted text of the participants responses, it is found that JavaScript simulation help student to understand the concept image of fundamental planes along with visualization, and helped them to understand the equation of fundamental planes in symbolic notation, which is the beauty of mathematics, and these are given as the equation of normal plane is

$$(\mathbf{R}-\vec{r}).\vec{T} = 0$$

the equation of rectifying plane is

$$(\mathbf{R}-\vec{r}).\vec{N} = 0$$

the equation of osculating plane is

$$(\mathbf{R}-\vec{r}).\vec{B} = 0"$$

In these equations, R vector, as general variable, represents position of arbitrary point on the normal plane, rectifying plane and osculating plane respectively.

Using all quotations as references, the result of this study showed that, Java Script simulation let students to manipulate the parameter through and able to understand the dynamics of fundamentals vectors one by one and the interplay of three fundamental planes. So major discussions and findings of this study are presented in the next headings.

Discussion and Findings

The results of this case based qualitative study has showed a significant insight to both educators and researchers that "how do a student learn mathematics and conceptualize textbook based "concept definition" through an interactive simulations or dynamic visualizations and own "concept-definition".

Though this study is based on a content of university level mathematics, which is "fundamental vectors, fundamental planes and associated dynamics", this study has opened a new area for further research and discussions, if similar approach can be utilize in other content area both in school and university level, it can accelerate students "concept-definition" of their learning for better understanding.

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During this study, the data analysis revealed that students got benefited to sync abstract symbolic representations of their “concept-definition” and formed own “concept-image” of the dynamics of “fundamental planes and fundamental vectors”.

In a literature Kim & Md-Ali (2017) have discussed that “technology supports mathematics learning, because technology supports in enhancing understanding, fostering higher-order thinking skills through visualization, and so it can increase student learning engagement” (Kim & Md-Ali, 2017). This opinion has also been justified by this research, because it is seen that approximately 70% (9 of 13) of the research participants of this research who were initially struggled to correctly identify the directions of fundamental vectors, they became able to understand the dynamics easily after the manipulating the JavaScript visualizations.

In some literature like Herring et al (2014) and Mishra et al (2007), they have mentioned that “kinesthetic interaction enhances learning conceptualization”. This opinion has been further justified by this research study. It is seen that JavaScript visualization supported the essence of cognition theory, suggesting that kinesthetic interaction enhances conceptualization, based on TPACK theory (Herring et al., 2014; Mishra et al., 2007), which suggest that technology integration is essential in this 21st century learning experience.

Notably, the students after utilizing the Java Script simulation, they demonstrated improved conceptual understanding on fundamentals of space curve, particularly the fundamental vectors and fundamental planes. This kinds of saying that “technology integration can support in student mathematics learning” is also discussed in number of literatures by the author itself (Dhakal, 2019, 2023).

In a research by Sugilar et al. (2023), they have reported that “students demonstrate good metacognition abilities when solving high-level mathematical problems with computer applications, benefiting from their speed, efficiency, accuracy, and visualization capabilities” (Sugilar et al., 2023). This findings alliance with this research however, the symbolic nature of mathematics, which is the core-cruX of mathematics, must align together while presenting visualization. This study found that, the visualization helps students to derive symbolic equations of fundamental planes and fundamental vectors. For evidence, approximately 40% of the respondents (5 out of 13), who initially struggled, now became able to derive symbolic equations independently.

This is due to the overall engagement with the interactive materials of JavaScript simulation.

This study found an important aspect for technology use in pedagogy, as mentioned by TPACK and connectivism. It affirms the saying that technology use can support student centered learning, let student learn independently, it is also discussed by Weinhandl et al. (2020). Further, the integration has been promising in teacher education programme, for example, number of research work have mentioned about it (Becerra-Romero et al., 2019; Navarro-Ibarra et al., 2017; Phoong et al., 2020). It is said that technology integration essential because it support on conceptual clarity and also prepare learners for workplace to be able to effectively use technology in their work performance (Sugilar et al., 2023).

Therefore, the study found that student-engagement-first approach is essential in pedagogy. This approach is also supported by the vision of design-based teaching methodologies, for example Anderson & Shattuck (2012) and Wang & Hannafin (2005). They have argued that symbolic only teaching should be complemented by engaged and interactive visualizations. It is also exemplified that technology use supports learner to be prepared in manifesting 21st-century learning opportunities, as mentioned by Kim & Md-Ali (2017).

However, it is known that there are despite potential benefits of using technology in pedagogy, for example as mentioned by TPACK and connectivism, but there should be clear synchronization in task (Weinhandl et al., 2020), the task should enable for "optimal thinking" or a deeper understanding of mathematical concepts to support effective problem-solving (Sugilar et al., 2023), and these task should be rapped by personal in-person feedback from teachers and classmates (Weinhandl et al., 2020).

Based on the findings of this research and discussing with number of sources, future studies are essential to further understand and optimize the role of technology, particularly JavaScript based visualizations. Among the several, in which existing research basically focuses on geometry and calculus, future studies should investigate JavaScript's effectiveness in different learning areas to understand whether these findings generalize to other students or different cultural contexts, or in different content areas.

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