LINEAR OSCILLATION OF THE INTER-CONNECTED SATELLITES UNDER THE INFLUENCE OF MAGNETIC FORCE OBLATENESS OF THE EARTH AND THE SHADOW OF THE EARTH DUE TO SOLAR RADIATION PRESSURE ABOUT THE POSITION OF EQUILIBRIUM FOR SMALL ECCENTRICITY NEAR THE MAIN RESONANCE =1

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ABSTRACT

This paper is devoted to study the non-linear oscillation along the stable position of equilibrium where the system oscillates about the position of equilibrium like a dumb-bell satellite with constant amplitude and phase varying with true anomaly. B.K.M. method has been exploited to get the general solution valid at and near the main resonance n = 1.

KEYWORDS : Okhotsimsky, non-linear, Equilibrium, Resonance, Oscillatory, Orbit, Solar radiation, gravitational.

INTRODUCTION

The Russian Mathematician Okhotsimsky and Saricher (1963) made significant studies on the effect of different perturbative forces on a cable connected satellites system. Similar problems have also been studied in details by Singh; R.B. (1972).

1. Linear Oscillation of the system about the position of equilibrium for small eccentricity.

The equation of motion of a satellite in the central gravitational field of force of oblate earth together with magnetic force and shadow of the earth due to solar pressure in polar form is given by

$$(1 + e\cos v)\Psi'' - 2e\Psi' \sin v + 3 \sin\Psi \cos\Psi = 2\, e \sin v + 5A\,(1 + e\cos v)^2 \sin\Psi\ \cos\Psi + B_0P^3\,(\cos\alpha\sin\Psi - \sin\alpha\cos\Psi) + C[1 + e\cos v) \sin\Psi - e \sin v \cos\Psi] \ldots (1)$$

Where, $A =$ - (const.) oblatness parameter

$B_O =$

B = Solar pressure parameter

C = Magnetic force parameter

$P =$

Where e is the eccentricity, p is the focal parameter, v is true anomaly of the centre of mass and das has denote differentiation w.r. to v

The equilibrium position is given be

$$\phi = \phi_0 = 0 \text{ and } \Psi = \Psi_0 = \frac{\dfrac{-B \sin.\theta \sin\alpha}{\pi}}{3 - 5A - C\dfrac{B \sin\theta \cos\alpha}{\pi}}$$

$$= \frac{B_0 \sin\alpha}{3 - 5A - C - B_0 \cos\alpha} \ldots \ldots(3)$$

The equation of small oscillation about the position of equilibrium obtained by putting $\Psi = \Psi_0 + \eta$ upto first order infinitesimal is given by

$$\eta'' + n^2\eta = e\,[2\sin v - \eta'' \cos v + 2\eta' \sin v + 10A\left\{\eta - \frac{B_o\sin\alpha}{3 - 5A - C - B_0\cos\alpha}\right\}\left\{1 + \frac{\eta B_o\sin\alpha}{3 - 5A - C - B_0\cos\alpha}\right\}\cos v + c\cos v$$
$$\left\{\eta - \frac{B_0 \sin\alpha}{3 - 5A - C - B_0\cos\alpha}\right\} + 3B_o\sin\alpha\left\{1 + \frac{\eta B_o\sin\alpha}{3 - 5A - C - B_o\cos\alpha}\right\}\cos v - C\sin v\left\{1 + \frac{\eta B_o\sin\alpha}{3 - 5A - C - B_o\cos\alpha}\right\} \ldots (4)$$

Where,

$$n^2 = (3 - 5A)\left\{1 - \frac{B_0^2 \sin\alpha}{(3 - 5A - C - B_0\cos\alpha)^2}\right\} - B_o\cos\alpha + \frac{B_o^2\sin^2\alpha}{3 - 5A - C - B_o\cos\alpha} - C \ldots (5)$$

Now let us construct the general solution of the oscillatory system based on B.K.M. method which will be valid at and near the main resonance n = 1. Assuming 'e' to be a small parameter, the solution in the first approximation of the equation (4) at the main resonance n = 1 can be sought in the form :

$$\eta = a\cos k$$

$$\text{Where, } k = v + \theta \ldots(6)$$

Where amplitude 'a' and phase 'θ' must satisfy the system of ordinary differential equations –

$$\left.\begin{aligned} \frac{da}{dv} &= e\,A_1(\alpha, \theta) \\ \frac{d\theta}{dv} &= n - 1 + eB_1(a, \theta) \end{aligned}\right\} \ldots (7)$$

Where amplitude 'a' and phase must satisfy the system of ordinary differential equations

Here $A_1(a, \theta)$ and $B_1(a, \theta)$ are periodic solutions periodic with respect to θ of the system of partial differential equations:

$$(n - 1)\frac{\partial A_1}{\partial\theta} - 2anB_1 = \frac{1}{\pi}\int_o^{2\pi} f_o(v, \eta, \eta', \eta'')\cos k\,dk$$
$$\text{and } a(n - 1)\frac{\partial B_1}{\partial\theta} + 2nA_1 = \frac{1}{\pi}\int_o^{2\pi} f_o(v, \eta, \eta', \eta'')\sin k\,dk \ldots (8)$$

Where $f_o(v, \eta, \eta', \eta'') = 2\sin v + an^2\cos k\cos v - 2an\sin k\sin v + 10\,A\,a\cos k\cos v$

$$\left\{1-\frac{B_o^2\sin^2\alpha}{(3-5A-C-B_o\cos\alpha)^2}\right\}\left\{\frac{1-B_o^2\sin^2\alpha}{(3-5A-C-B_o\cos\alpha)^2}\right\}$$

$$-3B_0 a\cos k\cos v\cos\alpha+\frac{3B_0^2\cos v\sin\alpha\cos\alpha}{3-5A-C-B_0\cos\alpha}+3B_o\cos v\sin\alpha+\frac{3B_0^2\sin^2\alpha\, a\cos k\cos v}{3-5A-C-B_o\cos\alpha}+c\,a\cos k\cos v$$

$$-\frac{CB_o\cos v\sin\alpha}{3-5A-C-Bo\cos\alpha}-\frac{CB_o\sin v\sin\alpha\ a\cos k}{3-5A-C-Bo\cos\alpha}c\sin v \quad \ldots.. (9)$$

Substituting the value of $f_0(v, \eta, \eta', \eta'')$ from (9) on the R.H. side of (8) and then integrating, we get from (8)

$$\left.\begin{aligned}(n-1)\frac{\delta A_1}{\delta\theta}-2anB_1&=-\mu\sin\theta+v\cos\theta\\ \text{and } a(n-1)\frac{\delta B_1}{\delta\theta}+2nA_1&=-\mu\cos\theta+v\sin\theta\end{aligned}\right\}\ldots (10)$$

Where $\mu = 2 - c$

and $\nu = 3B_o\sin\alpha + \left(\frac{3B_0\sin\alpha\cos\alpha - B_0C\sin\alpha}{3-5A-c-B_0\cos\alpha}\right)$ …. (11)

Now, the periodic solution with respect to θ of the system of equation (10) can be easily obtained as

and
$$\left.\begin{aligned}A_1&=-\frac{1}{(n+1)}(\mu\cos\theta+v\sin\theta)\\ B_1&=\frac{1}{a(n+1)}(\mu\sin\theta-v\cos\theta)\end{aligned}\right\}\ldots\ldots\ldots(12)$$

Using (12) in (7), we obtain,

$$\left.\begin{aligned}\frac{da}{dv}&=\frac{-e}{(n+1)}(m\cos\theta-v\sin\theta)\\ \text{and }\frac{d\theta}{dv}&=(n-1)-\frac{e}{(n+1)}(-\mu\sin\theta+v\cos\theta)\end{aligned}\right\}\ldots\ldots.(13)$$

The system of equations (13) can be written as

$$\frac{d\alpha}{d\nu}=\frac{1}{a}\frac{\partial\phi}{\partial\theta}=-\frac{1}{a}\frac{\partial\phi}{\partial\theta}\ \ldots\ldots (14)$$

Where $\phi=\frac{\alpha e}{n+1}(\nu\cos\theta-\mu\sin\theta)-\frac{(n-1)a^2}{2}$

Clearly, $\phi=\phi_0$ is the first integral

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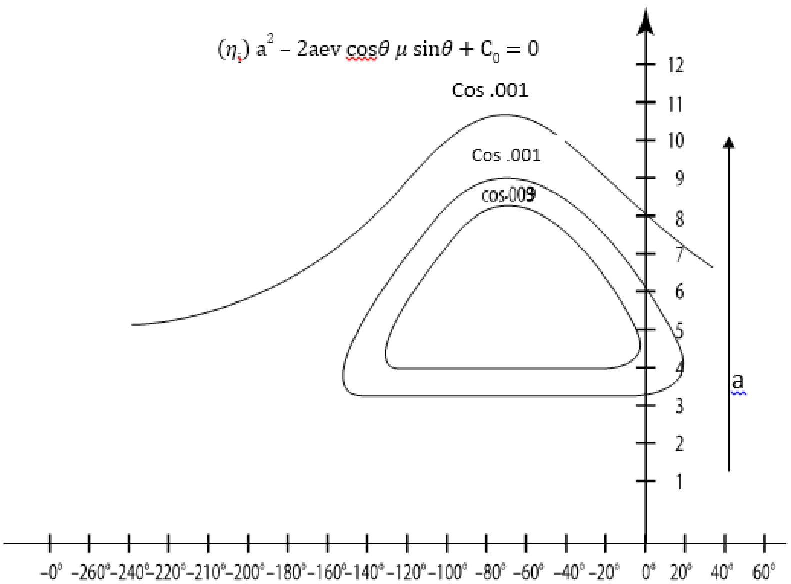
In order to examine the stability, the integral curve (14) have been blotted in phase plane (a, v) in the form

$$(n^2 - 1)\, a^2 - 2a\, e(\nu\cos\theta - \sin\theta) + Co = C \quad \ldots\ldots \quad (15)$$

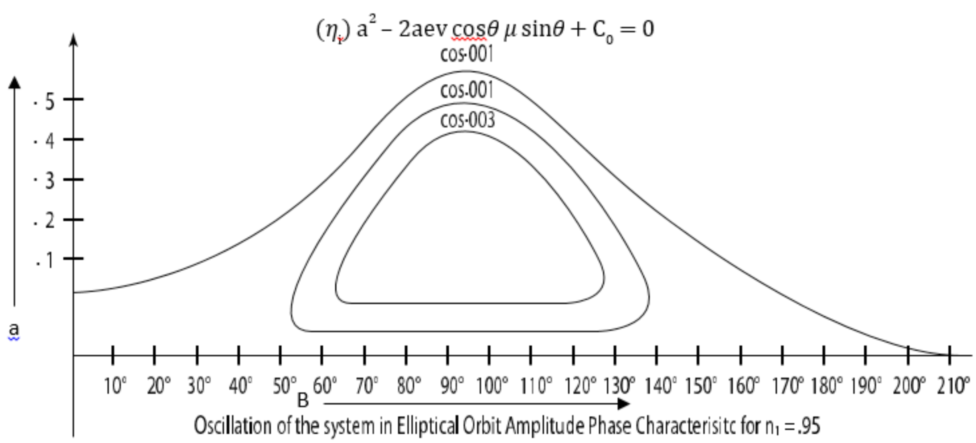
Where Co = 2 (n = 1) ϕ_0 = constant

Integral curves have been plotted in fig 1 and fig 2 for n=.95 and n=1.2 respectively for different values of a,e,ν,μ and Co. Since both curves are closed so we get the stability.



Oscillation of the system in Elliptical Orbit Amplitude Phase Characterisitc for $n_1 = 1.2$

Figure: 2



Oscillation of the system in Elliptical Orbit Amplitude Phase Characterisitc for $n_1 = .95$

Figure: 1

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