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Bayesian Estimation and MCMC-Based Analysis of the Inverse Exponentiated Exponential Poisson Distribution

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Abstract

This study takes a Bayesian approach to estimating and analyzing the parameters of the Inverse Exponentiated Exponential Poisson distribution (IEEP), using Markov Chain Monte Carlo (MCMC) sampling. To make sure the MCMC chains are mixing well, we used convergence diagnostics like the Gelman-Rubin diagnostic and trace plots. These checks confirm that our posterior estimates for both the rate and shape parameters are reliable and well-behaved, showing clear unimodal distributions and credible intervals that give us a probabilistic range of the estimates. We also performed residual analysis along with normality tests like Shapiro-Wilk and Anderson-Darling, which showed the residuals follow a normal distribution. So our model's assumptions hold up. Besides, posterior predictive checks and various models fit techniques demonstrated that the Bayesian model captures the underlying data distribution effectively. Overall, these results emphasize the robustness of Bayesian inference when modeling the IEEP distribution, supporting its usefulness and validity for both statistical analysis and real-world data applications.

Keywords: Bayesian inference, exponential poisson distribution, Gelman-Rubin diagnostic, posterior predictive checks, residual analysis.

Introduction

Statistical modeling plays a key role when it comes to understanding complex data and making trustworthy conclusions. One particular probability model that's been getting more attention lately is the Exponential-Exponentiated Power Inverse (EEPI) distribution. It's loved for its flexibility and reliability, especially when working with lifetime data, validity studies, or real-world situations. Traditional methods like Maximum Likelihood

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Estimation (MLE) can sometimes run into issues, especially if you're dealing with small sample sizes or don't have much prior information. That's where Bayesian inference comes in-it's a powerful alternative because it combines what we already believe (prior knowledge) with what the data shows (observed information) to give us better estimates. Now, Bayesian methods rely on something called the posterior distribution. Think of it as the result of merging your initial beliefs with new data, following Bayes' theorem. This approach helps us get more stable parameter estimates and is especially handy when dealing with complex models that are tough to solve analytically. Since deriving exact solutions for these posterior distributions can be tricky or sometimes impossible, we often turn to Markov Chain Monte Carlo (MCMC) techniques. Methods like Gibbs sampling and the Metropolis-Hastings algorithm allow us to efficiently approximate these distributions, leading to more accurate inferences.

Literature Review

There are various approaches to estimate parameters of distribution with a common approach as Bayesian inference. It is very useful if there is more uncertainty. One of the traditional, classical approach is the frequentist approach, but Bayesian methods incorporate what we already know called prior knowledge updating our estimates as new data comes in. Bayesian approach has been widely used across different fields like survival analysis, reliability testing as well as financial risk assessment (Gelman et al., 2013). Some of the important fields of Bayesian approaches are as follows:

Bayesian Methods for Statistical Inference

Bayesian methods are regularly changing and being used in various field of statistical models. The theoretical foundation of the Bayesian Method was laid out by Box and Tiao (1973) describing how Bayesian analysis can effectively be used for uncertainty about parameters. Then Gelman and colleagues (2013) elaborated its application and uses in hierarchical models developing computational methods such as Markov Chain Monte Carlo (MCMC). Gibbs sampling by Geman & Geman (1984) including the Metropolis-Hastings algorithm by Metropolis et al. (1953) has made supportive for working with complex probability distributions using Bayesian methods.

Bayesian Methods for Probability Distributions

Bayesian approach has been used by peoples in different types of probability models to estimate the parameters. Watanabe (2010) and Murphy (2012) explained the use of Bayesian methods for estimating parameters of probability distribution and claimed that it is more effective than other methods like maximum Likelihood methods (MLE) when some prior information about parameters are known and the sample size are small. Al-Awadhi et al. (2020) explained that Bayesian models provide a better option for estimating parameters of reliability and lifetime distributions which will be more accurate predictions compared other traditional and classical methods.

Bayesian Analysis for Model Validation and Residuals

Bayesian model can be verry effectively used for checking the accuracy of model fit and understanding the residuals. Vehtari et al. (2017) worked on posterior predictive checks as well as diagnostic measure such as Gelman-Rubin diagnostic (Gelman & Rubin, 1992) to make it possible for MCMC algorithms whether it converge properly or not. They also focused on Residual analysis and normality tests, such as the Shapiro-Wilk and Anderson-Darling tests.

Spiegelhalter et al. (2002) also focused saying that thorough residual diagnostics can be greatly improved making interpretations of models more reliable for decision-making. Bayesian methods have been successfully used on various probability distributions for estimating the parameters, one of area of its use is to estimate parameters for the Inverse Exponentiated Exponential Poisson (IEEP) distribution. This study mainly focusses to fill that gap created under classical providing explanation of Bayesian estimation of IEEP models. This study also focusses on MCMC methods. Mainly, this study aims to estimate the shape and rate parameters including check for reliability of the results obtained by ensuring the MCMC. Also, normality tests and residual analysis will be used to verify that model assumptions holds or not.

Objective of the Study

This study adds to the literature on Bayesian method. Study basically analyzes IEEP model using Bayesian method which has not been explored earlier. Study uses MCMC techniques to estimate the parameters more precisely than the This other traditional and classical methods. Study also demonstrates the reliability of the Bayesian estimates with convergence diagnostic. Furthermore, the study, provides a framework that can be extended to more complex probability model

Inverse Exponentiated Exponential Poisson (IEEP) Probability Distribution

The IEEP model used here is a flexible probability distribution that can be used for generalizing several existing lifetime and reliability distributions. Introduced by Telee & Kumar (2023), the IEEP distribution can be used in fields such as modeling, data analysis, engineering, medical sciences, and financial risk modeling. Present studies have concentrated on parameter estimation methods, with Bayesian techniques proving

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to be mainly effective in covering the distribution's tail behavior as well as ensuring robust predictions (Kumar & Singh, 2021). Here, we have tried to explore the Bayesian estimation of IEEP parameters based on MCMC techniques, focusing the need for further studies.

The Cumulative distribution function (CDF) and probability density function (PDF) of the IEEP are,

$$F(x;\alpha,\beta,\lambda)=1-\frac{1}{\left(1-e^{-\lambda}\right)}\left[1-\exp\left\{-\lambda\left(1-e^{-\beta/x}\right)^{\alpha}\right\}\right];x>0,(\alpha,\beta,\lambda)>0 \quad (1)$$

$$f(x)=\frac{\alpha\beta\lambda}{\left(1-e^{-\lambda}\right)}\left(\frac{e^{-\beta/x}}{x^2}\right)\left(1-e^{-\beta/x}\right)^{\alpha-1}\exp\left\{-\lambda\left(1-e^{-\beta/x}\right)^{\alpha}\right\}, x>0. \quad (2)$$

The reliability function and the hazard rate functions are given by eq (2) and eq (3)

$$R(x;\alpha,\beta,\lambda)=\frac{1}{\left(1-e^{-\lambda}\right)}\left[1-\exp\left\{-\lambda\left(1-e^{-\beta/x}\right)^{\alpha}\right\}\right] \quad ;\text{x}>0 \quad (.3)$$

$$h(x;\alpha,\beta,\lambda)=\frac{\alpha\beta\lambda e^{-\beta/x}\left(1-e^{-\beta/x}\right)^{\alpha-1}\exp\left\{-\lambda\left(1-e^{-\beta/x}\right)^{\alpha}\right\}}{x^2\left[1-\exp\left\{-\lambda\left(1-e^{-\beta/x}\right)^{\alpha}\right\}\right]} \quad ;\text{x}>0 \quad (4)$$

To demonstrate the flexibility of the model, we have plotted the pdf and hazard rate curves for various values of α false and β false taking λ = 2 are displayed in figure 1.

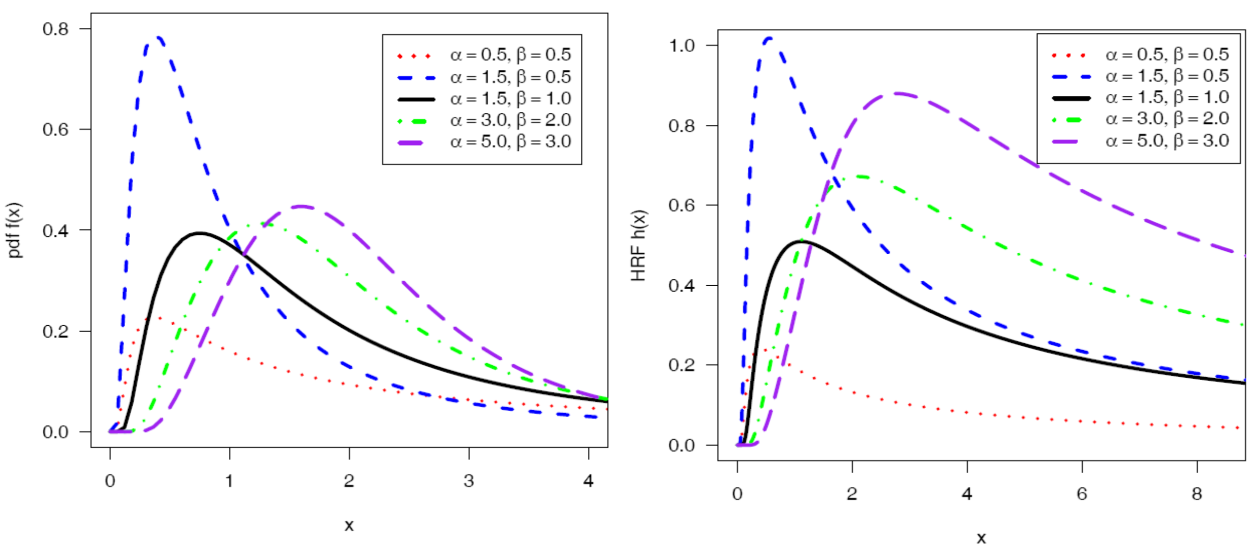

Figure 1: The PDFs (Left) and HRF(Right) at λ = 2

The pdf curves confirms that the pdfs are unimodal and the hazard rate curves shows that the hazard rate is inverted bathtub and increasing and decreasing in nature.

Data Set and Exploratory Data Analysis

In this section of the study, we have checked the practical applicability of the model taking a real data set. The data set is an accelerated life test that includes 59 conductors [Schafft et al. (1987); Nelson and Doganaksoy (1995)] where failure time is measured in hours with no any censoring of the observations.

4.700, 6.545, 9.289, 7.974, 8.799, 7.683, 7.224, 7.543, 6.956, 8.687, 2.997, 8.591,6.129, 11.038, 5.381, 6.958, 4.288, 6.522, 4.137, 7.459, 7.495, 6.573, 6.538,5.589, 5.807, 7.945,6.869, 6.352, 6.087, 6.725, 8.532, 9.663, 6.369, 7.024, 8.336, 9.218, 6.948, 9.254, 5.009, 7.489, 7.398, 6.033, 10.092, 7.496, 7.365, 6.923, 6.492, 5.459, 8.120, 4.706, 5.640, 5.434, 6.476, 6.071, 10.491, 5.923 , 7.937, 6.515, 4.531.

Summary statistics of the dataset provides overall nature of the data. Summary Statistics of the data is displayed in table 1. Also, summary statistics indicates that the data set is non-normal and positively skewed.

Table 1

Summary Statistics of the Dataset

Minimum	1st Qu	Median	Mean	3rd Qu	Maximum	Skewness	Kurtosis
2.997	6.052	6.923	6.980	7.941	11.038	0.193	3.087

To demonstrate the nature of the data, Boxplot is displayed in figure 2(left).

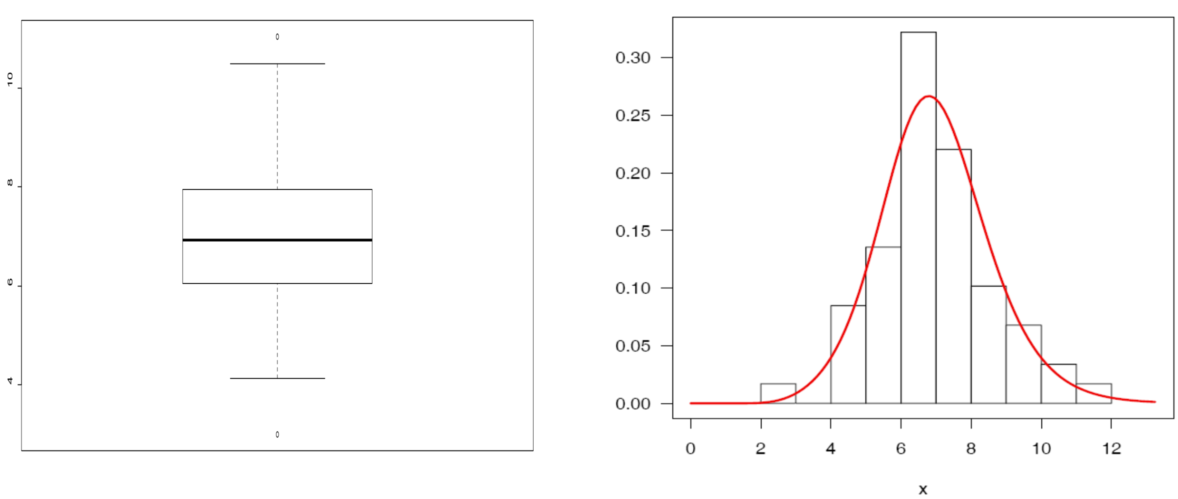

Figure 2: Boxplot (Left Panel) and Histogram and Density Plot (Right Panel)

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Maximum Likelihood Method of Estimation (MLE)

As for comparative study of Bayesian analysis, we have also estimated the parameter of the IEEP using MLE method. The MLEs, standard error and 95% confidence Interval (CI) for parameters obtained using R software (R core Team, 2024) as well as the quasi-Newton-Raphson algorithm in R [Rizzo, 2000] for MLE are given in Table 2.

Table 2

MLE, Standard Error and 95 Percent C.I.

Parameters	MLEs	Standard Error	95% C.l.
alpha	40.58683	4.860	(31.068, 50.113)
beta	22.75533	2.049	(18.739, 26.771)
lambda	2.996829	1.251	(0.5450, 5.4490)

Furthermore, the histogram and fitted pdfs are displayed in figure (1) right to shows the accuracy of the fitted mode graphically.

Bayesian Gamma Distribution Modeling of the IEEP Distribution

Model Overview and Prior Distribution

In this section of the study, we have modeled the data with a Gamma distribution using Bayesian analysis. Although under complex version of the model must include an extra parameter beta, but for simplicity, we have left beta just focusing on two parameters defining gamma distribution: shape and rate parameters. Here, alpha is the shape and lambda is the rate parameter. Gamma distribution is most commonly used under positive continuous variable. Taking 5000 samples from independent MCMC chains, we have analyzed posterior distributions for both the parameters. Following setting under Gamma priors are used for both the parameters as

Shape(α) ~ dgamma (1, 0.01) and Rate(λ) ~ dgamma (1, 0.01) where, model assumes that the dataa x[i] follows gamma distribution with parameters alpha and lambda. That is

$$x[i] \sim \text{Gamma}\ (\alpha, \lambda)$$

MCMC methods is used here for sampling the posterior distribution for α and λ. This is achieved through Just Gibbs Sampler (JAGS) software.

Bayesian Analysis

This section explains the Bayesian approach to estimate the parameter of IEEP distribution using MCMC sampling methods. Three separate MCMC chains are taken

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for 5000 samples and we looked at the posterior distribution for both parameter alpha and lambda. The iterations went from 2001 to 7000, with a thinning interval of 1.

Posterior Summary Statistics

Summary statistics of parameters under Bayesian estimation including mean, SD, 95% credible interval and quantities are given in table 3.

Table 3

Summary Statistics Parameters

Parameters	Mean	2.5%	Q1(25%)	Q2(50%)	Q3(75%)	97.5%	SD	95%C.I.
α	18.76	12.97	16.54	18.61	2097	24.98	3.14	(12.97,24.98)
λ	2.69	1.85	2.37	2.67	3.01	3.59	0.46	(1.85,3.59)

Trace Plots

Trace plots for λ and α are shown in figure 3. It is seen that trace plot looks well mixed indicating that better settlement of the Markov chains. Since the fluctuations is mostly random and shows that there is no any strong pattern so there is no sign of correlation sticking around. This suggests that MCMC algorithm made a better job exploring the posterior distribution for IEEP model

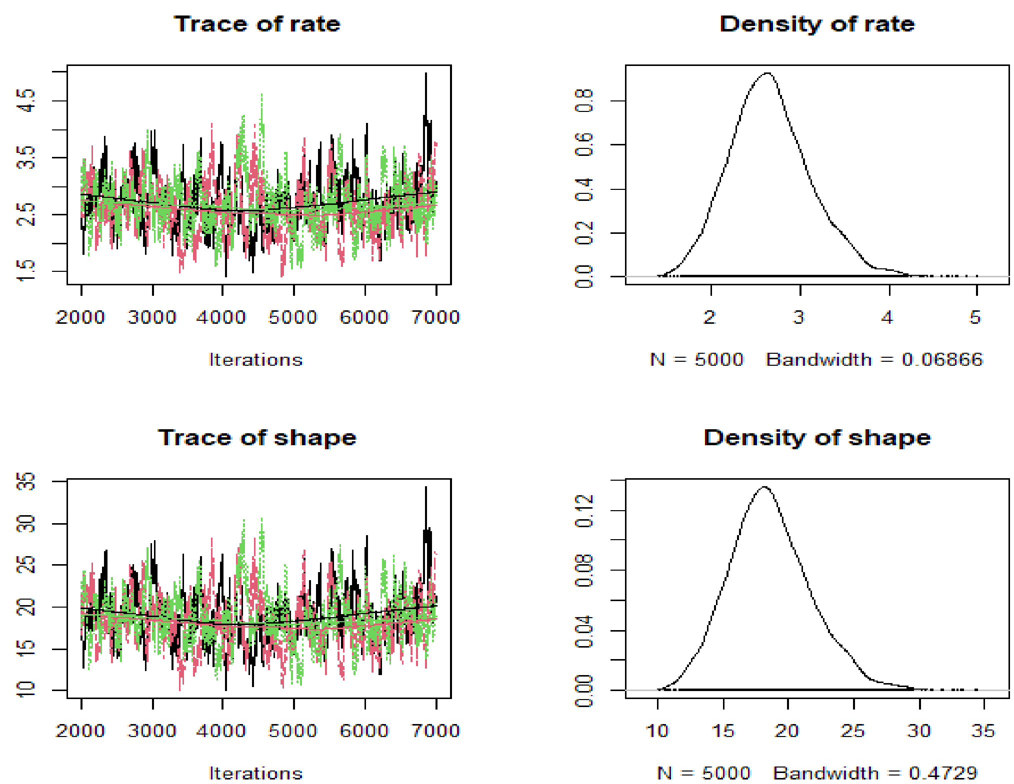

Figure 3: Trace and posterior density plots.

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Posterior Density Estimates

Right hand side of the figure 3 displays the density estimates for parameters. This shows how sure we are about estimate for level of certainty. Since both of these distributions have single peak and smooth stating that the Bayesian inference give well defined and clear parameter estimates. That is, it is seen that Bayesian inference confirms that the IEEP model parameters are well-identified as well as the results explore good insights into the data distribution. Also, credible intervals show a probabilistic range within which the true parameter values lie which also reflect the inherent uncertainty in estimation.

Auto-correlation

High initial and decreasing gradually over increasing lags, indicates some level of chain mixing but still with persistent correlations (figure 4).

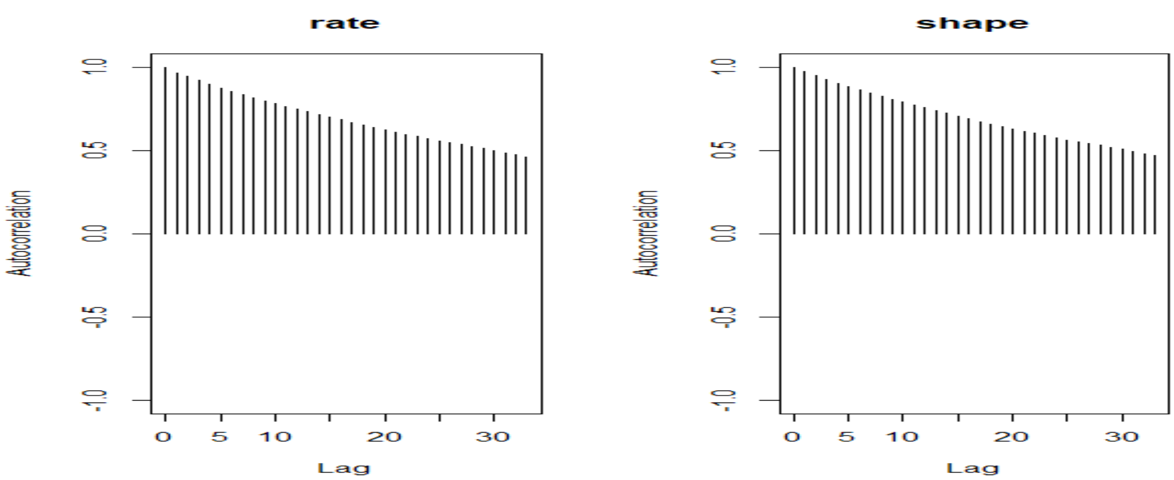

Figure 4: Auto-correlations Plots

Gelman-Rubin Diagnostic for Convergence

The Potential Scale Reduction Factor (PSRF) also named as Gelman-Rubin diagnostic, assesses the convergence of MCMC chains for lambda and alpha are calculated. It is seen that PSRF for lambda is 1.01 with Upper C.I. = 1.01. Similarly, for alpha, PSRF = 1.01 and Upper C.I. is1.02 Also, multivariate PSRF is 1.00. Since, both PSRF are close to 1 which indicates that MC have mixed well and are sampling from larger posterior distribution. Since upper C.I is very near to 1 which indicates no chain is significantly different from others. It is also seen that joint distribution has reached equilibrium across chains supporting convergence. All these evidence and results shows that analysis convergence and the posterior estimates will be reliable for estimates.

Posterior Predictive Distribution and Model Fit Assessment

Histogram in figure 5(left) shows a right skewed distribution which is expected for Gamma-distributed variable. The modal value is around 7 values ranging from 4 to 7 approximately. Also, variability in the posterior predictive distribution indicates that the model captures some uncertainty and the spread shows that the predicted values capture a reasonable range aligning with the observed data.

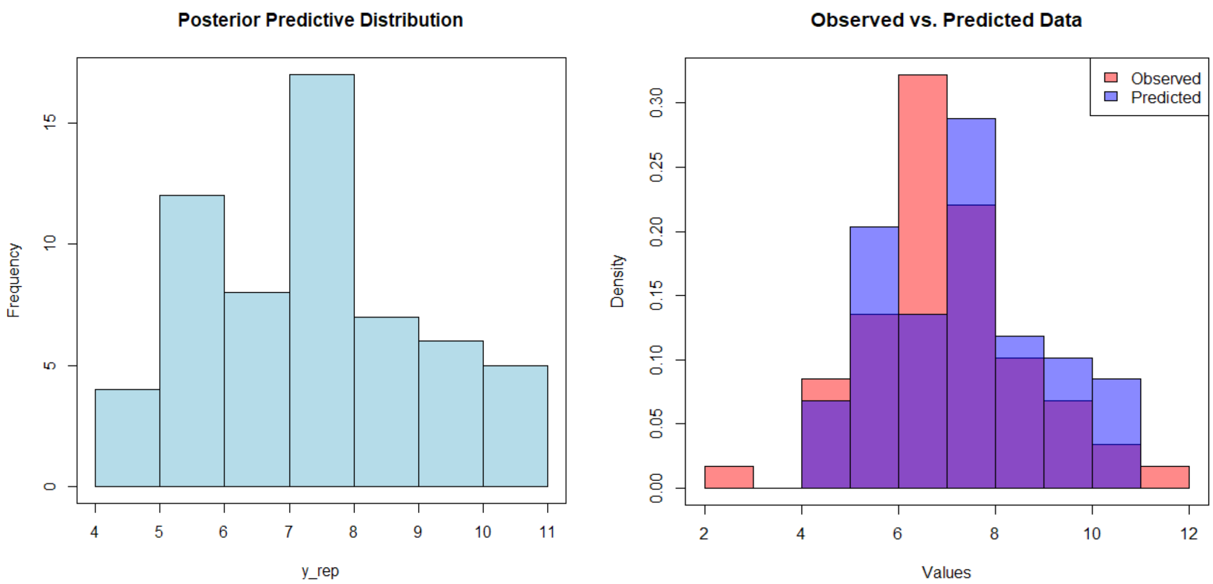

Figure 5: Posterior Predictive Distribution and Model Fit Assessment

Significant part colored with red of the data overlaps with predicted data indicated by blue color forming purple reasons which indicates that general shape of data is very well captured by the model. Also peak around 6-8 indicates model predicts central values approximately. Approximate lower value of 3-5, the observed data has a slightly higher density than predicted values. Furthermore, at higher value of 9-11, the predicted distribution extends observed value which indicates that model is overestimating larger values. The posterior P values is 0.559 which is around 0.5 indicated a better fit indicating that model generated data is same to the observed data.

Residuals Analysis

Figure 6 is residual plots for checking goodness of fit. Histogram is mostly symmetric around zero. This shows that model does not exhibits significant bias for its prediction. Also range of residual approximating from -8 to 6 clustering around zero indicates that model's prediction is generally close to the observed values. It is seen that assumption of normality reasonably met as the shape of the residual distribution is almost normal in

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shape. Also, it is seen, there is no clear sign of skewness or extreme outliers supporting the adequacy of the model. The Q-Q plots also approximately verify the normality

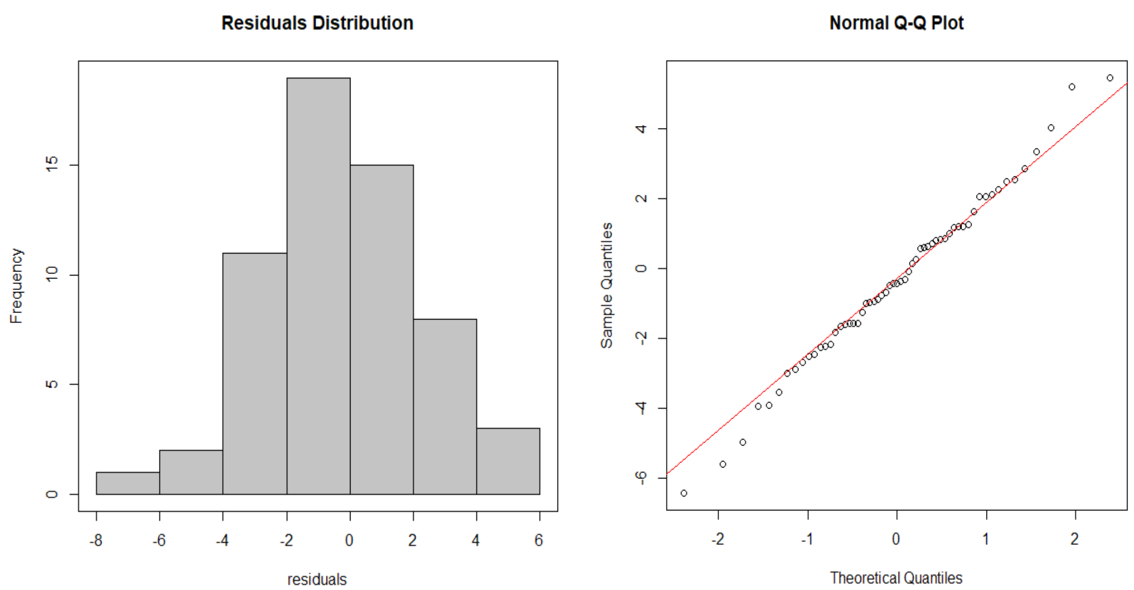

Figure 6: Residual Plots (Left) and QQ Plots (Right)

Normality Test of Residuals

Shapiro-Wilk Test (W) of 0.99274 with p-value as 0.9789 indicates that we fail to reject the null hypothesis suggesting that the residuals follow a normal distribution. Furthermore, Anderson-Darling Test (A) with value 0.1508 and p value 0.9594 confirming that the residuals do not significantly deviate from normality.

Findings Implications of the Study

Findings

The study's key findings include of this study include

1. A successful Bayesian estimation of EEPI parameters for posterior means for λ and α which were 2.69 and 18.76, as well the 95% credible intervals for the parameters which concluded strong evidence of estimation precision.
2. MCMC chains exhibited good convergence and Autocorrelation analysis showed some persistence.
3. Study revealed a good model fit using posterior predictive checks.
4. Residual analysis validated model assumptions of normality.

Implications of the Study

This study uses advancing Bayesian Methods for IEEP Distribution that had limited attention and explains MCMC for getting reliable and valid posterior compared with respect to traditional popular frequentist method. This study also highlights and emphasizes the posterior predictive checks as well as the interactions of credible, residuals and convergence diagnostics. Also, the Normality checks of residuals using the Shapiro-Wilk and Anderson-Darling tests strengthens the assumption that the Bayesian model is well-calibrated. Furthermore, this study elaborates improved parameter estimation in Reliability and Lifetime Analysis. Also, this will help to get idea about application of Modelling in Financial Risk Modeling as well as enhancing machine learning and predictive analysis. Findings of the study also focus on how Bayesian method is useful in Decision making Under Uncertainty.

Limitations of the Study

As study focuses on Bayesian analysis considering only two parameters lambda and alpha. That a key limitation of the study is to applying Bayesian inference on only two parameters alpha and lambda although the distribution IEEP has three parameters so, future research can extend this model by incorporating β into the Bayesian framework and examining its impact on parameter estimates and model fit. This study has significant implication in both practical and theoretical fields of Bayesian inferences, statistical modeling and decision making. Further refinement, applications on other models and study for larger sample size is necessary for to get more precise results.

Future Research Directions

This study focuses mainly on application of Bayesian analysis on IEEP probability model. It is clear that future study should apply the Bayesian IEEP model to real-world data set in different fields, such as manicuring, health, finance as well as research. Also, comparative analysis of IEEP with other published probability models are necessary. Further study may be focused on study of sensitivity analysis on prior distributions.

Conclusion

This study mainly focusses on Bayesian inference to estimate the parameters lambda and alpha of the Inverse exponentiated exponential (IEEP) distribution. Study demonstrates the effectiveness of MCMC methods in handling complex probability distributions. The study also analyzed and concluded that the Bayesian estimates were reliable & valid, as evidenced by using convergence diagnostics, posterior predictive checks, and residual analysis. Furthermore, the findings reinforce the applicability of Bayesian methods

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and its applications in statistical modeling, data analysis as well as in decision making, particularly when handling uncertainty in parameter estimation.

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